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Shear deformable beam model for buckling analysis of laminated beam-type structures

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Abstract

This paper introduces a shear deformable numerical model for conducting nonlinear stability analysis of beam-type structures. The incremental equilibrium equations for a straight thin-walled beam element are derived using the updated Lagrangian formulation, taking into account the nonlinear displacement field of cross-sections. This formulation considers both restrained warping and large rotation effects. Shear deformation effects are incorporated into the composite cross-section by considering the coupling effects of bending-bending and bending-warping torsion. The cross-section properties are calculated based on the reference modulus, allowing for the modeling of different laminate configurations. A numerical algorithm is developed to calculate the geometric properties of the composite cross-section. Various material configurations are examined, and several benchmark examples are presented for verification purposes. The obtained results demonstrate that the proposed model is free from shear locking issues and performs reliably.

Keywords: thin-walled, composite cross-section, beam model, buckling, large displacement, nonlinear stability analysis

1 Introduction

Load-bearing composite structures often consist of slender beam structural elements with thin-walled cross-sections. As a result, the response of these optimized structures to external loading becomes considerably more complex, with an increased susceptibility to deformation instability and buckling [1-3]. Instability in beam structures can manifest in various forms, including pure flexural, pure torsional, torsional-flexural, or lateral deformation. Therefore, when designing such structures, it is crucial to accurately determine the limit state of deformation stability, known as the buckling strength.

While analytical solutions are available for simpler cases [4], the need for numerical solutions becomes imperative. The development and application of numerical methods are thus essential. In [5-10], geometric nonlinear analyses of composite beam structures with shear deformation influence are presented. These works also account for bending-bending and bending-warping torsion coupling shear deformation effects, which occur in asymmetric cross-sections where the principal bending and shear axes do not align.

In the authors' previous publication [10], composite frames were presented using geometrically nonlinear beam elements that consider shear deformation effects. However, this study only focused on cross-ply laminated composite structures. The aim of the present work is to perform a large displacement nonlinear analysis of thin-walled beam-type structures, taking into account shear deformation effects and material inhomogeneity in the form of angle-ply laminates. The analysis will rely entirely on the numerical model developed by the authors, and the results will be compared to those obtained from other relevant sources.

2 Methods

To incorporate shear deformation effects, Timoshenko's theory for non-uniform bending and modified Vlasov's theory for non-uniform or warping torsion are employed in this formulation. Additionally, an improved shear-deformable beam formulation is presented in this work, accounting for bending-bending and bending-warping torsion coupling shear deformation effects [5-11]. These effects arise in asymmetric cross-sections where the principal bending and shear axes do not coincide [12]. The beam member is assumed to be prismatic and straight, while external loads are considered conservative and static.

The element's geometric stiffness is derived using the updated Lagrangian (UL) incremental formulation [13,14], incorporating the non-linear displacement field of the cross-section. This field includes second-order displacement terms to account for large rotation effects. Consequently, the incremental geometric potential of the semitangential moment is obtained for the internal bending and torsion moments, ensuring moment equilibrium conditions are maintained at the frame joint, where beam members with different spatial orientations are connected [15,16]. By employing cubic interpolation for deflections and twist rotation, as well as an interdependent quadratic interpolation for slopes and the warping parameter, which considers shear-deformable effects, a locking-free beam element is obtained. This element, known as a super-convergent element, eliminates the need for reduced integration techniques to prevent shear locking [17]. In terms of the incremental-iteration scheme, the generalized displacement control method [16] is utilized, and nodal orientation updating at the end of each iteration is performed using the transformation rule applicable to semitangential rotations [18,19]. The force recovery phase follows a conventional approach [19,20].

To account for material inhomogeneity in the form of a composite cross-section, a separate numerical model is employed for calculating cross-sectional properties. These properties are weighted by the reference modulus [12]. All cross-sectional properties are defined for the middle surface of the cross-sectional branch and are currently applicable only to balanced and symmetrical angle-ply laminates.

3 Results

The computer program THINWALL v.17 has been developed based on the finite element procedure, incorporating all the methods discussed in the previous section. This program is equipped with the capability to handle both linearized and nonlinear stability analyses. For nonlinear stability analyses, the generalised displacement control method is employed, and a small perturbation load is introduced to initiate buckling. This load-deformation approach allows for investigating stability issues by evaluating the structural behavior across the entire range of interest. It includes the pre- and post-buckling phases by plotting the loading of the structure as a function of deformation. This approach provides more reliable information for real structures and loading conditions compared to the eigenvalue approach.

In order to investigate the impact of shear deformability on the stability behavior of the analyzed structural members, two model comparisons are conducted. The first comparison involves ignoring shear deformability effects entirely, denoted as 'SR' in the presented results. The second model takes into account shear deformation effects using the methodology outlined in this paper and is labeled as 'SD' in the presented results. The material under analysis is graphite-epoxy (AS4/3501), with the following properties: longitudinal elastic modulus (E_1) = 144 GPa, transverse elastic modulus (E_2) = 9.65 GPa, shear modulus (G_{12}) = 4.14 GPa, and Poisson's ratio (ν_{12}) = 0.3.

In the first example, a cantilever column with a length of L = 50 cm is considered, subjected to an axial force F. The cross-section shape being analyzed can be observed in Figure 1. For the asymmetric channel profile, the column experiences buckling in a torsional-flexural mode. Each branch of the cross-section is composed of a symmetric and balanced laminate with a $[\theta/-\theta]_{2S}$ stacking sequence, where all plies have the same thickness. Figure 1 depicts the relationship between the buckling load and the ply orientation θ. It can be observed that the structure's buckling strength is highest when the ply angle is 15°.

To conduct the nonlinear stability analysis, a small perturbation force with an intensity of $0.001F$ is introduced, acting in the X-direction. Figure 2 presents the nonlinear convergence study for four different mesh configurations, consisting of one, two, four, and eight beam elements. The results in Figure 2 are normalized by the critical buckling force obtained from the laminate shell solution using NX Nastran software. The critical buckling force obtained for a ply orientation of θ = 0° is F_{CR} = 40.5 kN, and for a ply orientation of θ = 15°, it is F_{CR} = 44.5 kN. It is evident that the

SD model accurately identifies the buckling state across all the mesh configurations used, while the SR model overestimates the buckling strength by approximately 10%.

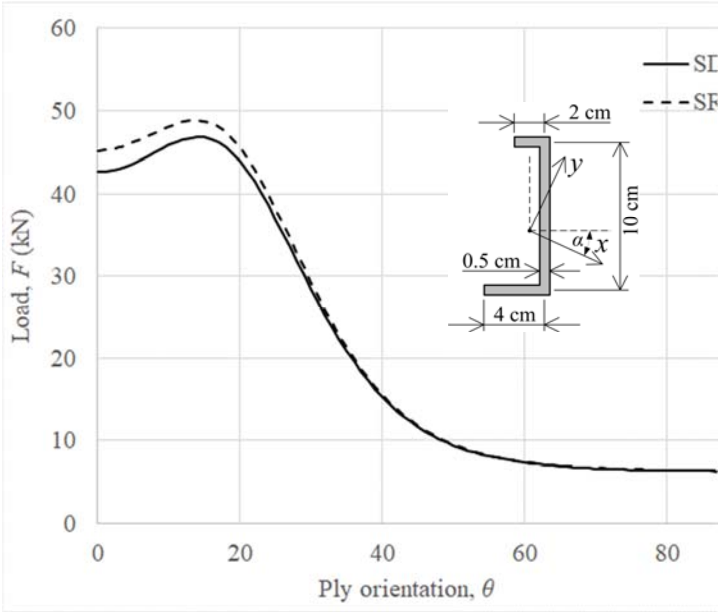

Figure 1: Cross-section shape and buckling load versus ply orientation (θ) for the cantilever column.

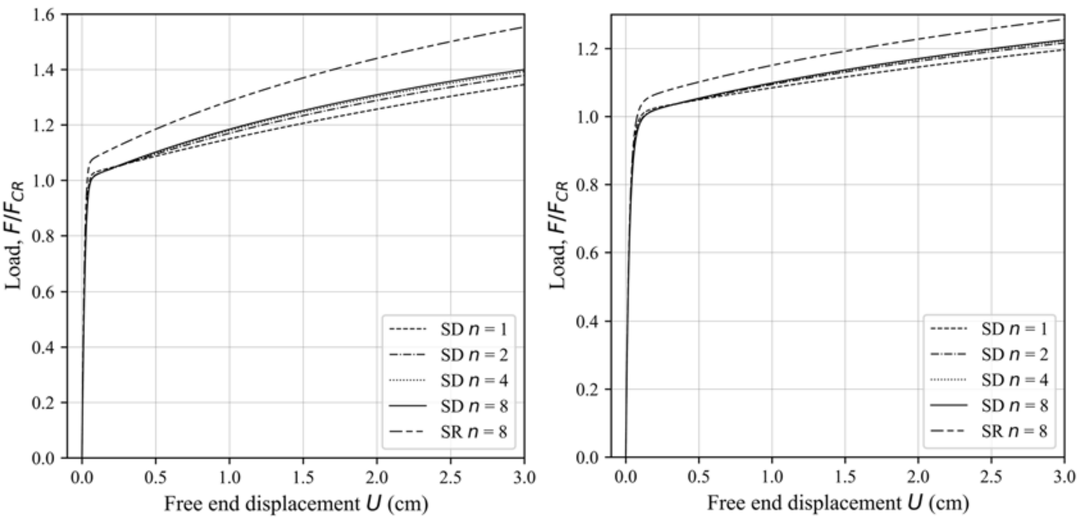

Figure 2: Convergence analysis for the cantilever column, prebuckling response. Left: Ply orientation $\theta = 0$. Right: Ply orientation $\theta = 15°$.

The second example involves an L-frame subjected to a load F in the Y-direction, applied through the centroid of the cross-section at the free end, as depicted in Figure 3. A mono-symmetric channel profile is utilized, and it is assumed that full warping restraint is present at the frame corner. In the case of the mono-symmetric channel profile, the frame experiences buckling in a lateral-torsional mode.

In this example, each branch of the cross-section is composed of a symmetric and balanced laminate with a $[\theta/-\theta]_{2S}$ stacking sequence, where all plies have the same thickness. Figure 4 illustrates the relationship between the buckling load and the ply orientation θ. Similar to the first example, it is observed that the buckling strength is highest when the ply orientation is 15°. However, in this case, the influence of shear deformations is significantly greater, resulting in a more pronounced effect on the buckling behavior.

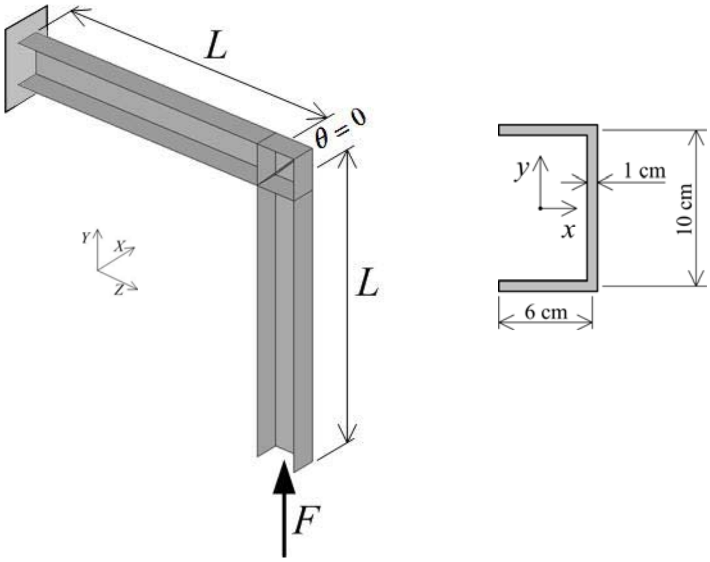

Figure 3: L-frame configuration under a load (F) in the Y-direction at the free end.

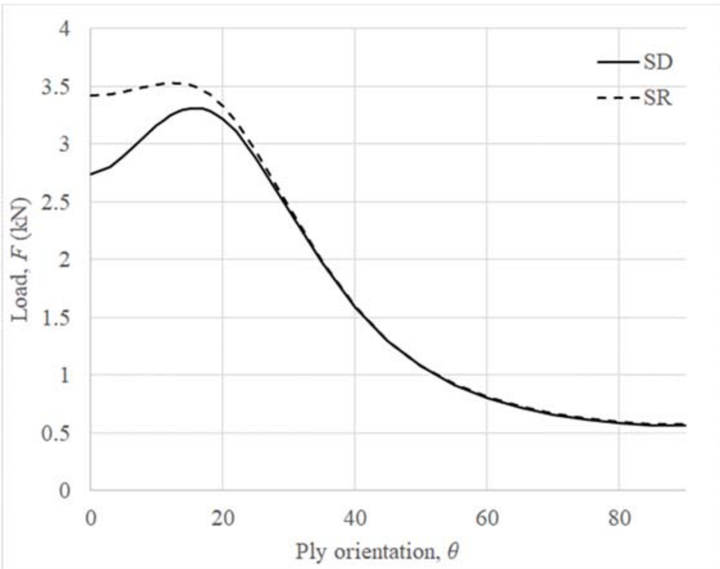

Figure 4: Buckling load versus ply orientation (θ) for the mono-symmetric channel profile in the L-frame.

For the nonlinear stability analysis, a small perturbation force with an intensity of $0.001F$ is applied in the X-direction. The results presented in Figure 5 and 6 are normalized by the critical buckling force obtained using the laminate shell solution from NX Nastran software. The critical buckling force obtained for a ply orientation of $\theta = 0°$ is $F_{CR} = 2.77$ kN, and for a ply orientation of $\theta = 15°$, it is $F_{CR} = 3.39$ kN.

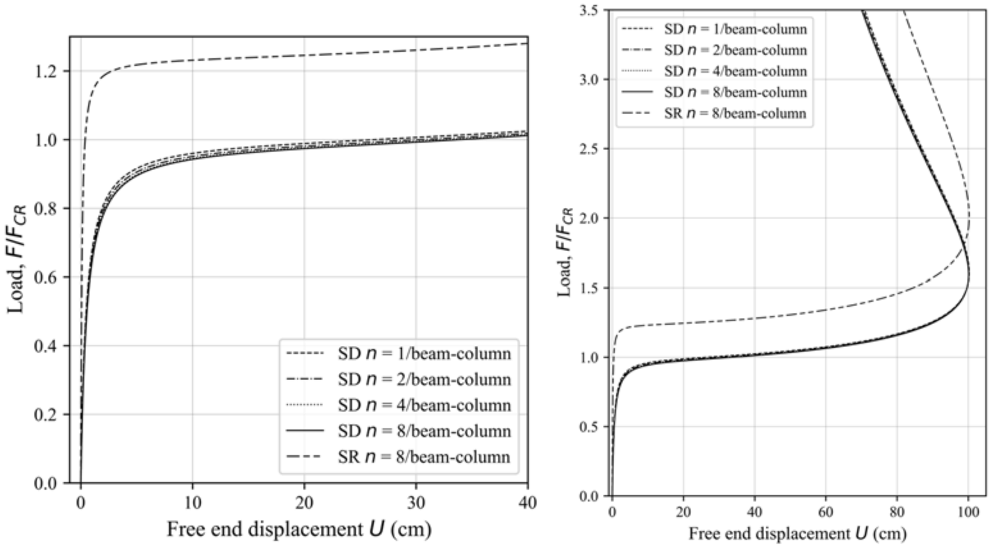

Figure 5: Convergence analysis for the L-frame ($\theta = 0°$). Left: Prebuckling response. Right: Postbuckling response.

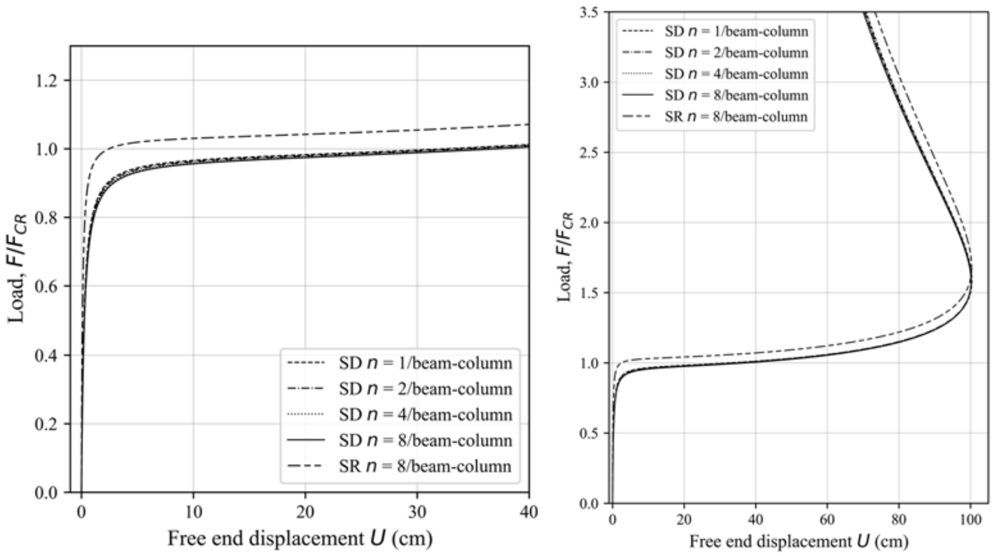

Figure 6: Convergence analysis for the L-frame ($\theta = 15°$). Left: Prebuckling response. Right: Postbuckling response.

Figure 5 and 6 showcase the nonlinear convergence study for four different mesh configurations, with each configuration consisting of one, two, four, and eight beam elements per frame member. It is evident that the SD model accurately identifies the buckling state across all the mesh configurations used, while the SR model significantly overestimates the buckling strength.

4 Conclusions and Contributions

A refined shear-deformable beam formulation has been proposed for the geometrically nonlinear stability analysis of composite frames. This formulation takes into account the effects of shear deformation resulting from the non-uniform bending and torsion of thin-walled beams with asymmetric cross-sections. Benchmark examples have been presented to validate the model's performance. The significance of considering shear deformations in the formulation is evident from the notable reduction in stability strength compared to models that neglect shear effects.

The developed model provides flexibility in controlling the warping restraints at nodes. This can be achieved either globally, by enforcing warping prevention for all beam elements connected at a common joint, or locally, by utilizing warping transformation parameters that represent the local warping restraint conditions for specific beam elements at a node.

To ensure the accuracy and reliability of the model, shear-locking tests have been conducted using different mesh configurations. These tests confirm that shear locking does not occur in the model, indicating its effectiveness in capturing shear deformation behavior.

Additionally, the influence of different material configurations has been analyzed, and their impact on the critical load has been demonstrated and verified through selected examples.

Acknowledgements

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