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A beam model for the buckling analysis of functionally graded open-section beams under thermal loads

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Abstract

A beam model for thermal buckling analysis of thin-walled functionally graded (FG) open-section beams is presented. The Euler-Bernoulli-Navier bending theory and Vlasov torsion theory are employed. The finite element equilibrium equations are developed by updated Lagrangian formulation considering a non-linear displacement cross-section field that includes the effects of warping torsion and large rotations. Material properties are assumed to be graded across the wall thickness and considered as a function of temperature. Three cases of the temperature distribution across the thickness of the cross-section walls are considered, which are uniform, linear and nonlinear, and linear temperature distribution along the beam length. The numerical results for thin-walled FG beam with I-section and channel-section are obtained to investigate the effects of various values of power law index *p,* FGM configurations and different types of boundary conditions, clamped-clamped (CC), clamped-simply supported (CS), and simply supported (SS), on the critical buckling temperature and post-buckling behaviour. The accuracy and reliability of the beam model are tested by comparison with the results obtained by applying shell finite element models from established packages. It is shown that all of the mentioned effects affect the thermal buckling analysis of open-section beams.

Keywords: thermal buckling, post-buckling, numerical analysis, functionally graded material, thin-walled, temperature distribution

1 Introduction

Thin-walled beams and structures are increasingly used in engineering branches as independent elements or as part of more complex structures due to their high strength-to-weight and stiffness-to-weight ratio. However, these structures show susceptibility to buckling failure [1,13]. Therefore the analysis of buckling and post-critical buckling response of such structures has been the subject of many researchers [10,11,14]

FG materials are a relatively new class of composite materials originally developed by Japanese researchers in the mid-1980s [7] and have begun to be intensively developed and used in various constructions in the last three decades due to the numerous advantages that such materials provide. With the development of industry and modern production processes, structural elements are increasingly found in the environment under high temperatures. FG materials, as a combination of ceramics and metal, show outstanding mechanical properties, especially thermal and corrosion resistance. Many papers have considered the buckling of FGM structures, but only a few of them are cited here [2,4,5,8,9,16]. In contrast to the large number of works related to the thermal buckling of solid sections, the literature investigating the thermal buckling of thin-walled structures is rather scarce [6,17].

In the present work, a beam model for the thermal buckling of FG thin-walled open-section beams is discussed. The model is based on Euler-Bernoulli-Navier bending theory and Vlasov torsion theory, assuming large displacement and small strains. The equilibrium equations of the finite elements are developed by an updated Lagrangian formulation. As an incremental iterative solution scheme, the Newton-Raphson method is used. Material properties are assumed to be graded across the wall thickness. Three cases of the temperature rise over wall thickness are considered, which are uniform, linear, and nonlinear, such as linear distribution along the beam length. The numerical results are obtained for FG beams with different boundary conditions, FGM configurations, and temperature distributions to investigate the effects of the power-law index on the critical buckling temperature and post-buckling response.

The main objective of the paper is to present the developed beam model for the thermal buckling analysis of FG thin-walled beam structures considering open cross-sections and to discuss the influence of the temperature distribution and FGM on critical temperatures. The analysis is based on the numerical model developed by the authors [3,15] and verified by benchmark shell examples.

2 Methods

Assume that the beam is made of a functionally graded material. The material properties are assumed to vary continuously through the wall thickness according to the power law distribution [12]:

$$P\,(n,T)=[P_o\,(T)\,-P_i\,(T)\,)]\cdot\,V_c\,(n)+P_i\,(T) \tag{1}$$

where P represents the effective material property such as Young's modulus E, shear modulus G, and coefficient of thermal expansion α, respectively. The subscripts i and o represent the inner and outer surface constituents while V_c is the volume fraction of the ceramic phase and there can be many variants of material distributions across the wall thickness. In this work, the material properties are assumed temperature-independent.

The stress-strain relation in terms of generalized Hooke's law can be written as follows:

$$\sigma_z = E\,(n,T) \cdot [\varepsilon_z - \alpha\,(n,T) \cdot \Delta T],$$
$$\tau_{zs} = G\,(n,T) \cdot \gamma_{zs} \tag{2}$$

where σ_z and τ_{zs} are stress components, ε_z, and γ_{zs} are strain components; n, s denote the flange normal and transverse directions, while z is parallel to the beam axis; ΔT is a temperature change.

The beam is subjected to a uniform, linear, and nonlinear temperature distribution over the beam wall thickness as well as linear along the beam length. In the case of linear temperature rise along the beam length, if the axial beam displacements are prevented, the temperature can be defined as

$$T\,(z) = T_A(z) \cdot (1 - z/L) + T_B(z) \cdot z/L, \tag{3}$$

where indexes A and B represent the end nodes of the beam while L is a beam length.

3 Results

Consider a thin-walled FG I-section beam with the length l = 6 m, height h = 0.2 m, width b = 0.1 m, and wall thickness t = 0.005 m, Fig 1. The beam is subjected to linear distributed temperature change along the beam length.

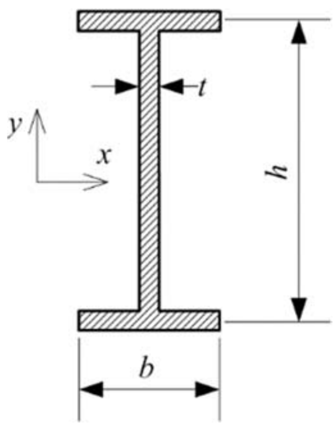

Figure 1: I-section beam.

The web of the I-section is made of ceramic core and FG skins in the ratio 3:4:3. The top skin varies from a ceramic-rich to a metal-rich surface, while the bottom skin varies from a ceramic-rich to metal-rich surface. The volume fraction of the ceramic V_c can be given by:

$$V_c = [(n - t_3)/(t_2 - t_3)]^p, \quad t_2 \leq n \leq t_3,$$
$$V_c = 1, \quad t_1 \leq n \leq t_2,$$
$$V_c = [(n - t_0)/(t_1 - t_0)]^p, \quad t_0 \leq n \leq t_1. \tag{4}$$

Flanges of the I-section are made of ceramic-rich bottom skin and FG skin at the top in the ratio 3:7. V_c can be determined as follows:

$$V_c = 1, \quad t_0 \leq n \leq t_1$$
$$V_c = [(n - t_3)/(t_1 - t_3)]^p, \quad t_1 \leq n \leq t_3 \tag{5}$$

Two different FG materials were considered: ZrO_2/SUS304 and ZrO_2/Ti-6Al-4V. The ceramic component is ZrO_2 (E_c = 168 GPa, $\alpha_c = 1.86e^{-5}$ 1/°C), and the metal is either SUS304 (E_m = 207 GPa, $\alpha_m = 1.53e^{-5}$ 1/°C) or Ti-6Al-4V (E_m = 105 GPa, $\alpha_m = 6.94e^{-6}$ 1/°C). The Poisson's ratio is assumed to be constant $\nu = 0.3$.

The thermal buckling of the beam was analysed for three different boundary conditions (clamped-clamped, simply supported, and combination) and different values of the power-law index p. The results of the author's beam model were verified with a numerical model based on shell finite elements. FG material was simulated by homogeneous layers. Table 1 shows a comparison of the critical temperatures obtained with the beam and shell model for different boundary conditions and different power-law values p. It can be seen that the current solutions are in excellent agreement with the results of the shell model.

BC		C-C		C-S		S-S	
p	*Material*	*Shell*	*Present*	*Shell*	*Present*	*Shell*	*Present*
0	ZrO_2/SUS304	49,03	49,28	25,05	25,17	12,15	12,30
	ZrO_2/Ti-6Al-4V	49,03	49,28	25,05	25,17	12,15	12,30
0,5	ZrO_2/SUS304	50,01	50,17	25,55	25,63	12,39	12,52
	ZrO_2/Ti-6Al-4V	54,82	54,71	28,01	27,95	13,58	13,66
1	ZrO_2/SUS304	50,51	50,66	25,8	25,89	12,51	12,65
	ZrO_2/Ti-6Al-4V	57,58	57,4	29,14	29,32	14,26	14,33
5	ZrO_2/SUS304	51,60	51,76	26,36	26,44	12,78	12,92
	ZrO_2/Ti-6Al-4V	63,63	63,29	32,5	32,33	15,74	15,8
10	ZrO_2/SUS304	51,86	52,01	26,49	26,57	12,85	12,98
	ZrO_2/Ti-6Al-4V	65,12	64,72	33,26	33,06	16,11	16,16

Table 1: Critical buckling temperatures

To initiate buckling, a lateral perturbation force $\Delta F = 0.001F$ is applied incrementally in the x-direction at the mid-span for C-C and S-S, and at $0.7L$ from the fixed end, where the largest displacement is expected, for C-S. Figure 2 shows a graphical comparison of the critical temperatures for the clamped- simply supported beam and power-law index $p = 10$ considering both FG materials. Material containing metal SUS304 achieves lower buckling temperatures. The diagram shows a good agreement between the nonlinear response curve and the eigenvalues.

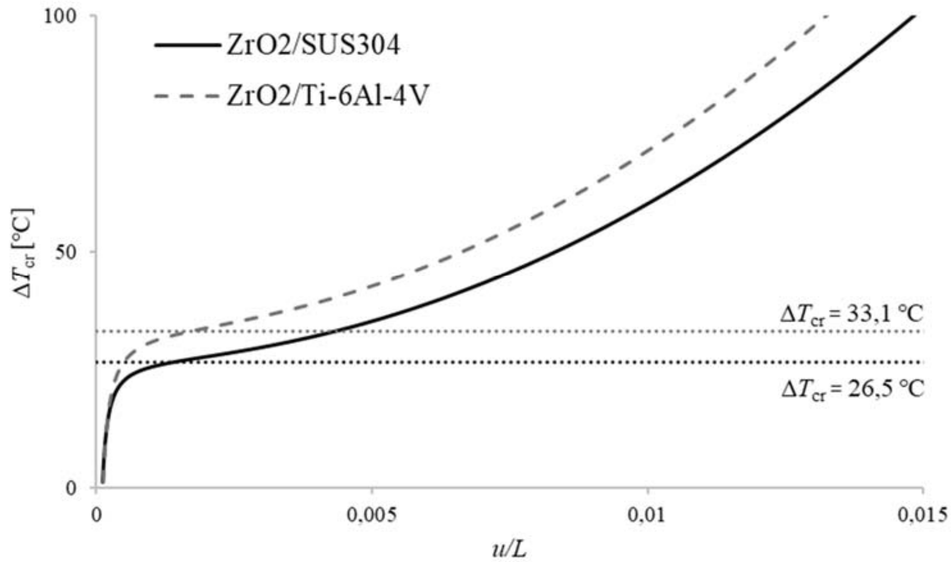

Figure 2: Lateral displacement vs. temperature for different FG materials

4 Conclusions and Contributions

A beam model for thermal buckling analysis of thin-walled FG open-section beam is presented. For various boundary conditions, the influence of the power-law index magnitude on the critical buckling temperature and post-buckling responses is observed. The efficiency of the proposed algorithm has been tested with benchmark examples.

The critical buckling temperature increases with increasing the power-law index p and this relationship is recognised for all boundaries considered. As expected, the beam clamped at both ends has the highest thermal buckling resistance, and the beam with simply supported ends has the lowest.

The authors' further research activities in this area will focus on the extension of a numerical model to simulate the thermal buckling of FG beam-type structures and frames subjected to different temperatures taking into account temperature-dependent material properties.

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