Computational Methods
for
Acoustics Problems

Saxe-Coburg Publications on Computational Engineering

Mesh Partitioning Techniques and Domain Decomposition Methods
Edited by: F. Magoulès

Programming Distributed Finite Element Analysis:
An Object Oriented Approach
R.I. Mackie

Object Oriented Methods and Finite Element Analysis
R.I. Mackie

Finite Element Mesh Generation
B.H.V. Topping, J. Muylle, P. Iványi, R. Putanowicz and B. Cheng

Parallel and Distributed Processing for Computational Mechanics:
Systems and Tools
Edited by: B.H.V. Topping

High Performance Computing for Computational Mechanics
Edited by: B.H.V. Topping and L. Lämmer

Computational Mechanics using High Performance Computing
Edited by: B.H.V. Topping

Innovative Computational Methods for Structural Mechanics
Edited by: M. Papadrakakis and B.H.V. Topping

Parallel Finite Element Computations
B.H.V. Topping and A.I. Khan

Computational Methods for Acoustics Problems

Edited by
F. Magoulès

© Saxe-Coburg Publications, Kippen, Stirlingshire, Scotland

published 2008 by
Saxe-Coburg Publications
Civil-Comp Ltd, Dun Eaglais, Station Brae
Kippen, Stirlingshire, FK8 3DY, Scotland

Saxe-Coburg Publications is an imprint of Civil-Comp Ltd

ISBN: 978-1-874672-30-2

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Cover: Modes of vibration for a seat belt safety lock from Figure 11.9 page 306. For more information see Chapter 11 by O. von Estorff, M. Markiewicz and O. Zaleski.

Copyright Permisssions: For copyright permisssions and other acknowledgements see page 337.

Disclaimer: The computer codes and information provided in this book are intended for research and educational purposes only. The codes are provided on an "as is" basis and no warranty is given for any professional or commercial purpose whatsoever. The authors, editor and publisher have neither liability nor responsibility to any person or entity with respect to any loss or damage arising from the information contained in this book.

Printed in Great Britain by Bell & Bain Ltd, Glasgow

Contents

Preface **ix**

1 Boundary Integral Equations Methods in Acoustic Scattering **1**
A. Bendali and M. Fares
1.1 Typical Scattering Problems in Acoustics and Brief Review of some approaches to their Numerical Solution 2
1.1.1 Some Model Scattering Problems in Acoustics 2
1.1.1.1 Simple Scattering Problems 2
1.1.1.2 Variable Coefficient Scattering Problem 3
1.1.2 Brief Review of some Numerical Methods for Solving Acoustic Scattering Problems 4
1.2 Boundary Integral Equations 5
1.2.1 The Basic Framework 5
1.2.1.1 The Integral Representation 5
1.2.1.2 Properties of the Potentials 6
1.2.1.3 General Principles Governing the Construction of a BIE 7
1.2.2 Direct formulations 7
1.2.2.1 Definition and general features 7
1.2.2.2 BIEs constructed from an Interior Dirichlet Condition 9
1.2.2.3 Combined Integral Equations 11
1.2.3 Indirect Formulations 12
1.2.3.1 Brakhage-Werner Formulations 12
1.2.3.2 Other Recent Formulations 13
1.3 Boundary Element Methods 15
1.3.1 General Presentation 15
1.3.2 Some Numerical Results 17
1.4 Finite Element-Boundary Element Methods 17
1.4.1 Standard Solution of the FE-BE system 17
1.4.1.1 The FE-BE Linear System 17
1.4.1.2 A Schur Complement Procedure 20
1.4.2 Domain Decomposition Methods 21
1.4.2.1 Uncoupling FE and BE Solutions 21

1.4.2.2 Treatment of a Generalized Impedance Boundary Condition . . . 23
1.4.2.3 Coupling a DDM with an Adaptive Radiation Condition . . . 27
1.5 Conclusions . . . 29

2 Infinite Elements **37**
J. Astley
2.1 Introduction . . . 37
2.2 The Unbounded Helmholtz Problem . . . 39
2.2.1 The Governing Equations . . . 39
2.2.2 The Sommerfeld Radiation Condition . . . 40
2.2.3 Separable Solutions in Spherical Coordinates . . . 40
2.2.4 The Atkinson-Wilcox Expansion . . . 41
2.2.5 Spheroidal Coordinates. . . . 42
2.3 Discretising the Exterior Domain . . . 43
2.3.1 The Variational Formulation . . . 43
2.3.2 The Discrete Model . . . 44
2.3.2.1 The Subdivision into Elements . . . 44
2.3.2.2 The Trial Solution . . . 45
2.3.2.3 The Discrete Equations . . . 45
2.3.2.4 The Finite Element Matrices . . . 46
2.3.2.5 The Infinite Element Matrices . . . 46
2.3.2.6 Mapped and Separable Elements . . . 46
2.3.2.7 Conjugated and Unconjugated Formulations . . . 47
2.4 A Simple Separable Infinite Element . . . 47
2.4.1 Element Geometry and Shape Functions . . . 47
2.4.2 The Unconjugated Element . . . 49
2.4.3 The Conjugated Element . . . 50
2.5 Mapped Infinite Elements . . . 50
2.5.1 The Element Mapping . . . 51
2.5.2 The Trial Solution . . . 52
2.5.3 The Unconjugated 'Bettess' Formulation . . . 53
2.5.4 The Conjugated 'Astley-Leis' Formulation . . . 53
2.6 Performance . . . 54
2.6.1 Conditioning . . . 57
2.6.2 Accuracy and stability . . . 59
2.7 Infinite elements in the time domain . . . 62
2.8 Conclusions . . . 65

3 Perfectly Matched Discrete Layers for Unbounded Domain Modeling **69**
M.N. Guddati, K.W. Lim and M.A. Zahid
3.1 Introduction . . . 69
3.2 Perfectly Matched Discrete Layers: Fundamentals . . . 72

3.2.1 Edge PMDL . . . 72
3.2.2 Corner PMDL . . . 76
3.2.3 Relationship to other absorbing boundary conditions . . . 77
3.2.4 Extension to Evanescent Waves and Dispersive Media . . . 78
3.2.5 Extension to Elastic Wave Propagation . . . 78
3.3 Formulation and Implementation of Elastic PMDL . . . 80
3.3.1 Formulation . . . 80
3.3.2 Finite Element Discretization . . . 85
3.3.3 Time Stepping . . . 87
3.4 Numerical Examples . . . 87
3.4.1 Scalar Waves in Square Computational Domain . . . 88
3.4.2 Polygonal Domain with Oblique Corners . . . 91
3.4.3 Elastic Half-Space . . . 91
3.4.4 Elastic Layer . . . 93
3.5 Concluding Remarks . . . 93

4 Stabilized Time-Discontinuous Galerkin Methods with Applications to Acoustics 99
L.L. Thompson, P. Kunthong and S. Subbarayalu
4.1 Introduction . . . 100
4.2 Time-Discontinuous Galerkin Method . . . 104
4.2.1 Multi-Pass TDG Algorithm . . . 106
4.3 Generalized Time-Discontinuous Galerkin Method . . . 108
4.3.1 Singly Diagonal TDG/GLS Multi-Pass Iterative Algorithm . 110
4.3.2 Numerical Example . . . 113
4.4 Spectral Decomposition . . . 116
4.5 Local Space-Time Adaptive Discontinuous Galerkin Methods . . . 118
4.6 Conclusions . . . 122

5 Boundary Conditions and Iterative Schemes for the Helmholtz Equation in Unbounded Regions 127
E. Turkel
5.1 Introduction . . . 127
5.2 Discretisation . . . 129
5.2.1 Accuracy . . . 129
5.3 Absorbing Boundary Condition (ABC) . . . 130
5.4 Iterative Solver . . . 141
5.4.1 Preconditioning . . . 143
5.5 Conclusion . . . 150

6 Domain Decomposition Methods for Fluid Structure Coupling in Vibro-acoustic Problems 159
F. Magoulès and F.-X. Roux
6.1 Introduction . . . 159
6.2 The Vibro-acoustic problem . . . 161

6.2.1 Formulation of fluid-structure coupling in vibro-acoustics . 161
6.2.2 Preconditioning strategies for the global problem 162
6.2.3 Optimal local preconditioner 164
6.2.4 Solution strategy via code coupling 166
6.3 Interface management for the coupling operators 167
6.3.1 Discretization of coupling operators 167
6.3.2 Computation of interface projections 168
6.3.3 Interface management 169
6.3.4 Computation of the matrix-vector product for the coupling matrices 169
6.3.5 Description of the fluid-structure interface 170
6.4 Numerical experiments 171
6.4.1 Fluid cavity with a thin wall 171
6.4.2 Car compartment 171
6.5 Conclusion 177

7 Theory and Numerical Methods for Eigenvalue Problems 181
K. Meerbergen
7.1 Motivation and Definitions 181
7.1.1 The Vibration Problem 182
7.1.2 The Helmholtz Equation 183
7.1.3 Motivation 185
7.1.4 Linear algebra basics 185
7.1.5 Theory for Spectral Approximation 187
7.2 Classification 188
7.2.1 The Symmetric Definite Eigenvalue Problem 188
7.2.2 The Quadratic Eigenvalue Problem 189
7.2.2.1 Proportional Damping 190
7.2.2.2 Palindromic Eigenvalue Problems 191
7.2.2.3 Hamiltonian Eigenvalue Symmetry 191
7.3 Krylov Methods for the Definite Generalised Problem 192
7.3.1 The Spectral Transformation 192
7.3.2 The Sylvester Theorem and Matrix Inertia 194
7.3.3 The Spectral Transformation Lanczos Method 195
7.3.4 Orthogonality Property and Maintaining Orthogonality . . . 196
7.3.5 Eigenvalue Computation 197
7.3.6 Shifting Strategy and Locking 198
7.4 Krylov Methods for the Quadratic Eigenvalue Problem 200
7.4.1 Linearisation 200
7.4.2 Partial Schur Form 200
7.4.3 Exploitation of Structure 201
7.4.4 Reliability of the Solution of the QEP 201
7.5 Software 201
7.6 Bibliographical Note 202

8 Advances in the On-Surface Radiation Condition Method: Theory, Numerics and Applications **207**
X. Antoine
8.1 Introduction . . . 207
8.2 Solving Scattering Problems by Integral Equations . . . 208
8.2.1 Scattering Problems . . . 208
8.2.2 Integral Equation Representations . . . 209
8.3 The OSRC Method: Theoretical Issues . . . 211
8.3.1 Principles of the OSRC Method . . . 211
8.3.2 The Formal Construction of Kriegsmann *et al.* . . . 212
8.3.3 Going Further . . . 214
8.3.4 Solutions for more robust Frequency Range OSRCs . . . 216
8.3.5 OSRC Formulations and Other Developments . . . 218
8.4 The OSRC Method: Numerical Issues . . . 219
8.4.1 Variational OSRC Formulation . . . 219
8.4.2 Finite Element Approximation . . . 220
8.4.3 Solution of Linear Systems . . . 220
8.4.4 Numerical Validation . . . 221
8.4.5 The Case of Concave Obstacles . . . 224
8.5 Some Applications in Computational Acoustics . . . 224
8.5.1 Artificial Boundary Condition Techniques . . . 224
8.5.2 Generalized Impedance Boundary Conditions for Penetrable Bodies . . . 225
8.5.3 Generalised Burton-Miller Integral Equation Formulations . . . 225
8.5.4 Preconditioning Techniques for Integral Equations by Open Structures . . . 226
8.6 Conclusion . . . 226

9 Fast Frequency Response Function Computation by Model Reduction Methods **233**
K. Meerbergen
9.1 Introduction . . . 233
9.2 Theory for Linear System Solvers . . . 234
9.3 Modal Truncation . . . 235
9.3.1 Modal Damping . . . 237
9.3.2 Rayleigh Damping . . . 238
9.3.3 General Situation . . . 238
9.4 Shifted Linear Systems . . . 238
9.4.1 Arnoldi's method . . . 239
9.4.2 The Shifted Equation Solved by Arnoldi's method . . . 240
9.4.3 Characterisation of the Convergence . . . 241
9.4.4 Vector-Padé Approximation . . . 242
9.4.5 Undamped Problems . . . 245
9.4.6 Comparison with Modal Truncation . . . 245

9.4.7 Rayleigh Damping 247
9.4.8 Quadratic parameter problem 248
9.4.9 Higher Order Polynomials 253
9.5 Model Reduction 257
9.6 Implementation and Software 259
9.7 Bibliography 259
9.8 Conclusions 260

10 Inverse Acoustic Problems **263**
R. Djellouli
10.1 Introduction 263
10.2 Preliminaries 266
10.2.1 Nomenclature and Assumptions 266
10.2.2 The Direct Acoustic Scattering Problem 268
10.3 The Inverse Obstacle Problem (IOP) 268
10.3.1 Problem Statement 269
10.3.2 Mathematical Results 269
10.3.2.1 Uniqueness 269
10.3.2.2 Regularity 270
10.4 The Regularized Newton Method 272
10.4.1 Parametrization 272
10.4.2 The Regularized Newton Equation 273
10.4.3 Evaluation of the Fréchet derivatives 274
10.4.4 Multistage Solution Procedure 275
10.4.5 Reduced Data - Intensity 276
10.5 Fast Solution of Direct Acoustic Scattering Problems 276
10.6 Numerical Results 278
10.6.1 Problem I: Parameter Identification Problem 279
10.6.2 Problem II: Blind Test Problems 282
10.6.3 Problem III: Incomplete Parametrization Problems 284
10.7 Summary and Conclusion 286

11 Validation of Numerical Methods in Acoustics: What can we expect? **295**
O. von Estorff, M. Markiewicz and O. Zaleski
11.1 Introduction 295
11.2 Theoretical Background 296
11.2.1 Helmholtz Equation 296
11.2.2 Acoustic Finite Element Method 296
11.2.3 Acoustic Boundary Element Method 297
11.2.4 Coupling 298
11.3 Comparing Measurements and Computations 299
11.3.1 Uncoupled Sound Radiation Problems 299
11.3.1.1 “Historical Examples” 299
11.3.1.2 Safety Lock of a Seat Belt 303

11.3.1.3 Crank Case of an Engine 308
11.3.2 Coupled Sound Radiation Problems 311
11.3.2.1 “Historical Example” 312
11.3.2.2 Crankshaft Pulleys 313
11.3.2.3 Washing Machine 317
11.3.3 Sound Transfer Problems 321
11.3.3.1 Honeycomb Plate 322
11.3.3.2 Aircraft Side Wall Panel 325
11.4 Conclusion 330

Author Index **333**

Keyword Index **334**

Copyright Permissions **337**

Preface

The computation of acoustic phenomena is regarded today as one of the most challenging in scientific computation. This field has been an area of active research for decades. A major current difficulty lies in the effective treatment of unbounded domains by standard domain-based methods. Finite element methods cannot directly handle such configurations in an effective way. An artificial boundary that truncates the unbounded domain is used to form a bounded computational domain. Special techniques are then required to reduce the spurious reflection of waves that impinge on this artificial boundary, including infinite elements, boundary element, etc. Additional techniques enable an increase in the accuracy of the numerical schemes and the computational efficiency as well as the robustness of the methods. In this manuscript, computational methods for acoustics problems are presented in eleven chapters. Each chapter, written by different authors, presents a state of the art of well established or innovative methods, techniques or algorithms. A large bibliography is included at the end of each chapter.

The chapter authored by A. Bendali and M. Fares presents some new techniques, specially adapted to high performance computing, for solving acoustic scattering problems involving a bounded zone filled by a heterogeneous medium. The discretization in this zone is carried out by using a standard finite element method. The rest of the computational domain is dealt with by using a boundary integral equation. The final linear system resulting from the coupling of two solution procedures is partly sparse and partly dense. Serious difficulties arise when the solution has to be tackled on a parallel platform. The authors start by presenting a condensed but systematic way to obtain the most used boundary integral equations for acoustic scattering problems. Then, they show how the non overlapping domain decomposition methods, usually used for the Helmholtz equation, can be used to overcome the difficulties induced by the coupling. They finally present an efficient procedure, which requires the solution of only a small linear system of finite element equations at each iteration, to solve the scattering problem. It is mainly based on the utilization of a non overlapping domain decomposition method, but this is in the context of a nodal finite element method with appropriate treatment of the nodes being shared by more than two sub-domains. Numerical experiments illustrate the methods presented.

The chapter authored by J. Astley presents the infinite element concept for acoustic problems. Infinite elements were first proposed in the 1970s and 80s for the propagation of surface waves on water, but were not developed specifically for acoustics

until the 1990s. They are now widely used and are particularly attractive for inclusion in commercial finite element codes since they largely preserve the structure of conventional finite element models. They are able to act as a high order non-reflecting boundary condition at the outer boundary of a truncated finite element model for an exterior problem, while also predicting directly the far field solution. The method is presented for mapped and for separable elements and for symmetric (unconjugated) and non-symmetric (conjugated) formulations. Spherical and spheroidal formulations are discussed. The accuracy and conditioning of the various formulations is investigated in some detail and the extension to transient problems is outlined.

The chapter of M.N. Guddati, K.-W. Lim and A. Zahid presents Perfectly Matched Discrete Layers (PMDL), a new method that combines the advantages of various existing methods for modeling unbounded domains. PMDL links the seemingly disparate techniques of PML and local absorbing boundary conditions (ABCs). PMDL is a special discretization of PML, which preserves the property of perfect impedance matching even after discretization. Thus, PMDL is as flexible as PML and is in fact an improvement. PMDL is also shown to be equivalent to rational approximation of the exact impedance, thus inheriting the desirable accuracy properties of local ABCs. Furthermore, unlike the existing methods, PMDL is applicable not only to rectangular computational domains, but also to polygonal domains. The chapter contains the basic derivation of PMDL as well as the illustration of its superior performance for acoustic as well as elastic wave propagation problems.

The chapter authored by L.L. Thompson, P. Kunthong and S. Subbarayalu discusses original Time-Discontinuous Galerkin (TDG) methods which provide high-order accuracy and stability for second-order hyperbolic systems including those governing structural dynamics and acoustics. Generalized gradients of residuals of the governing equations are added to the standard TDG variational equation to provide high-order accuracy and stability properties of the parent TDG method while gaining significant reductions in computational cost comparable with standard second-order accurate single-step/single-solve (SS/SS) time-stepping algorithms. Using this stabilized framework, together with optimal design of temporal approximations and time-scales, efficient multi-pass iterative solution algorithms are developed which: maintain C- and L-stability; provide high-order accuracy in only two or three iterative passes; and can easily be implemented in standard finite element codes. Alternative decoupling strategies are also developed using spectral decomposition of the time arrays to provide fast solves of space-time matrix equations resulting from the TDG method which are ideal for parallel implementation. In addition, new space-time finite element strategies based on the TDG method are presented including high-order accurate non-reflecting boundary conditions. The time-discontinuous Galerkin (TDG) variational method is used to divide the time-interval into space-time slabs, the solution advances from one slab to the next. Within each global space-time slab subdivisions of multiple local space-time elements are allowed. This gives the flexibility required to change local time-step size for different elements in the spatial mesh in a truly local self-adaptive space-time methodology. By maintaining orthogonality of

the space-time mesh and pre-integrating analytically in the time-dimension through each local element in the time-slab, an efficient yet robust adaptive method is obtained which accommodates any standard spatial element without modifications. The resulting technologies provide significant advances in accuracy, efficiency, and reliability over standard time-stepping methods, especially for long-time simulations which track response over large distances and long time intervals.

The chapter of E. Turkel surveys absorbing boundary conditions and iterative methods for the numerical solution of the Helmholtz equation in unbounded regions. For acoustic scattering around a body it is necessary to enclose the infinite region by an artificial surface. One needs to set a boundary condition that absorbs outgoing rays. The chapter discusses several approaches for such boundary conditions for various shapes of the artificial surface. After discretizing the total system one needs to solve a large but sparse system that is neither positive nor Hermitian. For three dimensional problems or high frequencies one must use an iterative method, usually a Krylov space method. To converge within a reasonable number of steps it is necessary to precondition the system. A survey of different preconditioners is also presented in this chapter.

The chapter authored by F. Magoulès and F.-X. Roux presents a general methodology to solve coupled fluid-structure problems with non-matching grids arising from vibro-acoustics problems. The coupling is ensured through the boundary conditions defined along the fluid-structure interface. Here the coupled quantities are integrated over a set of quadrature points defined on the fluid-structure interface. This integration involves the computation of nodal values and interpolation at given Gauss points which allows any hp-refinement. Several preconditioned sub-structuring methods and domain decomposition preconditioning techniques are proposed. Implementation aspects of these methods are provided in details and validated on two vibro-acoustic problems. Numerical results illustrate the efficiency, robustness and performance of the proposed preconditioning techniques applied to the global coupled problem.

The chapter by K. Meerbergen on theory and numerical methods for eigenvalue problems reviews the eigenvalue problems that arise in the analysis of vibrations. The best known problem is the definite generalized eigenvalue problem of the finite element discretization of the Helmholtz equation, which produces the eigenfrequencies and eigenmodes. For this problem the Lanczos method is a success story and this chapter discusses some aspects of the method without dwelling on implementation details. When damping is present, the Helmholtz equation often becomes a quadratic eigenvalue problem. The solution of this problem is still under investigation but good progress has been made over the last decade. The author shows some particular features of this problem and some differences with the definite generalized eigenvalue problem.

The chapter of X. Antoine proposes a complete overview of the theoretical and numerical developments related to the On-Surface Radiation Condition (OSRC) method in computational acoustic scattering since its introduction in the middle of the 1980s. This asymptotic technique tends to produce some approximate and fast numerical

computations of scattered fields and far-field patterns for large wavenumbers and various boundary conditions. The proposed discussion follows the chronological developments and explains the strengths and future applications of the OSRC for prospecting large scale problems. Numerical treatments are discussed in details for three-dimensional problems using surface finite element methods. Finally, several applications are pointed out such as the construction of artificial boundary conditions and generalized impedance boundary conditions, the development of well-posed and well-conditioned new integral equations for scattering problems as well as preconditioning techniques for integral equations by open structures.

Frequency response computations over a frequency range are relatively expensive because a large number of linear systems has to be solved. The chapter of K. Meerbergen on fast frequency response function computation by model reduction presents the application of Krylov methods for this purpose. They build rational approximations to the frequency response as a function of the frequency. In general, modal superposition often is the fastest method, but the method is not always applicable, for example the case of damped problems or in the presence of infinite elements. These Krylov methods are relatively new to the community concerned with computational acoustics, but they are very reliable and show excellent speed-ups to the direct approach.

The chapter authored by R. Djellouli discusses an original computational methodology for solving inverse acoustic scattering problems. The problem considered here is an inverse obstacle problem where the objective is to determine the shape of an obstacle, or a part of this shape, from the knowledge of some scattered far-field patterns, and assuming certain characteristics of the surface of the obstacle. Although this problem is one of the simplest problem arising in the inverse scattering field, the issues and ideas discussed here are relevant to many applications such as sonar, radar, geophysical exploration, medical imaging and nondestructive testing. The proposed solution methodology is a regularized iterative-based method, that distinguishes itself from similar Newton-type procedures by a sensitivity-based and frequency-aware multi-stage solution strategy, a computationally efficient usage of the exact sensitivities of the far-field pattern to the specified shape parameters, and a numerically scalable domain decomposition method for the fast solution in a frequency band of three-dimensional direct acoustic scattering problems.

The chapter authored by O. von Estorff, M. Markiewicz, and O. Zaleski discusses a number of representative examples where results obtained using the finite element and boundary element computations are compared to measured values. The considered systems are of different complexity levels and include academic as well as rather sophisticated industrial examples. The contribution covers coupled and uncoupled radiation problems, followed by two transmission loss investigations. The major objective is the validation of the models and to show their limitations and accuracy. Moreover, the authors try to share parts of their wide experience with measurements and computation in the fields of acoustics and vibro-acoustics. The chapter is concluded with a number of essential hints and remarks to provide some help for possible

validation activities planned by the reader.

Naturally, the present book cannot provide a complete record of the many approaches, applications, features, and schemes related to computational methods for acoustics problems. However, it does provide a good tutorial on the most important methods used for academic and industrial research. This book will be of interest to engineers, computer scientists and applied mathematicians. The editor wishes to thank the authors for their willingness to contribute to this edited book dedicated to computational methods for acoustics problems.

Frédéric Magoulès
Applied Mathematics and Systems Laboratory
Ecole Centrale Paris, Grande Voie des Vignes
92295 Châtenay-Malabry Cedex
France