



Proceedings of the Seventeenth International Conference on
Civil, Structural and Environmental Engineering Computing
Edited by: P. Iványi, J. Kruis and B.H.V. Topping
Civil-Comp Conferences, Volume 6, Paper 8.2
Civil-Comp Press, Edinburgh, United Kingdom, 2023
doi: 10.4203/ccc.6.8.2
©Civil-Comp Ltd, Edinburgh, UK, 2023

Uncertainty of material parameters for clay and its influence on simulation results

T. Janda

**Czech Technical University in Prague
Prague, Czech Republic**

Abstract

The paper presents how the uncertainty of material parameters of fine grained soils propagate to the results of numerical simulation. In particular, the posterior distribution describing the uncertainty of the five basic parameters of the hypoplastic model for clay is considered in two scenarios. In the first scenario, only the marginal distributions material parameters are considered independently and no correlation between them are assumed. In the second approach the complete multivariate posterior distribution is assumed in which the parameters are mutually correlated. This posterior distribution shows relatively low correlation between most of the parameter pairs with exception to the slope of primary consolidation line φ and its intercept N . The significantly higher variance of the simulation results obtained for the uncorrelated marginal distribution show that the correlations within the posterior distribution need to be considered in stochastic geotechnical simulations.

Keywords: model calibration, parameter uncertainty, constitutive model for clay, stochastic simulation, triaxial shear test, oedometer test

1 Introduction

A numerical analysis of geotechnical structure typically combines data from various areas of geotechnical survey. These data often show some degree of uncertainty. The goal of a stochastic analysis is to propagate the uncertainty of the input variables to the quantities of interest, i.e. structure deformation or distribution of internal forces.

The uncertainties of the data entering a simulation of geotechnical problems come of number of different sources [1], with the most obvious one being the heterogeneity of soil profiles [2, 3].

Even for homogeneous material, the parameters derived from laboratory or field tests can be relatively scattered [4, 5]. This is given by the variability of the tests themselves as well as the calibration process that interprets the laboratory and in-situ test results and determines the parameters of chosen constitutive law. The choice of the calibration process can also range from relatively straightforward parameter sensitivity analysis [6, 7] to a general optimisation techniques such as machine learning [8].

In some cases, the calibration uncertainty can be directly implemented into the constitutive laws [9] but it is more common to acknowledge the uncertainty directly to the material parameters – or other data entering the simulation – and perform stochastic simulations. The commonly used approaches to stochastic analysis in geotechnics are the Monte-Carlo method [2, 10, 11] and Latin hypercube sampling [12]. Another approaches exploit random sets [13] or crate meta models based either on Bayesian networks [14, 15] or on the polynomial chaos expansions [16, 17].

Out of the many different sources of uncertainty in material parameters this paper focused on one: the uncertainty due to limited laboratory data available in calibration procedure. The term *calibration* here means a deterministic algorithm that consumes the data obtained form laboratory tests and returns material parameters of chosen material model. And since the the user can supply different amount of laboratory data, the material parameters obtained from the calibration procedure also differ.

This uncertainty quantified in form of joint probability distribution is used here as input for numerical simulation of two basic geotechnical test: undrained shear test and 1D compression test also known as oedometric test.

2 Methods

The hypoplastic model for clay [18] with five basic parameters φ , κ , λ , N and ν is considered in this study. The utilised calibration procedure combines a direct parameter determination with optimisation techniques. A full description of the calibration algorithm is provided in [19, 20]. To calculate the parameters of the hypoplastic model for clay the procedure requires the data of at least two different laboratory test. Neverthe-

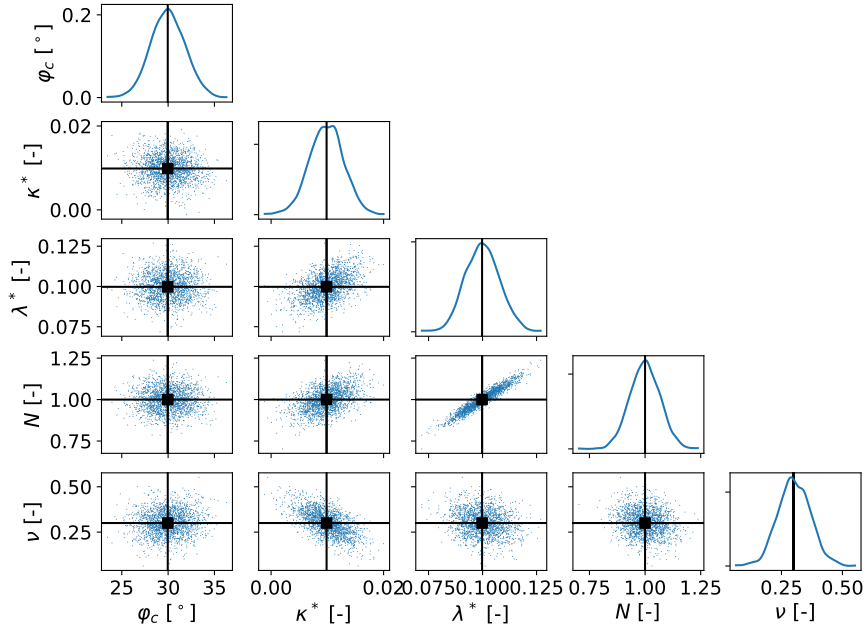


Figure 1: Posterior predictive distribution of five material parameters of the hypoplastic model for clay. The plots on diagonal show kernel densities (smoothed histograms) of marginal distributions whereas the scatter plots visualise the correlations of the parameters.

less, if data from more tests are supplied, all of them are considered equally weighted. As indicated above, this arbitrariness in the choice of calibration input is the source of uncertainty considered here.

A hierarchical stochastic model was developed and its parameters were inferred in [21]. The Bayesian model inference makes easy to obtain posterior predictive distribution of material parameters with known mean. This means that entire joint distribution is obtained for a chosen set of material parameters. A pair plot of five basic material parameters of the hypoplastic model for clay is shown in Figure 1. The points in the scatter plots are the elements of the Markov chains, i.e the direct result of the Bayesian inference, and serve as the input for the numerical simulations.

Two very simple simulations were performed with these scattered material parameters: the undrained triaxial shear test and the constrained 1D compression test. The undrained triaxial shear tests starts from isotropic stress of 100 kPa and the specimen is loaded up to compressive vertical strain of 0.05. The constrained 1D compression test starts from unstressed state and the specimen is loaded up to compressive strain of 0.1.

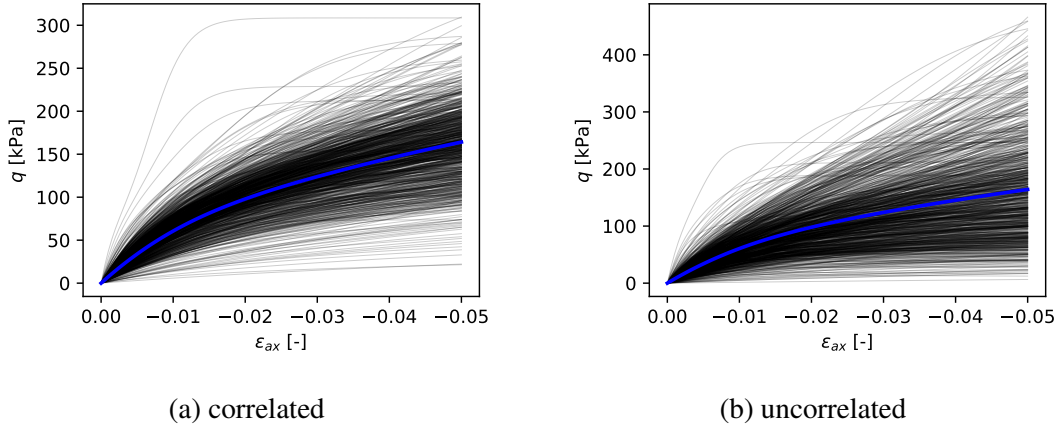


Figure 2: Results of the simulations of undrained triaxial shear tests. Equivalent deviatoric stress (von Mises stress) q is plotted as a function of the axial strain ε_{ax} .

3 Results

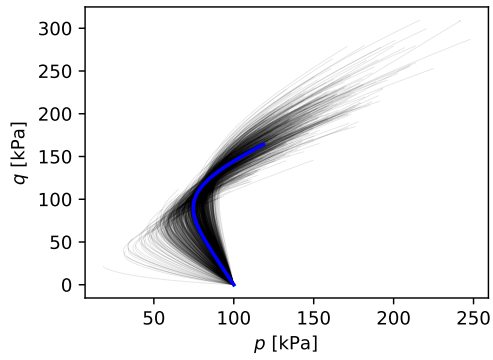
The numerical simulation of the two laboratory tests resulted in scattered curves, as displayed on Figures 2–5. Each curve correspond to a simulation with different set of material parameters and characterise the mechanical response corresponding to the individual point in the posterior predictive distribution on Figure 1. These points are drawn from the posterior predictive distribution around the mean values $\mu_{\varphi_c} = 30^\circ$, $\mu_{\kappa^*} = 0.01$, $\mu_{\lambda^*} = 0.1$, $\mu_N = 1.0$, $\mu_\nu = 0.3$. The blue curve in each figure shows the results of the simulation with these mean values. The particular mean values of μ_{φ_c} , μ_{κ^*} , μ_{λ^*} , μ_N and μ_ν were chosen somewhat arbitrary to represent some typical values for low plasticity clay.

The simulations of the undrained shear tests are plotted in Figures 2 and 3. The results in figures labelled as *correlated* are obtained directly for the points in the posterior predictive chains, i.e. the points displayed in Figure 1. On the other hand, the results in the figures labelled as *uncorrelated* are generated from the marginal densities, i.e. from the 1D distributions displayed on diagonal of Figure 1. The marginal distribution are therefore assumed as uncorrelated.

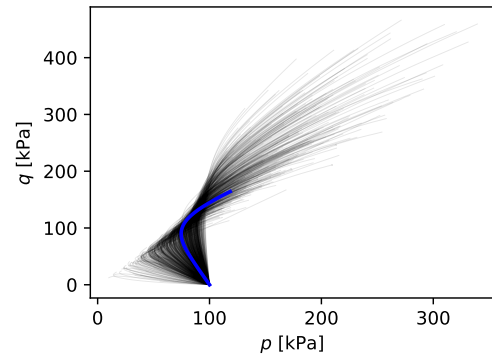
The results obtained for uncorrelated marginal posterior distributions are presented below.

4 Conclusions

Several conclusions can be drawn from the results in this study. First, the results show that the complete joint probability distribution including the correlations between the parameters should be used as an input for stochastic analysis. When independent uncorrelated marginal distribution are used as the input of the stochastic simulations,

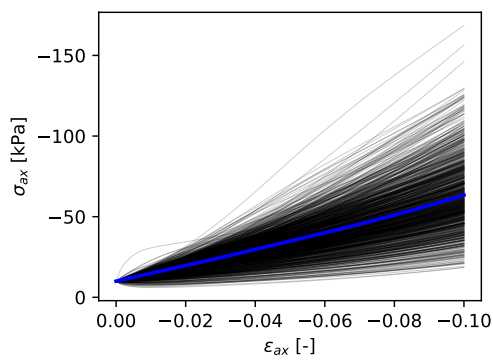


(a) correlated

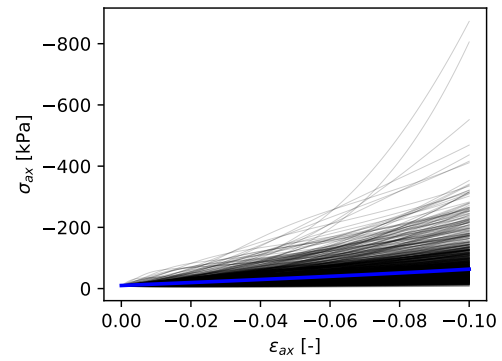


(b) uncorrelated

Figure 3: Stress paths of the simulations of undrained triaxial shear test. Equivalent deviatoric stress (von Mises stress) q is plotted as a function of the mean stress p .



(a) correlated



(b) uncorrelated

Figure 4: Results of the simulations of constrained 1D compression test. Axial stress σ_{ax} is plotted as a function of the axial strain ϵ_{ax} .

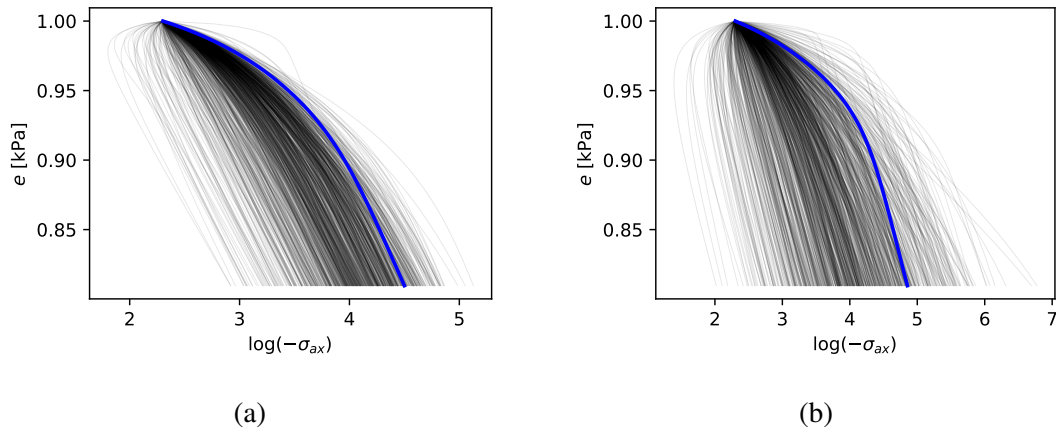


Figure 5: Results of the simulations of constrained 1D compression test. Void ratio is plotted e is plotted as a function of the logarithm of the axial stress σ_{ax} .

the variance of the results, i.e. the dispersion of the stress strain curves and the stress paths, increased substantially. This finding therefore discourages for using any kind of confidence or credible intervals as a sole mean to communicate the uncertainty of material parameters. The correlation coefficients between the material parameters should not be neglected.

Second conclusion is about the theoretical critical state line that in case of the hypoplastic model for clay passes through the origin of $p \times q$ space and touches the end point of the stress path in Figure 3. The variance of the slope of the critical state line is relatively low, when compared to the variance of stress-strain curves in Figure 2.

Finally, it can be concluded especially from Figure 4 and especially from Figure 5 that the mean of the simulation results does not correspond to the simulation results obtained for the mean material parameters. In other words, the blue curve obtained for the "most typical" parameters set is not most typical curve within in the observed bundle. This systematic bias is the consequence of the nonlinear nature of the material model response to loading.

Acknowledgements

The financial support provided by the Czech Grant Agency, project No. 22-12178S is gratefully acknowledged.

References

- [1] J. L. Favre, Errors in geotechnics and their impact on safety, *Computers & Structures* 67 (1) (1998) 37–45. doi:10.1016/S0045-7949(97)00154-5.

- [2] R. Suchomel, D. Mašín, Probabilistic analyses of a strip footing on horizontally stratified sandy deposit using advanced constitutive model, *Computers and Geotechnics* 38 (3) (2011) 363–374. doi:10.1016/j.compgeo.2010.12.007.
- [3] Y. Tao, H. Sun, Y. Cai, Bayesian inference of spatially varying parameters in soil constitutive models by using deformation observation data, *International Journal for Numerical and Analytical Methods in Geomechanics* 45 (11) (2021) 1647–1663. doi:10.1002/nag.3218.
- [4] M. I. Lingwanda, A. Prästings, S. Larsson, D. L. Nyaoro, Comparison of geotechnical uncertainties linked to different soil characterization methods, *Geomechanics and Geoengineering* 12 (2) (2017) 137–151. doi:10.1080/17486025.2016.1184761.
- [5] J. Pieczyska-Kozowska, I. Bagiska, M. Kawa, The Identification of the Uncertainty in Soil Strength Parameters Based on CPTu Measurements and Random Fields, *Sensors (Basel, Switzerland)* 21 (16) (2021) 5393. doi:10.3390/s21165393.
- [6] J. Wesseling, J. Kroes, T. Campos Oliveira, F. Damiano, The impact of sensitivity and uncertainty of soil physical parameters on the terms of the water balance: Some case studies with default R packages. Part I: Theory, methods and case descriptions, *Computers and Electronics in Agriculture* 170 (2020) 105054. doi:10.1016/j.compag.2019.105054.
- [7] J. Wesseling, J. Kroes, T. Campos Oliveira, F. Damiano, The impact of sensitivity and uncertainty of soil physical parameters on the terms of the water balance: Some case studies with default R packages. Part II: Results and discussion, *Computers and Electronics in Agriculture* 170 (2020) 105072. doi:10.1016/j.compag.2019.105072.
- [8] P. Zhang, Y.-F. Jin, Z.-Y. Yin, Machine learning-based uncertainty modelling of mechanical properties of soft clays relating to time-dependent behavior and its application, *International Journal for Numerical and Analytical Methods in Geomechanics* 45 (11) (2021) 1588–1602. doi:10.1002/nag.3215.
- [9] M. Lacour, N. A. Abrahamson, Stochastic constitutive modeling of elastic-plastic materials with uncertain properties, *Computers and Geotechnics* 125 (2020) 103642. doi:10.1016/j.compgeo.2020.103642.
- [10] J. Ching, *Practical Monte Carlo Based Reliability Analysis and Design Methods for Geotechnical Problems*, 2011. doi:10.5772/15136.
- [11] J. Ching, K.-K. Phoon, Effect of element sizes in random field finite element simulations of soil shear strength, *Computers & Structures* 126 (2013) 120–134. doi:10.1016/j.compstruc.2012.11.008.
- [12] R. Boushehri, R. Motamed, K. Ellison, K. Stanton, Estimating epistemic uncertainty in soil parameters for nonlinear site response analyses: Introducing the Latin Hypercube Sampling technique, *Earthquake Spectra* 38 (4) (2022) 2422–2450, publisher: SAGE Publications Ltd STM. doi:10.1177/87552930221101413.
- [13] M. Oberguggenberger, W. Fellin, Reliability bounds through random sets: Non-parametric methods and geotechnical applications, *Computers & Structures*

- 86 (10) (2008) 1093–1101. doi:10.1016/j.compstruc.2007.05.040.
- [14] A. Cañavate-Grimal, A. Falcó, P. Calderón, I. Payá-Zaforteza, On the use of stochastic spectral methods in deep excavation inverse problems, *Computers & Structures* 159 (2015) 41–60. doi:10.1016/j.compstruc.2015.06.009.
- [15] A. Prästings, J. Spross, R. Müller, S. Larsson, W. Bjureland, F. Johansson, Implementing the Extended Multivariate Approach in Design with Partial Factors for a Retaining Wall in Clay, *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering* 3 (4) (2017) 04017015. doi:10.1061/AJRUA6.0000918.
- [16] Q. Pan, X. Qu, L. Liu, D. Dias, A sequential sparse polynomial chaos expansion using Bayesian regression for geotechnical reliability estimations, *International Journal for Numerical and Analytical Methods in Geomechanics* 44 (6) (2020) 874–889. doi:10.1002/nag.3044.
- [17] M. Mohammadi, A. Mosleh, M. Razzaghi, P. Alves Costa, R. Calçada, Stochastic analysis of railway embankment with uncertain soil parameters using polynomial chaos expansion, *Structure and Infrastructure Engineering* 0 (0) (2022) 1–20. doi:10.1080/15732479.2022.2033277.
- [18] D. Mašín, Clay hypoplasticity with explicitly defined asymptotic states, *Acta Geotechnica* 8 (5) (2013) 481–496. doi:10.1007/s11440-012-0199-y.
- [19] T. Kadlíek, T. Janda, M. Šejnoha, D. Mašín, J. Najser, S. Beneš, Automated calibration of advanced soil constitutive models. Part I: hypoplastic sand, *Acta Geotechnica* 17 (8) (2022) 3421–3438. doi:10.1007/s11440-021-01441-0.
- [20] T. Kadlíek, T. Janda, M. Šejnoha, D. Mašín, J. Najser, S. Beneš, Automated calibration of advanced soil constitutive models. Part II: hypoplastic clay and modified Cam-Clay, *Acta Geotechnica* 17 (8) (2022) 3439–3462. doi:10.1007/s11440-021-01435-y.
- [21] Tomáš Janda, Veronika Pavelcová, Alena Zemanová, Michal Šejnoha, Uncertainty in calibration of hypoplastic model for clay attributed to limited number of laboratory tests, *Computers and Structures* (under review) (2023).