The Fourteenth International Conference on
Computational Structures Technology
23–25 August, 2022 | Montpellier, France

Proceedings of the Fourteenth International Conference on
Computational Structures Technology
Edited by B.H.V. Topping and J. Kruis
Civil-Comp Conferences, Volume 3, Paper 3.2
Civil-Comp Press, Edinburgh, United Kingdom, 2022, doi: 10.4203/ccc.3.3.2
©Civil-Comp Ltd, Edinburgh, UK, 2022

Wave propagation at the coupling boundary of Peridynamics and finite elements

M. Yu and G. Noh

School of Mechanical Engineering, Korea University
Seoul, Republic of Korea

Abstract

This research investigates the optimal use of time integration methods to solve transient wave propagation problems in the coupling domain of bond-based Peridynamics with finite elements. We respectively study the numerical dispersions of bond-based Peridynamics and finite elements with the central difference method and the Noh-Bathe time integration method. It reveals that the difference in numerical wave speed between FE and PD domain causes spurious wave reflection leading to inaccurate solutions. The study of the numerical dispersion reveals that the Noh-Bathe scheme can reduce the wave reflection, whereas the central difference method still causes the wave reflection between the domains. In the practical wave propagation problems, the Noh-Bathe scheme provides more accurate solutions than the central difference method.

Keywords: wave propagation, direct time integrations, numerical dispersion, coupling, Peridynamics, finite element method.

1 Introduction

Peridynamic (PD) theory has been developed to simulate the fracture of structures efficiently [1,2]. In opposition to classical continuum theory such as the finite element method (FEM), Peridynamics does not assume the differentiability of displacement to obtain the forces, and it easily represents the discontinuity in the displacement field without special treatments.

However, PD theory suffers from some drawbacks:

- PD methods are more computationally inefficient than those based on classical continuum theory since every point of PD grid interacts with all other points within its neighborhood [3].
- the enforcement of boundary conditions in PD grid may be complicated due to the problem of defining the boundary in a non-local theory [1].

In this regard, it would be effective to couple PD domain with FEM meshes to possess the advantages of both schemes. In most cases, cracks are likely to grow in a small part of the model, whereas the remaining parts follow the usual continuum theory, thus, the former would be discretized using a PD grid, and the latter would be modeled using a FE meshes. Among the studies, an effective strategy to couple PD grid with FEM was introduced by Zaccariotto et al. [4]. This coupling scheme can be easily implemented since there is no need to interpolate or transfer displacement and force.

An inherent drawback of the coupling scheme is the appearance of spurious wave reflections. Reflections are usually observed when the waves propagate across the different numerical models. For example, the coupling scheme of PD with FEM introduced by Zaccariotto et al. also produces a spurious reflection on the interface of PD grids and FEM meshes. Therefore, there are researches on developing a coupling scheme to remove spurious reflection [5-7]. Although, these schemes somehow use the central difference time integration or the velocity verlet scheme. These time integration methods exhibit the accurate solutions when the lumped mass is used. Still, these may produce a spurious reflection on the coupling interface due to the difference in numerical wave speed.

Therefore, this research aims to present the optimal use of direct time integration schemes to analyze wave propagations at the coupling boundary of bond-based Peridynamics and finite elements. We investigate the numerical dispersion of two time integration schemes with PD grids and FE meshes. The time integration schemes include the central difference method and the Noh-Bathe time integration method [8]. The theoretical performances of these time integration schemes to relieve spurious reflection are measured using the difference in numerical wave speed between PD grids and FE meshes. Then, to illustrate the practical performance of time integration schemes, we present several benchmark problems of 1D wave propagation.

2 Methods

To analyze the spurious reflection of the coupling interface, we first study the numerical dispersion of PD grids and FEM mesh. In the 1D case, we consider the nodes equally spaced Δx apart along the x-axis (Δx=h, h is the element size). Then the dispersion error caused by the spatial approximation is considered by the time-independent form of the wave equation and the associated system algebraic equation

$$\mathbf{KU} - (k_0 h)^2 \mathbf{MU} = \mathbf{0} \tag{1}$$

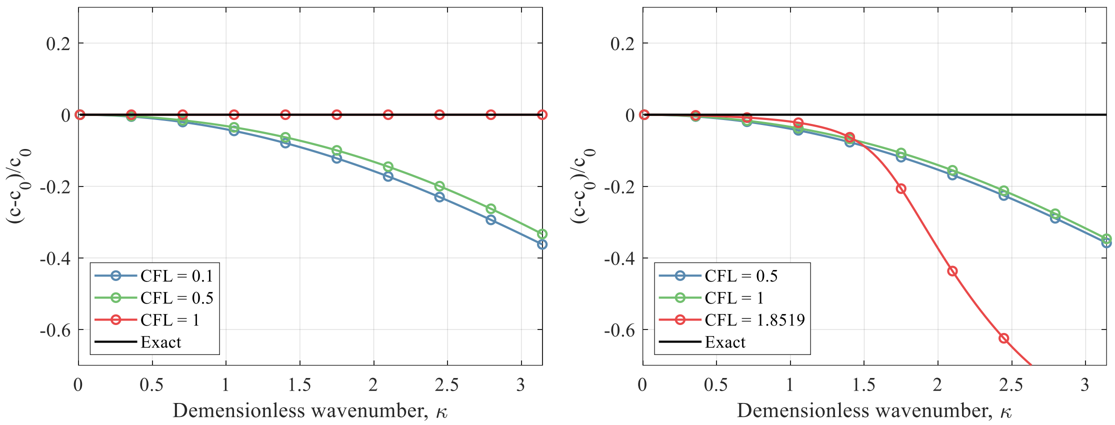
where **K**, **M**, and **U** are the stiffness matrix, mass matrix, the vector of unknown, and k_0 is the exact wave number. Note that the linearized form of PD grids and FE meshes is identically written as Eq. (1). However, the ways PD and FEM construct the stiffness matrix are different. The details to obtain the stiffness matrix for PD grids refer to Ref. [9].

We assume the solution of Eq. (1) to be the numerical sinusoidal waveform as

$$\mathbf{U}_I = \hat{\mathbf{U}} e^{ik\mathbf{n}\cdot\mathbf{y}_I} \tag{2}$$

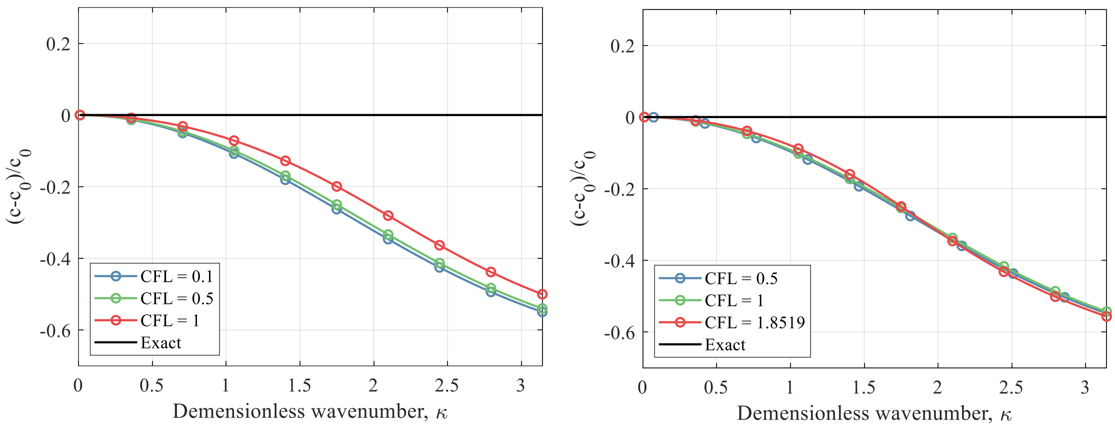
where $\hat{\mathbf{U}}$ is the amplitude, $\mathbf{y}_I$ is the position vector, and k is the solution wave number. Substituting Eq. (2) into Eq.(1), the algebraic system equation is rewritten as

$$(\mathbf{D}_{stiff} - (k_0 h)^2 \mathbf{D}_{mass})\hat{\mathbf{U}} = \mathbf{0} \tag{3}$$



(a) Central difference method (b) Noh-Bathe method

Figure 1: The numerical dispersion for the finite element method.



(a) Central difference method (b) Noh-Bathe method

Figure 2: The numerical dispersion for Peridynamics.

Thus, we obtain a relationship between k and k_0 from the determinant of Eq. (3)

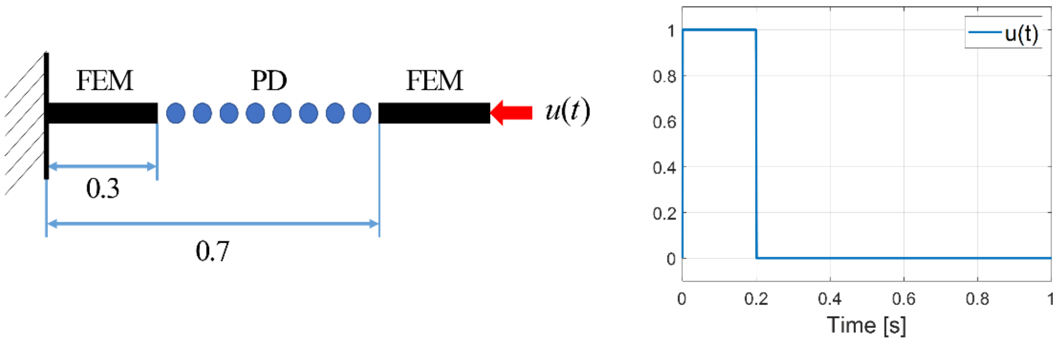
$$\det(\mathbf{D}_{stiff} - (k_0 h)^2 \mathbf{D}_{mass}) = 0 \tag{4}$$

Next, the scalar wave equation is considered to observe the dispersion error that originates from the time approximation. the solution of the scalar wave is governed by

$$\ddot{u} - c_0^2 \Delta u = 0 \tag{5}$$

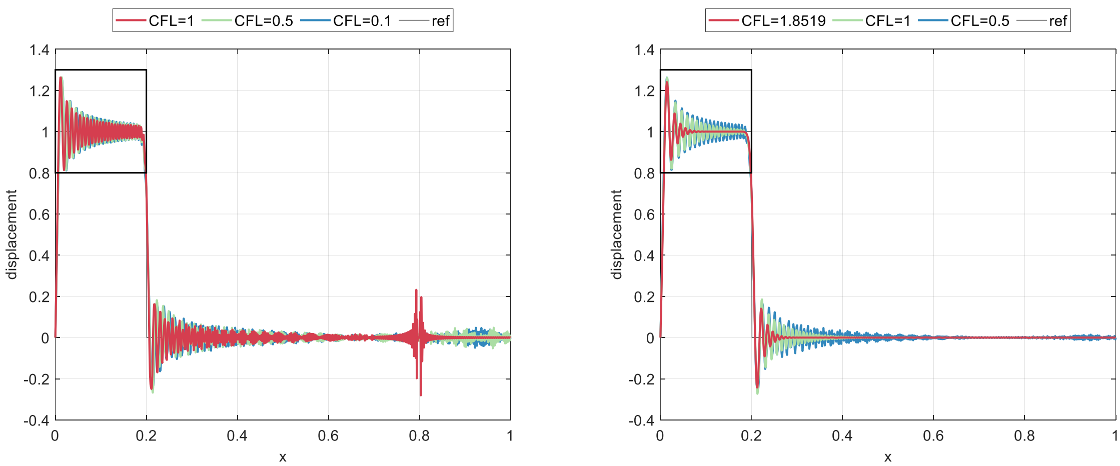
and the solution of the scalar wave is given by

$$u = Ae^{i(k_0 \mathbf{n} \cdot \mathbf{y} - \omega_0 t)} \tag{6}$$



(a) The description of 1D wave propagation (b) The applied displacement

Figure 3: 1D wave propagation problem.



(a) Central difference method (b) Noh-Bathe method

Figure 4: Displacements at t = 1s with various CFL numbers

where A is the amplitude, w_0 is the exact angular frequency, and t is time, and $\mathbf{n}$ and $\mathbf{y}$ are the unit vector along the wave direction and the position vector, respectively. Substituting Eq. (6) into Eq.(5), we consider the scalar wave equation as follows

$$\ddot{u} + k_0^2 c_0^2 u = 0 \tag{7}$$

and investigate the effect of the time integration method. The total numerical dispersion is then obtained by the errors from the spatial and time approximation. The details to obtain the dispersion error caused by the time integration scheme are introduced in Ref. [10].

3 Results

The numerical dispersions of PD and FEM are shown in Figs. 1 and 2. The central difference method exhibits good numerical wave speed errors in PD and FEM for each CFL number. In contrast, the Noh-Bathe method shows worse numerical wave speed errors in PD and FEM than the central difference method. However, the key point is that highly inaccurate wave modes are discarded when the Noh-Bathe method is used. Recalling that spurious reflection originates from the difference in wave speed at the coupling interface, the property of the Noh-Bathe method in which high modes are discarded effectively reduces the difference in wave speed at the interface of coupling.

Considering the 1D wave propagation problem shown in Fig. 1. We discretize the domain with the PD-FE coupling method proposed by Zaccariotto et al. [4]. The results in Fig. 4 show that the Noh-Bathe method performs better than the central difference method. In the central difference method, spurious wave reflection is observed at the coupling interface. Although the central difference method gives the exact wave speed in FEM, it provides the worst solution when CFL = 1 is used in the coupling domain since the central difference method does not delete high modes causing the reflection caused by the difference in wave speed. On the other hand, the Noh-Bathe method deletes the spurious reflection at the interface and gives an accurate solution when CFL = 1.8519 is used.

4 Conclusions and Contributions

In this research, we investigate the wave performance of direct time integration schemes at the coupling interface of Peridynamics and the finite element method. Spurious wave reflection originates from the difference in the wave speed, thus we respectively analyze the theoretical wave speed of Peridynamics and finite element method, and a difference in wave speed is observed.

The dispersion properties of PD and FEM reveal that, although the central difference method gives the exact wave speed in FEM, the Noh-Bathe method shows more stable solutions than the central difference method since the former removes high modes that cause the difference in wave speed at the coupling interface, and the latter does not.

In the 1D wave propagation problem, as we expected, Noh-Bathe method gives more stable and accurate solutions than the central difference method. Moreover, since the Noh-Bathe method can use a larger time step, we verify that the Noh-Bathe method is a better time integration method for the PD-FEM coupling grids.

References

[1] Silling, S. A. (2000). Reformulation of elasticity theory for discontinuities and long-range forces. Journal of the Mechanics and Physics of Solids, 48(1), 175-209. https://doi.org/10.1016/S0022-5096(99)00029-0

[2] Silling, S. A., Epton, M., Weckner, O., Xu, J., & Askari, E. (2007). Peridynamic states and constitutive modeling. Journal of elasticity, 88(2), 151-184. https://doi.org/10.1007/s10659-007-9125-1

[3] Chen, X., & Gunzburger, M. (2011). Continuous and discontinuous finite element methods for a peridynamics model of mechanics. Computer Methods in Applied Mechanics and Engineering, 200(9-12), 1237-1250. https://doi.org/10.1016/j.cma.2010.10.014

[4] Zaccariotto, M., Mudric, T., Tomasi, D., Shojaei, A., & Galvanetto, U. (2018). Coupling of FEM meshes with Peridynamic grids. Computer Methods in Applied Mechanics and Engineering, 330, 471-497. https://doi.org/10.1016/j.cma.2017.11.011

[5] Kulkarni, S., & Tabarraei, A. (2018). An analytical study of wave propagation in a peridynamic bar with nonuniform discretization. Engineering Fracture Mechanics, 190, 347-366. https://doi.org/10.1016/j.engfracmech.2017.12.019

[6] Giannakeas, I. N., Papathanasiou, T. K., & Bahai, H. (2019). Wave reflection and cut-off frequencies in coupled FE-peridynamic grids. International Journal for Numerical Methods in Engineering, 120(1), 29-55. https://doi.org/10.1002/nme.6099

[7] Wang, X., Kulkarni, S. S., & Tabarraei, A. (2019). Concurrent coupling of peridynamics and classical elasticity for elastodynamic problems. Computer methods in applied mechanics and engineering, 344, 251-275. https://doi.org/10.1016/j.cma.2018.09.019

[8] Noh, G., & Bathe, K. J. (2013). An explicit time integration scheme for the analysis of wave propagations. Computers & structures, 129, 178-193. https://doi.org/10.1016/j.compstruc.2013.06.007

[9] Macek, R. W., & Silling, S. A. (2007). Peridynamics via finite element analysis. Finite elements in analysis and design, 43(15), 1169-1178. https://doi.org/10.1016/j.finel.2007.08.012

[10] Kwon, S. B., Bathe, K. J., & Noh, G. (2020). An analysis of implicit time integration schemes for wave propagations. Computers & Structures, 230, 106188. https://doi.org/10.1016/j.compstruc.2019.106188