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Braced grid framework rigidity characterization

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Abstract

Bar and joint frameworks present models of engineering structures. The purpose is to find an efficient algorithm for deciding infinitesimal rigidity in differently braced three-dimensional. Using the bar-joint structure's symmetry to determine the rigidity is a problem of long-standing interest in kinematics, statics, and optimization. The algorithm has applications in robotics as an actuator-controlled mechanism and in material science as meta-materials and reconfigurable materials. The bar and joint framework have served as valuable models of the structure of metals, crystal states of matter, building science, and biological systems. Scaffolding, as repetitive objects, are helpful as preliminary structures of design. Applying some further bracing elements such as Cable, Strut, or Rod (bracing bar) the Scaffolding will be rigid. The given models describe the rigidity of the differently braced scaffolding frameworks and produce a graph theoretical characterization that provides an efficiently solvable graph or directed graph as the original structure.

Keywords: bar joint framework, cubic grid, rigidity, scaffolding, directed graph, computational complexity.

1 Introduction

Saving materials and energy is the basis of sustainable development, so creating optimal structures is our goal. The stability and mobility of new materials and their micro- and nano-structures is a pivotal issues during planning. Its implementation requires the coordinated application of thorough mechanical, mathematical, computing, and materials knowledge.

Primarily, we choose the kinematic description typical of rod-hinged structures, but we touch on the structural stability resulting from the deformability of real materials.

The mathematical characterization of the stiffness of bar joint structures was first described by Maxwell [1,2] with the rank condition given to the stiffness matrix of the structure. Let (x_i,y_i,z_i) be the coordinates of the joint P_i of a Bar and Joint structure. A bar between the joints P_i and P_j determines the distance from P_i to P_j. Since the distance is constant, differentiating its square leads to the equation:

$$(x_i - x_j)(\dot{x}_i - \dot{x}_j) + (y_i - y_j)(\dot{y}_i - \dot{y}_j) + (z_i - z_j)(\dot{z}_i - \dot{z}_j) = 0 \qquad (1)$$

The velocity coordinates $\dot{x}_i$, $\dot{y}_i$, $\dot{z}_i$ are the variables. If we use bars between joints, and the number of bars is e, we get a system with e pieces of equations.

The matrix representation of the equation system is:

$$A\boldsymbol{u} = 0 \qquad (2)$$

where $\boldsymbol{u}$ is the column vector of velocity, and A is an $e \times 3n$ rigidity matrix.

In the case of the infinitesimal rigid structure, equation (2) has the trivial solution only (i.e., the rigid body-like motions). Naturally, the rigid body motion of the joint keeps fixed the distance between the pairs of the joint. If the framework joints have infinitesimal motions that are different from the rigid-body-like motion, then the framework is not rigid. In this case, the $rank(A)<3n-6$. The framework is infinitesimally rigid if and only if the $rank(A)=3n-6$; see in [2]. Maxwell [1] gave this characterization, but with his result, the time complexity of deciding the rigidity is $O(e^3)$. We present a better characterization for Scaffolding structures using different bracing elements.

Laman [3] gave a condition only for planar minimally rigid structures in 1970, which requires an upper limit for the number of bars between the joints of any substructure of the structure. This characterization is computationally disadvantageous because the Laman condition must be checked in every substructure. According to the results of Lovász and Yemini [4], the minimal structure is rigid if, by doubling one of its bars, we can divide the bars into two groups so that we can get from any joint to any other along the bars belonging to the same group. This provides a quick solution for deciding the stiffness of generic planar structures. So far, finding a good characterization in space has not been possible. Therefore, examining more specialized systems, e.g. grid-like structures, also came to the fore. If we consider the cube grid as a structure in which the edges are replaced by bars and the grid points are replaced by ball joints, then the system will not be rigid yet; additional rods are needed, i.e., reinforcement is required; this is also called stiffness percolation. A fast algorithm is provided by Bolker and Crapo's [5] result for square grids. They construct a stiffening bipartite graph that is connected if and only if the diagonally reinforced square lattice structure is stiff. Baglivo and Graver [6] use cables and struts as bracing elements in the case of a tensegrity structure; it characterizes the stiffness of a square grid by the strong connectedness of directed graphs [7]. Claims similar to these and their consequences are examined in the following articles.

The structures consisting of perfect rods, cables, and struts connected to each other by ball joints are called tensegrity structures. Fuller [8] called tensegrity structures only those in which only struts and cables were used, so the struts could not be connected. If two points are connected with strut, the distance between the two points is not less than the length of the strut in the case of permitted movements of the points; similarly, if two points are connected with a cable, the connected joints can only move in such a way that the distance between them cannot be greater than the length of the cable. The bow or the violin string is the most trivial device considered as such a structure.

We can also explain the measurement principle of the Egyptian rope tensioners (Harpenodaptai) using the simplest rigid tensegrity: consider the distance between the two bodies as the strut (because in the case of two points of objects chosen arbitrarily, this is not less than their distance, the measuring rope they stretch, even though the cable of the tensegrity. The structure will be rigid when the length of the rope is equal to the distance to be measured (exactly when the taut rope will be as long as the distance to be measured).

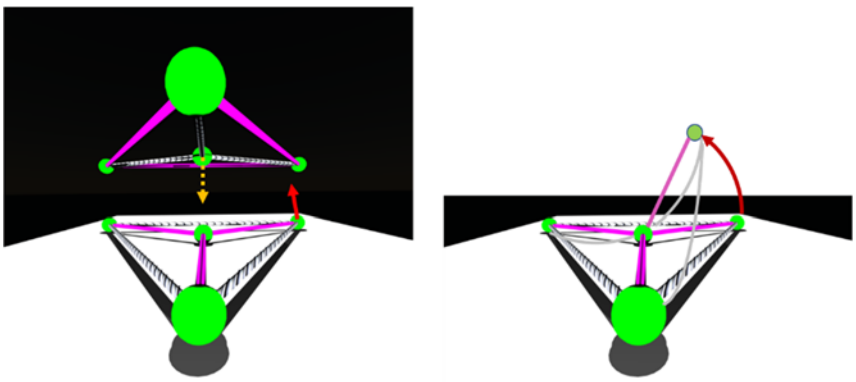

Figure 1. two tensegrity structures, the lower one is not rigid, while the upper one is rigid in space, but none of them is infinitesimally rigid in space. In the plane of structures, both structures are rigid and infinitely rigid.

Such three-dimensional tensegrity structures can be seen in Figure 1. We allow the struts can be connected, and we also enable rods in the system, which can be considered both a cable and a strut; i.e., if two points are connected with a rod, only movements of the points are allowed during which their distance remains constant.

2 Methods

Diagonal braces are excellent for preventing lateral movement of grid-like buildings and for avoiding loss of stability due to damage to individual supporting structural elements.

Figure 2. Node solutions in mega diagonal and tensegrity cases. The first picture is a mega diagonal building in London, and the second picture shows the slab supports of the 50 m pool of the BVSC swimming pool in Budapest.

In the case of a tall building, construction aspects during planning, the design of the structure traditionally consists of orthogonal steel or reinforced concrete elements (beams, columns), which work together through diagonal bracing, thus resisting wind loads, earthquakes that can be considered extraordinary, and natural disasters. Against such exceptional loads, not only static but also various dynamic devices are used. Instead of rigid diagonals, flexible diagonals, often actuators (which can control their own length), computer-controlled pistons, and cables are used [9,10] effects as well. Architects and designers used to try to keep the structural elements hidden by using external coverings, nowadays these structures that provide rigidity carry a special, sometimes eccentric appearance because the designers ostentatiously prefer to show and emphasize them Figure 2. Highlighting the structural elements on the facades of skyscrapers and thereby emphasizing their uniqueness distinguishes them and puts them at an advantage even over their taller counterparts.

Another distinguishing element can be if we give up the static nature of the building, allowing some parts or even the whole to move so that the stability of the building as a whole remains; this is a great challenge for architects [11]. David H Fisher's building in Dubai, which rotates some of its floors, or the Falkirk Wheel, a boat lift between two canals. In the performance, we combine these two emphasized elements [12], moving the cross-linked structural structure by changing the length of the diagonal cables used for bracing see Figure 3.

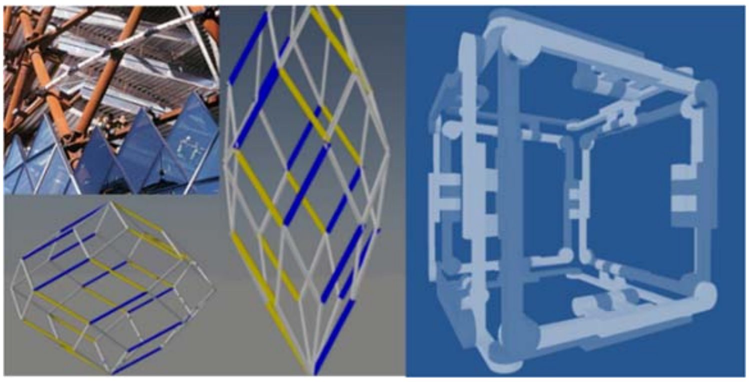

Figure 3: An infinitesimally rigid structure built on a polar zonohedron frame above. Below and besides are two phases of the motion of a simpler polar zonohedron. A construction that provides a zonohedron-like movement to the right in the case of a simpler zonohedron as the cube.

Very few descriptions in the literature use cables or struts in the case of mega diagonals; instead, intelligent actuators come to the fore [9,10]. They are not yet used instead of mega diagonals.

Some architects looking for a challenge look for the source of their shapes in nature. Thus, it is no coincidence that biological structures come into the field of vision of our architects, and this is now an area researched not only by architects and structural engineers but also in materials science. A new branch of science, structural biology, has emerged, which plays a significant role not only in organ tissues but also in the mechanical characterization of the more minor elements that make them up, such as collagens and associated calcites, as well as the cross-connections of microtubules and actin fibers found within cells [13,14].

In the presentation, we can describe the movement of three-dimensional grid-like rod-hinged structures using constraints called tensegrity elements (rods, struts, cables). Therefore, we state some necessary and sufficient conditions between the location and position of tensegrity elements and the movements and stiffness of the structures. We show applications and mechanisms to get to know the traces, skeletons, and essence of the natural and artificial works of materials science and construction, and biology.

The materials of our structures are not perfect, so we neglect them when describing them, on the one hand, so that we can describe the structures' behavior, movement, stiffness, and safety. On the other hand, it is because simulation finite element programs are the helpers of statics for determining deformations. To use finite element programs, the structure to be calculated must be rigid in that real elements are replaced by ideal constituent elements. The system is ideal if, even with arbitrary force and torque, the length of the bars does not change, does not bend, does not twist, the distance between the endpoints of the struts does not decrease, and the length of the cables does not increase during the permitted movements. Finite element programs determine the displacements for rigid structures consisting of ideal constituent elements from the given physical and geometrical properties of the real structural constituent elements (which can no longer be considered ideal) and based on the load. If the geometry of the structure is variable, i.e., the coordinates of the joints; and the position of the bracing rods, struts, and cables connecting specific pairs of these varies, then it is essential to quickly decide whether the given structure is infinitesimally rigid, geometrically rigid, or not rigid, i.e., it can be considered a mechanism. Impulses, forces, and gravity between structural elements can be dispensed with. Suppose we can provide a fast algorithm for determining the stiffness of such structures, which is linear depending on the number of input data, or at most a quadratic step number. In that case, we can say that we have given a good characterization of the stiffness task. This applies to the result of [1,2], according to which the definition of stiffness is the same as the number of steps required to calculate the rank of the stiffness matrix of the rod joint structure, uses Gaussian elimination for an $n \times n$ matrix $O(n^3)$.

3 Results

3.1 Rhombus tiling braced with rods or tensegrity elements

In 2010, we investigated the stiffness of rhombus tiling structures [15-17]. If we place ideal hinges at the vertices of a rhombus tiling or mosaic and ideal rods at the sides, we can characterize the movements of the resulting structure using the movements of some vectors. We were looking for what allowed movements are possible with the arrangement of the diagonals placed in the diagonals of the rhombus. We chose a similar model as [18,19]. In 2016, this result was improved in two directions. Instead of diagonal braces as bars, we allowed tensegrity elements. Using the results of Recski and Shai, we also found a mutually clear correspondence between the possible movements of the structure and the connection of the bracing graph. The generalization in the other direction characterizes hierarchical structures, for which the sample is e.g. the location of the cellulose bundles located in wood and the location of collagen fibers can be found in bony tissues [14]. We will apply the results of rhombus tiling rigidity to the stiffening of planar scaffolding with cables and struts.

3.2 Bracing scaffolding with rods

A good description of the stiffening of scaffolding can be found in [20], in which we also examined more general structures; we allowed to delete joints from the original structure with the rods connected to them so that the line parallel to the support planes fitted to the joints of the remaining structure is a single connected joint-rod-joint rod-...-rod-joint contains a series of joints so that these joints can only move along this line. These cannot have even an infinitesimal displacement in the direction of the support planes since they are connected to them through bars fixed to each other with joints. We called these stairs.

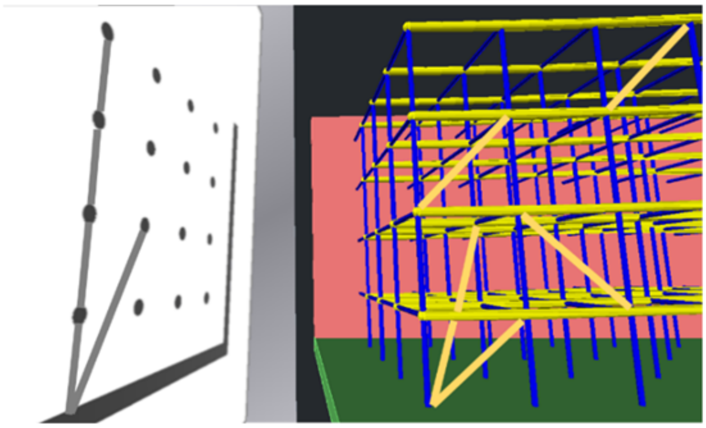

Figure 4. The figure shows a scaffolding on the right, which rests on the horizontal ground (green) and the pink wall. On the left side, you can see the shadow of the hinges and diagonal braces of the structure, as well as the shadow of the ground and wall, the Sun shines from the right side. The connectivity of the graph of the scaffolding stiffening is a necessary and sufficient condition for the infinitesimal stiffness of the structure. This statement is equivalent to the fact that along the shadows we can get from any shadow of the joints to the shadow of the ground or the wall.

The diagonal braces of the structure shown in the Figure 4 can be longer than the diagonal of the face of the unit cube.

3.3 Comparison of finite element analysis

A symmetrically braced 4×4 bar joint structure was investigated [21] with the finite element method under lateral loading. In the [22,23] article, we checked the calculations and calculated the stiffness of a 4×5 structure that is one level higher and therefore more general. In this structure, the realization of the mega diagonal is not trivial, and then we draw some conclusions regarding spatial structures.

We have also performed a finite element analysis of the cable-braced structure, and the results of this consideration will also be presented Figure 5.

Bracing with cables on the right pictures of Figure 5, the deflections will be twice as large as during bracing with rods since the structures are symmetrical about the vertical axis, so half of the symmetrically placed cables are stretched, but their symmetrical image does not work during loading, while all of the rods are deformed, stretched or is compressed.

3.4 Comparison of finite element analysis

The description of the directed graph concept of bracing can be seen in Figure 6. In the picture on the left side of the figure, there are three stiffening elements: cable J_{212},J_{153}; trust J_{222},J_{233} and rod J_{242},J_{323}.

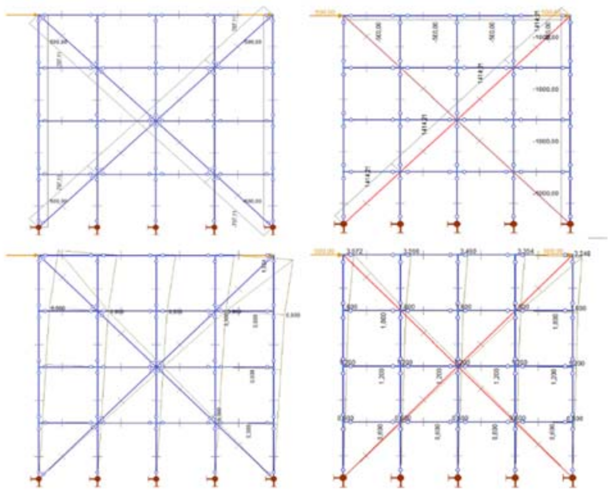

Figure 5. Loads of a scaffolding structure are indicated by the yellow arrows. The diagonals of the left structure have rods; the diagonals of the right structure have cables. Below you can see the deformation caused by the load. In the upper right stress diagram, there is no compressive stress in the cables connecting the upper left corner of the squares with the lower right corner, in contrast to the case on the left side, where there is compressive stress since we used rods for bracing.

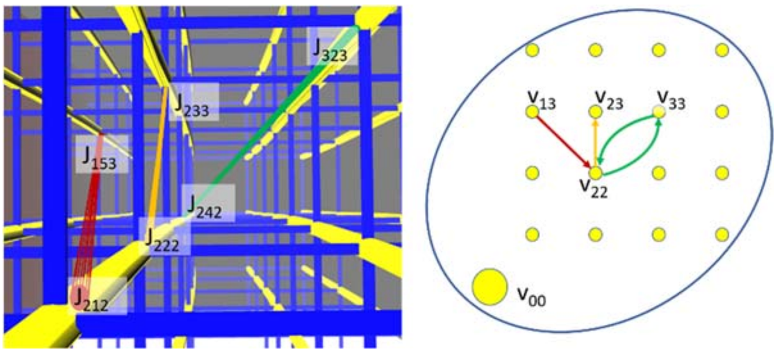

Figure 6. On the left side, you can see three braces, which represent constraints on the lateral displacement of the structure, i.e. perpendicular to the plane of the figure, since the wall is on the left side and is not visible. On the right side of the figure you can see the graph of the bracing of the scaffolding. The direction of the arrows is shown by the fact that if the joint with the smaller middle index is attached, the other joint can move closer in the case of a cable (burgundy), move away in the case of a strut (yellow) and move away in the case of a pole. Both arrows are kept because the distance does not change.

Bracing with cables, struts and bars, the strong connectedness, i.e., all points are reachable from all points along directed edges, of the directed bracing graph guarantees the infinitesimal stiffness of the system.

The above results, determining the strong connectedness of a directed graph instead of the original bar joints framework rigidity provide the computational complexity of deciding the strong connectedness of a directed graph at most quadratic to deciding the stiffness of the bar joint structure, which is at least cubic as a function of the number of bracing elements.

4 Conclusions and Contributions

Parallel to the previously mentioned theoretical design, finite element methods are mostly used to determine static stresses. However, the detailed planning and execution are much more challenging compared to traditional systems since the technical connection of the mega diagonal elements and their cooperation with the original structure has not yet really crystallized. In particular, the nodal solutions of structures can make the work of statics more difficult as we can see in [24-23].

This paper considered the infinitesimal rigidity of the Cubic grid framework as Scaffolding braced by cables, struts, and rods.

Applying an appropriately directed graph, a necessary and sufficient connection between the braced Scaffolding infinitesimal rigidity and its bracing directed graph strong connectedness was presented.

We can find an efficient algorithm for deciding the infinitesimal rigidity of 3-dimensional cable strut an rod-braced scaffolding.

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