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Damage-Based Criteria for the Combination of Offset Probabilistic Temporal Loads in Topological Optimization Designs

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Abstract

Topology optimization is a widespread and robust process to generate structures supporting a given loading state while subject to minimum volume and maximum stiffness specifications. This traditional framework, embodied by techniques such as SIMP or ESO, is lacking in some aspects. Firstly, loading conditions are assumed static, meaning the structure will only be optimized for that very specific layout, unable to adapt to any eventualities, e.g. vibrations, unexpected (impacts) or alternating loads. Secondly, loads are considered fixed in position, direction and modulus, which is often not the case in industrial applications. Loads can be misplaced, move and vary their

direction and modulus under certain conditions, including those linked to nonlinear effects (buckling, creep, fatigue). Lastly, as a direct consequence of the previous point, damage considerations are not usually taken into account. This absence misrepresents results as they do not reflect wear and tear over time (4D optimization). In this article, a novel attempt at solving these issues is presented. Uncertain and pseudo-dynamic loading is introduced and its long-term effects captured by a damage parameter based on the elastic energy at each step following a reinforced SIMP scheme. Future ramifications of this work are pondered, especially regarding metamaterial inverse design.

Keywords: topology optimization, fatigue damage, finite element method, multi-objective optimization, continuous media, probabilistic mechanics, metamaterials

1 Introduction

Since the dawn of the Industrial Revolution, structural design has focused on meeting both mechanical (stiffness, deformation) and design/economic constraints (weight minimization, i.e. material layout optimization). A work-minimizing solution for frame (discrete) structures was presented by Michell [1], perfected for flexural meshes by Rozvany [2] and formulated by him and Prager [3]; and ultimately conveyed as a form of material density distribution over a continuous dominion by Bendsøe and Kikuchi [4,5].

Topology optimization (TO) techniques define a mathematically robust framework by which material is distributed throughout a spatial dominion via minimization of elastic strain deformation (compliance, $c = uKu$) subjected to a volume fraction requirement $f = V_f/V_0$. It is an optimization process inasmuch as it improves the material layout according to the aforementioned constraints (compliance, light-weight); and topological as it changes the genus of the prototype, adding non-disconnecting cuttings along non-intersecting closed simple curves, i.e. ”holes”. Shape optimization can also be performed on the generated material regions as a refinement step, with no topological implications (genus remains unaltered) [6]. While closely related, these two processes (shape and topology optimization) are conceptually distinct [7,8]. Most common TO methods are solid isotropic microstructure/material penalization (SIMP) [5,9] - and (bi-directional) evolutionary structural optimization ((B)ESO).

Considering a plate with normalized thickness ρ, SIMP aims for a binary material (black, $\rho = 1$) versus void (white, $\rho = 0$) solution, based on a power rule with penalization p. Typically, intermediate states (grey, $0 < \rho < 1$) require more finely-tuned elimination to avoid checkerboard patterns [10], mesh dependence and convergence problems due to nonexistence of the solution, or ill-defined gradient searching for local minima instead of global ones. These issues can be alleviated via filtering, relaxation and continuation methods, respectively [11].

The ESO method, also known as sequential element rejections and admissions

(SERA), requires computing a representative parameter value (e.g.: von Mises' stress), eliminating the elements presenting its lowest value in each iteration, that is, setting them from full to void. The bi-directional version [12] adds extra elements near to high-value already existing ones. (B)ESO can yield better results if a global optimum is found from within a large number of trials [13]. This last method has been criticized for its completely heuristic nature and its questionable efficiency, since it does not guarantee an optimal solution - which could be singular [14] - but a (hopefully) near-optimal one. Similar techniques are based on stochastic procedures such as genetic algorithms, who keep the most robust designs out of a pool that shrinks with every evaluation, imitating natural selection (hence the name). They have been used extensively for trusses and lattice structures [14,15].

Other approaches based on homogenization, such as near-optimal microstructure (NOM) [4] or optimal microstructures with penalization (OMP) [7], are demonstrably under-performing due to insufficient penalization and their excessive complexity - they require multiple variables per element [16]. None of these methods takes into account mechanical properties explicitly, an issue some new approaches, such as the updated properties model (UPM) [17] have successfully addressed to some degree.

However, traditional TO studies are usually subjected to static loading, a rather deterministic framework where neither forces nor any other boundary conditions change over time in modulus or position. This hinders their generalization and applicability under dynamic loading in service and the inherent uncertainty in loading conditions (accentuated by vibration and wear, also ignored by most optimization schemes). Some inverse design proposals for discrete lattices exist [18], although bibliography tackling the continuum remains scarce. The modeller must also bear in mind that not all loads are applied at the same time, nor with the same frequency. A common example is that of an unexpected impact load: an eventuality which can still have a great impact on the structure's resistance, perhaps to the point of compromising it (failure). Reliability/performance-based topology optimization (RBTO/PBTO) and inverse optimum safety factors (IOSF) [19,20] strive for a more flexible, custom, probabilistic approach, compatible with multi-objective optimization (multiaxial, dynamic loading with damage implications) and not as vulnerable to printing irregularities [21].

The inclusion of damage constraints in the optimization process is still far from ubiquitous, since TO is conceived primarily as an inverse design tool, regardless of maintenance concerns. Whereas static loading obeys static failure criteria, easily implementable through stress-based formulations (Rankine, Saint-Venant, Tresca, von Mises, etc.); dynamic loads imply fatigue damage, which can be parametrized by stress (Wöller's diagram), strain (e.g., Smith-Watson-Topper's model) and/or energy criteria (e.g., fracture mechanics) integrated over time. Arguably, the latter come as more general and deeply naturally intertwined with TO - minimizing compliance, i.e. elastic strain energy. Some authors consider total strain energy (elastic and plastic) a parameter for cumulative fatigue damage in itself - including explicit fracture considerations (e.g., crack growth) [22,23], even suitable for damage prediction [24]. Other important additions of TO involve realistic material stress constraints [25], efficient FE2 approaches [26] and/or correctly separated multi-phase solutions [27].

After briefly introducing the state of the art in this first section, the proposed methodology will be presented in Section 2. Some preliminary results shall be analyzed in Section 3, following some conclusions and proposals in the fourth and last section.

2 Methodology

This article is based upon Andreassen et al.'s [28] 88-line adaptation of a famous previous version by Sigmund [29], whose initial SIMP approach was improved in computational efficiency (by a factor of 100), as well as numerical stability (setting a minimum non-null void Young's modulus E_{min}) and convergence (avoiding checkerboard patterns by density - and volume - filtering). In the present article, uncertainty was introduced in the load vector F, both in modulus and position, varying them according to a normal distribution. Additionally, two subprocesses were included in the main iteration loop. The innermost one considers every cycle j under a randomized approximation of a canonical load F_i, accumulating contributions c_j computed directly as a user-defined damage function γ involving c_e. The outer one alternates i loads included in F, accumulating their respective compliance contributions c_i. Such compliance contributions are averaged outwards at the cycle (c_j), load (c_i) and iteration level (c) through the damage parameter γ, which is computed locally for each cell.

Three damage criteria are considered. The first of them is an energy criterion, computing a damage parameter $\gamma = \Delta W_t$ containing a cyclic contribution (N_f) and a tensional contribution (C):

$$\gamma = \Delta W_t = \kappa N_f^{\alpha} + C = \kappa N_f^{\alpha} + \frac{1}{2E}\sigma_f^2 \tag{1}$$

where κ and α are adjustable parameters, N_f is the number of loading cycles, E is the base material's Young modulus and σ_f is the tension damage threshold.

The second criterion considers the 2D Finite Element structure's stress state, penalizing the cell with the highest principal stress ε_1 by a factor P multiplying the damage parameter ΔW_t so that $\gamma = P\Delta W_t$. If such maximum corresponds to traction, the resulting structure will be reinforced against such loadings, which is useful for cement-like materials withstanding little or no tension. On the contrary, should compression be maximum, penalization will protect the optimized architecture from buckling and other issues arising in metals and composites alike.

A third criterion could include the effects of the prototype's printing direction, as in [30,31], which of course distorts the original (assumed) isotropy of the optimized structure. This effect can be included by multiplying ΔW_t times the projection $proj$

of such printing direction r_{print} on the principal direction r_1:

$$\gamma = \Delta W_t proj(\vec{r_{print}}, \vec{r_1}) = \Delta W_t(1 + \beta(\vec{r_{print}} \cdot \vec{r_1})) \tag{2}$$

where β is a tuneable intensifying parameter. For that, the strain tensor ε is computed from the displacements $\mathbf{u}$ and $\mathbf{B}$ for 2D square elements:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} & 0 & \frac{\partial N_4}{\partial y} \\ \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_4}{\partial x} & \frac{\partial N_4}{\partial y} \end{bmatrix} \left\{\mathbf{u}^{(\mathbf{e})}\right\} \tag{3}$$

Which, assuming unitary side and centered nodes, becomes easy to compute if linear shape functions $N_i(x, y)$ are applied.

Lastly, a forth criterion could be directly von Mises' equivalent stress, accounting for the distortion energy. First, the stress tensor is computed from Lamé-Hooke equations:

$$\sigma = \lambda tr(\varepsilon) + 2\mu\varepsilon \tag{4}$$

Which allows the calculation of von Mises' stress for the 2D case as the damage parameter:

$$\gamma = \sigma_{vM} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy} + 3\tau_{xy}^2} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \tag{5}$$

These criteria are tailored for specific purposes (damage, stress, printing parameters and distortion), but they can be combined when needed.

3 Results

Different boundary conditions were examined via various loads and criteria, mainly a cantilever beam and a 3 point bending test. The cantilever beam was exposed to multiaxial loading: two static horizontal loads (5 N) were applied each 25 loading cycles, together with two alternating vertical loads around the upper tip and bottom middle, varying in modulus and position (within a 600x200 lattice) according to two different normal distributions $\mathcal{N}(1, 0.5)$ and $\mathcal{N}(550, 50)$. A penalization factor $p = 3$ and a filter radius $r_{min} = 2$ were considered. When compared to the vanilla configuration (top88 by [28]), the proposed probabilistic approach offers a quite different topology - as seen in Figure 1 -, more resistant to uncertainty as it spreads in several directions in a more organic and continuous manner, with more reinforcement as adjusting parameters K_f, σ_f, α in criterion 1 are refined.

The 3-point bending test is subjected to a central downward load with modulus given by $\mathcal{N}(1, 0.3)$. The objective is now to ponder the effects of each aforementioned damage criteria individually.

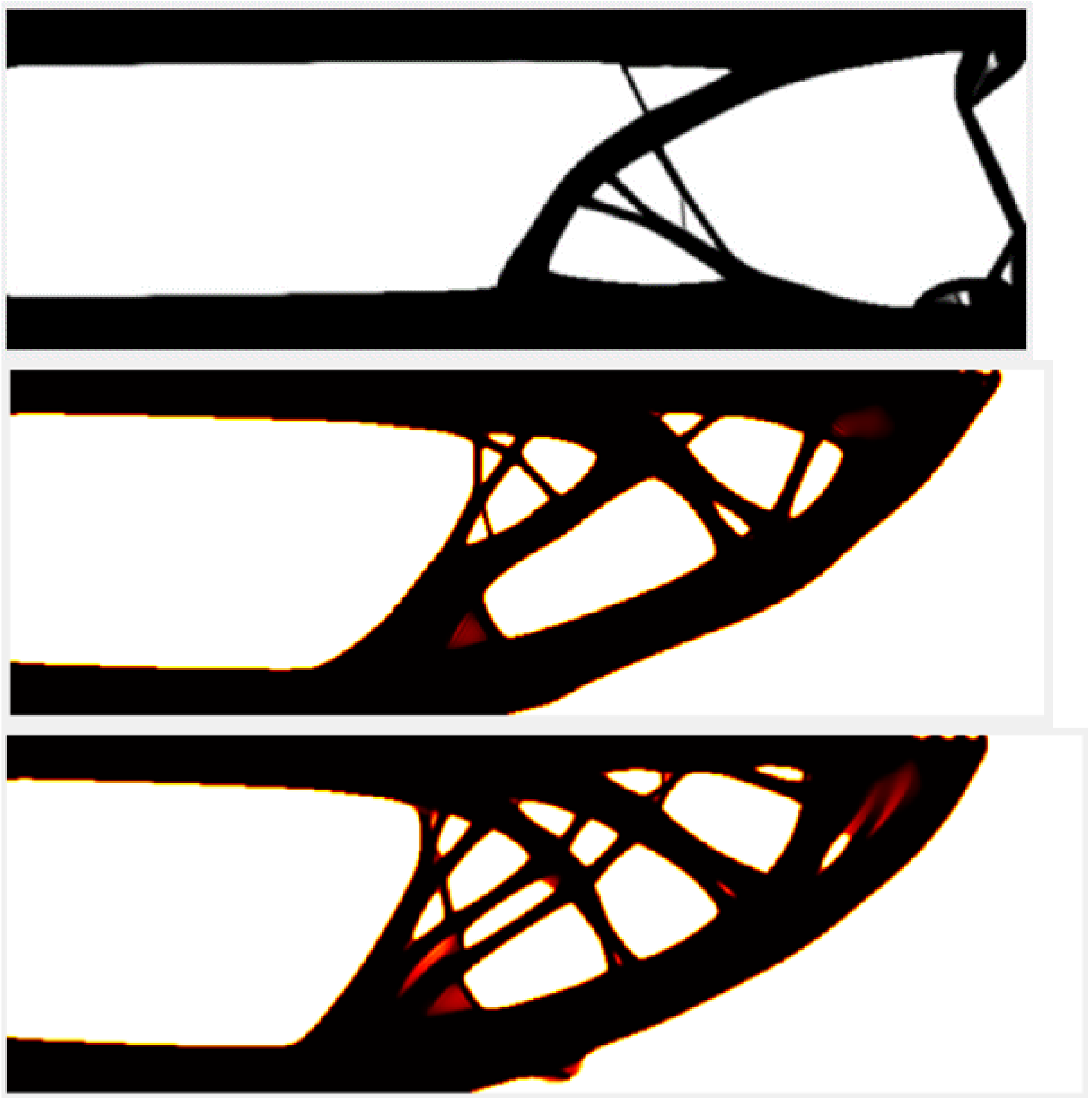

Figure 1: Multi-load case, position and modulus uncertainty: vanilla (up), configuration 1 with $\kappa = \sigma_f = 10^{-5}$ and $\alpha = 9 \cdot 10^{-2}$ (middle) and configuration 2 with $\kappa = 5 \cdot 10^{-5}$, $\sigma_f = 5 \cdot 10^{-2}$ and $\alpha = 9 \cdot 10^{-3}$ (bottom). Iteration 100

This comparison (Figure 2) shows how penalizing certain tensional states (traction/compression) reinforces the parts subjected to such conditions. Analogously, material along the printing direction will be prominent and broadened. The last criterion provides a minimum-material version of the isotropic case with homogeneous distortion energy (von Mises).

Units have been deliberately left undeclared, since linear elasticity is considered throughout the article and so scale is up to the designer, who will obtain the same results as long as the proportions between size and loads are kept. Experimental testing will be performed to evaluate the influence of the printing conditions on the designed

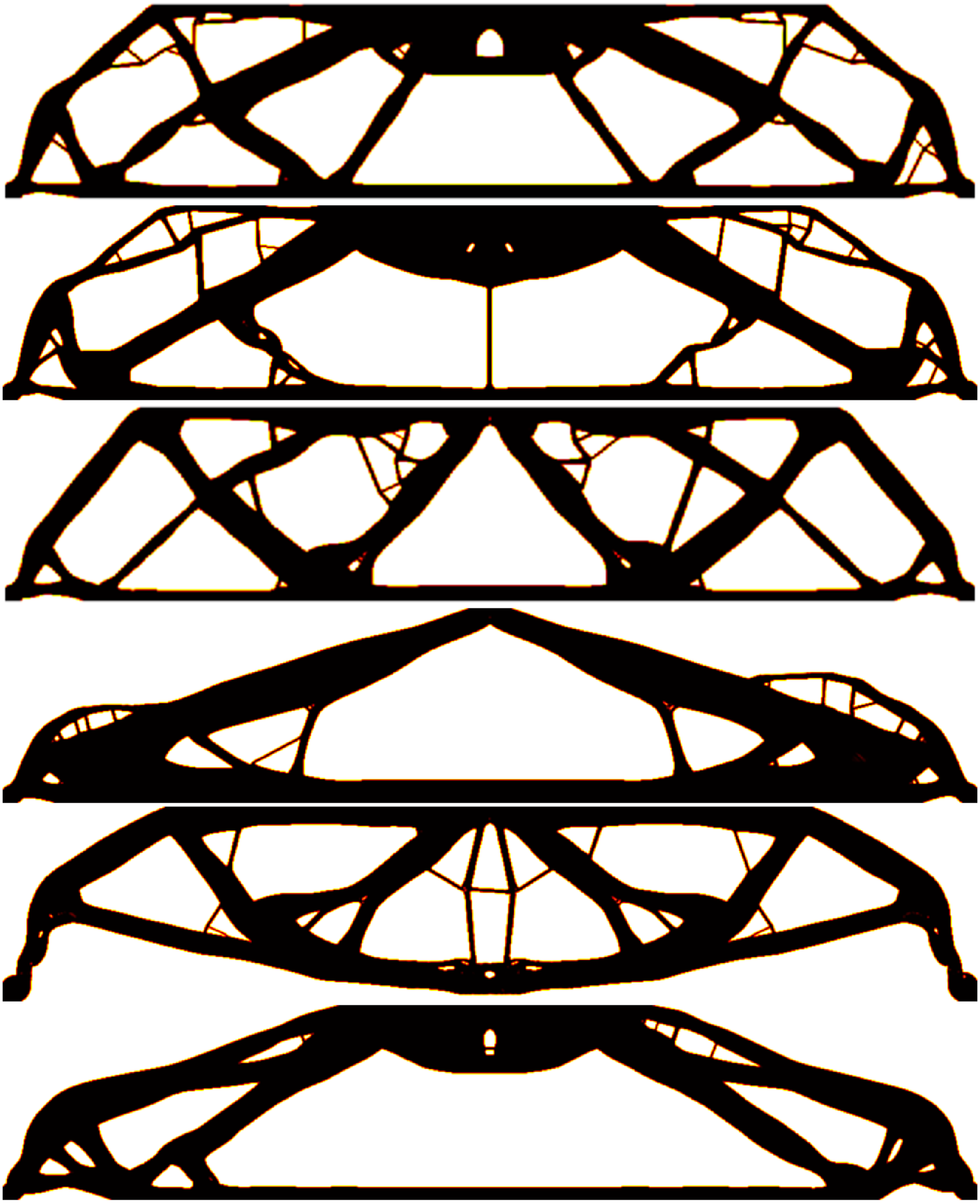

Figure 2: 3-point bending test with 10-element wide supports on the lower corners and configuration 3 ($\kappa = 5 \cdot 10^{-7}$, $\sigma_f = 5 \cdot 10^{-3}$ and $\alpha = -9 \cdot 10^{-5}$). Iteration 200 for the isotropic criterion (first), the tensional one penalizing compression (second) and traction (third) with $P = 10000$, printing angles of 0º (fourth) and 90º (fifth) with $\beta = 500$ and implementing von Mises' equivalent stress as a damage parameter (sixth). Size $1000x200$, $p = 5$.

prototypes.

4 Concluding remarks

In the light of the obtained results, this method has proven its robustness for inverse design, adding damage implications to the vanilla configuration [29,28]. This is coherent with the initial goal of providing uncertainty- and damage-proof alterantives to traditional deterministic designs, able to withstand multi-objective optimization (multiaxial, uncertain load, frequency and position) while offering different refining damage criteria: tensional (compression, tension, von Mises' equivalent stress) and operational (horizontal and vertical printing directions).

This upgrade opens up a plethora of new, sturdy, inverse designs. Remarkably, topology and shape optimization can be leveraged for the design and improvement of architectured materials, also known as metamaterials. Metamaterials (from "$meta$": beyond) are composed of micro-lattices tailored to showcase macro properties and behaviours unheard of in bulk materials (continuous media).

According to their field of application, metamaterials include 4 main types [32]: electromagnetic - hyperlenses [33], Left-Handed Materials with a negative refraction index [34,35] and optical cloaks [36] -, thermal [37], acoustic - absorption [38], cloaks [39] - and mechanical - auxetic [40] - presenting a zero [41], negative [42] or even tunable Poisson's ratio [43,44] -. Some metamaterials even combine several areas, such as mechanical and thermal [45] or acoustic and mechanical [46]. The finite element method (FEM) [47,48], as well as Model Order Reduction (MOR) [49,50] and Machine Learning (ML) techniques - mainly Convolutional [51], Recurrent [52] and Graph Neural Networks [53,54] - have been widely used for their design and characterization [55-58], including inverse design [59].

Different specific lattice-like continuum architectures (periodic or not) generated via TO allow for a discrete, tailored behaviour, like composites, except with lighter weight and more design flexibility overall. Some authors consider metamaterials a subset of multi-objective composites in which one of the phases is void [60]. One common trait of both is functional grading, i.e. different properties for different needs along sections of the prototype. Greater precision (down to one micron [61]) and lesser cost of manufacturing techniques [62] - mainly additive, i.e. 3D printing [63] - have ushered the study of new, exponentially more complex structural designs, with much needed multi-scale (hierarchical) implications [64]. Conversely, a sufficiently dense network could emulate the behaviour of a continuous medium if needed, only with a minimum weight, as trabecular bone does organically.

Focusing on mechanical metamaterials, they share some design constraints with macroscopic structures, depending on their geometry and loading cases, namely fatigue [65,66] - extensively studied for additively manufactured metamaterials [67-71], buckling [72] -although it can be leveraged [73] - and creep [74,75]. Topological optimization aims to improve the (meta)material's response to such boundary conditions by iteratively changing its geometry, namely by scrapping material where it is not entirely necessary to endure a given loading scenario. It can also be used to alleviate the aforementioned problems.

Since a metamaterial already largely consists of void, the TO process is expected to be simpler, and so, faster - both conceptually and computationally -, since light weight is already guaranteed to certain degree. Thus, such optimization approach would mostly focus on redistributing material to gain the desired observable properties (e.g.; auxeticity). It could even guide their (inverse) design [76-80], as a sort of inverse homogenization [81,82], thus circumventing the ill-posedness of such quests. Diverse data-driven attempts exist for inverse metamaterial design, both with the continuum [83-86] and discrete lattices [87,57] as starting points. All these aspects, as well as a 3D generalization, will be present in subsequent publications.

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