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# **Calibration of constitutive models for soils: uncertainties attributed to limited number of laboratory test**

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## **Abstract**

The basic formulation of the Mohr-Coulomb model does not reproduce several phenomena that a real soil exhibits including: a) the evolution of volumetric plastic strain upon isotropic compression, b) either compaction or dilatation of the soil upon shear, depending on the initial density, c) stiffness increasing with mean stress. These phenomena are inherently built into the critical state constitutive models such as the modified Cam clay (CC) model or the hypoplastic model for clay (HC). Although these constitutive models are present in many geotechnical finite element programs, their use is still rather limited. The reason for this is that the parameters of these models are less known, and are seldom provided by the geotechnical laboratory. The ExCalibre web application available at <https://soilmodels.com/excalibre-en/> was recently released to make the nontrivial process of calibration for these constitutive models easily available to everyone interested. To calibrate either CC or HC model, ExCalibre accepts an Excel worksheet with an arbitrary number of oedometric and undrained triaxial laboratory tests and fits the model response to the measured data. This contribution focuses on how the number of particular laboratory tests that the user chooses to upload to ExCalibre influence the obtained set of model parameters.

For both material models and all soil samples the results show that the mean value of the calibrations with reduced laboratory protocols tests does not differ from the reference value obtained for single calibration run taking into account all available laboratory tests. This observation suggests that doing the calibration several times with a limited number of laboratory tests, i.e. one oedometer test and one undrained

triaxial test, and then averaging the results yields very similar results as running the calibration just once with all available data.

Second observation is that for both material models and most of the soil samples the coefficient of variation is less than 10% for all material parameters with exception of Poisson's ratio whose coefficient of variation reaches 50% in case of the modified Cam clay model and 20% in case of hypoplastic model for clay.

**Keywords:** soil, constitutive model, calibration, uncertainty, laboratory tests.

## 1 Introduction

Various software based on the finite element method are used to design and assess all sorts of complicated geotechnical structures such as deep excavations, tunnels or foundations. One of the main challenges when creating the finite element model is to choose an appropriate constitutive model for each affected soil and determine its parameters so that the constitutive model properly simulates the soil's mechanical behaviour.

The Mohr-Coulomb model represents the simplest and - despite its history of more than two centuries - the most widely used constitutive model for soils. Its yield surface is determined by two parameters: the angle of internal friction and the cohesion. Together with the Young modulus and the Poisson ratio essentially became "the parameters of soil" that most geotechnical laboratories provide based on oedometric and direct shear tests.

Nevertheless, the basic formulation of the Mohr-Coulomb model does not reproduce several phenomena that a real soil exhibits including: a) the evolution of volumetric plastic strain upon isotropic compression, b) either compaction or dilatation of the soil upon shear, depending on the initial density, c) stiffness increasing with mean stress.

These phenomena are inherently built into the critical state constitutive models [1]. The modified Cam clay model (CC) [2] is a classical example based on theory of plasticity whereas the hypoplastic model for clay (HC) [3,4] is based on the theory of hypoplasticity [5,6] and formulates the constitutive law in incremental form only and does not distinguish between elastic and plastic parts of strain increments.

Although these constitutive models are present in many geotechnical finite element programs, their use is still rather limited. The reason for this is that the parameters of these models are less known, and are seldom provided by the geotechnical laboratory. The ExCalibre web application available at <https://soilmodels.com/excalibre-en/> was recently released to make the nontrivial process of calibration for these constitutive models easily available to everyone interested. To calibrate either CC or HC model, ExCalibre accepts an Excel worksheet with an arbitrary number of oedometric and undrained triaxial laboratory tests and fits the model response to the measured data. This contribution focuses on how the number of particular laboratory tests that the user chooses to upload to ExCalibre influence the obtained set of model parameters.

## 2 Methods

For CC model the ExCalibre app outputs five material parameters, namely the slope of the normal consolidation line  $\lambda$  and the slope of the unloading-reloading line  $\kappa$  in  $(\ln(p) \times e)$  space, maximal void ratio  $e_0$ , the Poisson ratio and the slope of the critical state line in  $(p \times q)$  space. Here  $p$  and  $q$  denotes the mean and equivalent deviatoric stress and  $e$  denotes the void ratio. The first four parameters predominantly control the stiffness while the last one controls the material's strength.

For the HC model the application returns also five material parameters: the slope of the normal consolidation line  $\lambda^*$  and the slope of the unloading-reloading line  $\kappa^*$  in  $(\ln(p) \times \ln(1+e))$  space, parameter  $N$  which is the logarithm of maximal specific volume  $v_0=1+e_0$ , the Poisson ratio and the angle of internal friction at critical state  $\varphi_c$ .

For clayey soils the input of the automatic calibration algorithm is a laboratory protocol containing a collection of an arbitrary number of oedometric (OED) tests preferably with both the loading and the unloading branches and the undrained triaxial shear tests with measured pore pressure excess (CIUP) performed preferably at different chamber pressures. The algorithm combines a direct calculation of the stiffness parameters from the slopes of the OED tests' chart and several optimization runs to determine the remaining parameters. Full description of the algorithm is provided in [7,8].

In the following we examine the influence of the number of OED and CIUP considered in the calibration on the variance of particular calibrated parameters. In particular, laboratory protocols of three soil samples presented on ExCalibre website were selected. The numbers of particular tests performed for each soil are listed in the following table.

Locality	USCS class	num. of OED tests	num. of CIUP tests
Hájek, Czech Republic	CL	2	3
Brno, Czech Republic	CH	1	4
Bangkok, Thailand	CH	3	5

Table 1: Selected soils, their USCS class and number of available tests in reference laboratory protocol.

A reference set of material parameters was obtained for the original laboratory protocol containing all the listed OED and CIUP tests. Then the calibration was performed for a number of reduced laboratory protocols containing different subset of the laboratory tests available in the original protocol. The mean value, standard deviation and the coefficient of variation (CoV) were calculated for the parameters obtained for the reduced laboratory protocols.

### 3 Results

The calibration was performed for two material models and three soils. The results obtained for the modified Cam clay model are shown in Table 2. Three values are presented for each soil and each model parameter. The reference value is the value of the material parameter obtained for the original laboratory protocol with all available laboratory tests. The mean value is the mean of the material parameter obtained for reduced laboratory protocols. Note that a reduced laboratory protocol contains only a subset of the laboratory tests available in the original protocol. Nevertheless, the reduced laboratory protocol has to contain at least one OED test and one CIUP test in order to be accepted by the automatic calibration algorithm. Finally, the coefficient of variation is the ratio of the standard deviation and the mean value of the parameter for reduced laboratory protocols.

Soil	Stat	$e_0$ [-]	$\lambda$ [-]	$\kappa$ [-]	$M_{cs}$ [-]	$\nu$ [-]
Hajek	ref.	0.905	0.057	0.008	1.41	0.2
	mean	0.901	0.058	0.0084	1.28	0.17
	<b>CoV [%]</b>	<b>3.1</b>	<b>3.5</b>	<b>25.9</b>	<b>5.5</b>	<b>49.8</b>
Brno	ref.	2.141	0.186	0.01	1.01	0.38
	mean	2.142	0.186	0.0098	1.02	0.31
	<b>CoV [%]</b>	<b>0.02</b>	<b>0</b>	<b>3.6</b>	<b>2.8</b>	<b>30.4</b>
Bangkok	ref.	8.26	1.107	0.01	1.055	0.2
	mean	8.2458	1.10712	0.01	1.069000	0.285000
	<b>CoV [%]</b>	<b>4.4</b>	<b>7.2</b>	<b>0</b>	<b>4.0</b>	<b>31.4</b>

Table 2: Cam clay model - reference value, mean value and CoV of material parameters for three selected soil samples.

The analogical results obtained for calibration of the hypoplastic model for clay are listed in Table 2. Note that the coefficients of variation for the hypoplastic model for clay are generally smaller than those for the modified Cam clay. This suggests that the calibration of the HC model is less sensitive to the inconsistencies in the laboratory data.

Soil	Stat	$N$ [-]	$\lambda^*$ [-]	$\kappa^*$ [-]	$\varphi_c$ [°]	$\nu$ [-]
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Hajek	ref	0.683	0.041	0.005	34.9	0.23
	mean	0.681	0.0412	0.0056	31.9	0.25
	<b>CoV [%]</b>	<b>2.5</b>	<b>1.6</b>	<b>7.6</b>	<b>5.0</b>	<b>20.1</b>
Brno	ref	1.536	0.129	0.011	25.6	0.28
	mean	1.536	0.129	0.011	25.75	0.285
	<b>CoV [%]</b>	<b>0.1</b>	<b>0</b>	<b>4.1</b>	<b>2.55</b>	<b>7.5</b>
Bangkok	ref	2.76	0.296	0.01	26.6	0.33
	mean	2.72	0.288	0.0093	27.1	0.33

Table 3: Hypoplastic clay model - reference value, mean value and CoV of material parameters for three selected soil samples.

#### 4 Conclusions and Contributions

An automatic calibration of a constitutive model, i.e. the deterministic process of finding the material parameters for a real soil sample tested in a laboratory, is inherently a task with uncertain results. The uncertainty is attributed to a) the difference in the laboratory data obtained for several runs of the same laboratory test performed on the same soil, b) the fact that the number of laboratory tests used for calibration may vary according to the user's decision. The differences in the resulting material parameters for the Cam clay model (CC) and the hypoplastic model for clays (HC) were presented in this paper for three different samples of clayey soils.

For both material models and all soil samples the results show that the mean value of the calibrations with reduced laboratory protocols tests does not differ from the reference value obtained for single calibration run taking into account all available laboratory tests. This observation suggests that doing the calibration several times with a limited number of laboratory tests, i.e. one oedometer test and one undrained triaxial test, and then averaging the results yields very similar results as running the calibration just once with all available data.

Second observation is that for both material models and most of the soil samples the coefficient of variation (CoV) is less than 10% for all material parameters with exception of Poisson's ratio. This conclusion is based on a limited number of soil samples and is by no means universal, however it might serve as a rough estimate of uncertainty of the material parameters obtained through automatic calibration application ExCalibre.

A parameter showing exceptional variance is undoubtedly the Poisson ratio whose coefficient of variation reaches 50% in case of the modified Cam clay model and 20%

in case of hypoplastic model for clay. The results suggest that special attention should be paid to the calibration process when the Poisson's ratio will play an important role in the subsequent finite element simulations.

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