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Relative entropy-based reliability assessment of cable structures

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Abstract

This paper is devoted to the reliability assessment of civil engineering cable structures subjected to various sources of environmental uncertainty as well as structural imperfections. Such an analysis is extremely important considering nonlinear character of their structural response, increasing statistical scattering of weather phenomena, small cross-sections and large deformations as well as usually very optimal character. This assessment has been completed using both the First Order Reliability Method (FORM) as well as the new apparatus based on the relative entropy approach proposed in Bhattacharyya mathematical theory. Numerical experiment has been completed here using the 10^{th} order iterative generalized Stochastic Finite Element Method in its displacement formulation.

Keywords: relative entropy, reliability assessment, guyed steel mast, stochastic perturbation method

1 Introduction

Cable structures became popular in various civil engineering applications [1] due to their optimal character, but are still very challenging in the context of designing numerical analysis [2] and also due to difficult and expensive experimental verifications. They gained huge popularity due to their wide applications in steel and composite bridges suspensions systems, roofs suspensions in the sport stadiums, line railways as well as the guys supporting higher masts; some cables serve exceptionally in the pushover tests in the full scale experiments with steel structures.

There is no doubt that they are exposed to extreme environmental conditions, where ice loading becomes important, and even exceptional temperature decrease during hot summer after accidental rainfalls needs to be accounted for. Due to an enormous ratio of the span to the vertical displacements and lack of bending rigidity with remarkable dead load, an efficient verification of the limit states is still an important task [3]. All these aspects make reliability assessment necessary and complex, where a lot of uncertainties should be considered at the same time while using stochastic perturbation method, semi-analytical approaches or one of the sampling techniques [4].

Therefore, the main aim in this work is some new reliability algorithm, which is based on the certain probabilistic divergence model [5]. This has been contrasted with the Cornell First Order Reliability Method (FORM) [6], where the first two probabilistic moments of the structural response have been determined using higher order iterative stochastic perturbation technique. An applicability of this new approach have been demonstrated for elasto-static deformation of some steel trusses [7], and also for elasto-dynamic response of steel halls [8]. Now it is checked in case of cable structures on the example of some high steel guyed mast.

2 Governing equations

The reliability index $\beta=\beta(t)$ is investigated in this work for the solid mechanics problem identified by the following dynamic equilibrium equations system in the domain Ω for the given time interval $[t_0,t_k]$:

- equation of motion

$$\rho\ddot{u}_i = \sigma_{ij,j} + \rho f_i, \quad \mathbf{x}\in\Omega, \quad t\in\left[t_0,t_k\right], \tag{1}$$

- constitutive equations

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}, \quad \mathbf{x}\in\Omega, \quad t\in\left[t_0,t_k\right], \tag{2}$$

- geometrical equations

$$\varepsilon_{kl} = \frac{1}{2}\left(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}\right), \quad \mathbf{x}\in\Omega, \quad t\in\left[t_0,t_k\right], \tag{3}$$

- boundary conditions

$$\sigma_{ij}n_j = \hat{t}_i, \quad \mathbf{x}\in\partial\Omega_\sigma, \quad t\in\left[t_0,t_k\right], \tag{4}$$

$$u_i = \hat{u}_i, \quad \mathbf{x}\in\partial\Omega_u, \quad t\in\left[t_0,t_k\right], \tag{5}$$

- and also initial conditions

$$u_i = u_i^0, \quad \mathbf{x} \in \Omega, \quad t = t_0, \tag{6}$$

$$\dot{u}_i = \dot{u}_i^0, \quad \mathbf{x} \in \Omega, \quad t = t_0. \tag{7}$$

The displacements $u_i(t)$, and stresses $\sigma_{ij}(t)$ are sought from this system.

Gaussian uncertainty sources considered here include an elastic moduli of mast guys, elastic moduli of mast shaft, thermal load uniformly applied to the structure as well as elevation-dependent dynamic wind pressure. All structural elements are modelled according to the Euler-Bernoulli beam theory considering the fact that the mast is designed with the use of tubular structural elements only; so that further simplifications within the initial equations system (1-6) can be provided.

The reliability indices analysed here are some functions of the first two probabilistic moments of the structural response; this analysis is restricted to two cases – the Ultimate Limit State (ULS), and also the Serviceability Limit State (SLS). These indices must fulfil the following inequality throughout the entire time domain:

$$\beta_{SLS} \geq \hat{\beta}_{SLS} \ \wedge \ \beta_{ULS} \geq \hat{\beta}_{ULS}, \quad \mathbf{x} \in \Omega, \quad t \in [t_0, t_k], \tag{8}$$

where $\hat{\beta}_{SLS}, \hat{\beta}_{ULS}$ stand for the admissible values in these two states recommended by the designing code Eurocode 0. According to the FORM one calculates

$$\beta(t) = \frac{E[R - E(t)]}{\sqrt{Var(R - E(t))}}, \quad \mathbf{x} \in \Omega, \quad t \in [t_0, t_k], \tag{9}$$

where $E[.]$ and $Var(.)$ denote the expected values and variances of the given response function, whereas R and $E(t)$ correspond to the structural resistance and extreme dynamical effort. This equation is most frequently presented in shorter form due to a lack of knowledge concerning any correlation function(s) between R and $E(t)$:

$$\beta(t) = \frac{E[R] - E[E(t)]}{\sqrt{Var(R) + Var(E(t))}}, \quad \mathbf{x} \in \Omega, \quad t \in [t_0, t_k]. \tag{10}$$

Some interesting alternative is the Bhattacharyya relative entropy measuring a probabilistic distance in-between two distribution functions (PDFs) of R and $E(t)$ in each discrete time moment. Such a relative entropy quantifies a distance of two different probability distributions [5] and may measure an interference of R and $E(t)$

in this case. It can be represented under the same assumptions by the following relation [9]:

$$
\begin{aligned}
H_B(t) = {} & \frac{1}{4}\frac{\left(E[R]-E[E(t)]\right)^2}{Var(R)+Var(E(t))} \\
& + \frac{1}{2}\ln\left(\frac{Var(R)+Var(E(t))}{2\sqrt{Var(R)Var(E(t))}}\right), \quad \mathbf{x}\in\Omega, \quad t\in[t_0,t_k]
\end{aligned}
\tag{11}
$$

This relative entropy may be relatively easily rescaled to the variability interval of the FORM index using a similarity in-between its first component and the FORM formula to be applicable in practical reliability assessment. The following rescaling procedure has been proposed:

$$
\beta'(t) \equiv \frac{\sqrt{H_B(t)}}{2}, \quad \mathbf{x}\in\Omega, \quad t\in[t_0,t_k]. \tag{12}
$$

An initial dynamic equilibrium equations (1-6) have been discretized using classical procedure inherent in the Finite Element Method where non-linear dynamic analysis has been made; the equations of motion integration has been carried out by the Hilber-Hughes-Taylor (HHT) algorithm [10]. Due to remarkable slenderness of such a structure, and its strong mechanical nonlinearities due to formulation of cable finite elements, some improvement of solution accuracy have been introduced by self-updating of stiffness matrix at each iteration. The HHT algorithm governing equation can be rewritten as

$$
\begin{aligned}
\mathbf{M}\ddot{x}_{n+1} + (1+\alpha_{HHT})\mathbf{C}\dot{x}_{n+1} - \alpha_{HHT}\mathbf{C}\dot{x}_n + (1+\alpha_{HHT})\mathbf{K}x_{n+1} \\
- \alpha_{HHT}\mathbf{K}x_n = \mathbf{F}(t_n + (1+\alpha_{HHT})\Delta t)
\end{aligned}
\tag{13}
$$

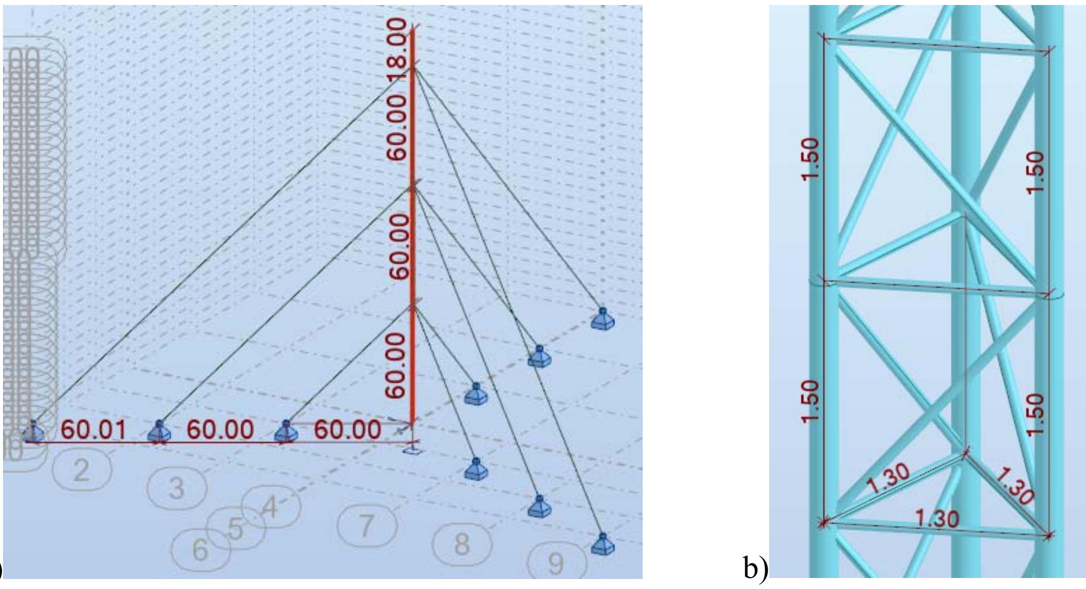
$$
\begin{cases}
x_{i+1} = x_i + \Delta t\cdot\dot{x}_i + (\Delta t)^2\left[\left(\frac{1}{2}-\beta_{HHT}\right)\cdot\ddot{x}_i + \beta_{HHT}\cdot\ddot{x}_{i+1}\right] \\
\dot{x}_{i+1} = \dot{x}_i + \Delta t\cdot\left[\left(1-\gamma_{HHT}\right)\cdot\ddot{x}_i + \gamma_{HHT}\cdot\ddot{x}_{i+1}\right]
\end{cases}
\tag{14}
$$

where **M**, **C** and **K** denote the structural mass, damping and stiffness matrices, Δt is the time increment, **F** is the external forces vector, while $x_i, \dot{x}_i, \ddot{x}_i$ stand for nodal displacements, velocities and accelerations, respectively. The coefficients β_{HHT} and γ_{HHT} in Eqn (14) are directly associated with numerical damping inherent in the HHT method. These parameters are related to the parameter α directly representing damping coefficient in the method, which can be taken from the interval [-0.3,0.0]. Lower bound of the parameter α_{HHT} corresponds to a greater numerical damping, which may prevent accuracy loss even with interferences of vibrations at higher frequencies. Their interrelations are introduced by the following formulas:

$$\beta_{HHT} = \frac{(1-\alpha_{HHT})^2}{4}, \; \gamma_{HHT} = \frac{1-2\alpha_{HHT}}{2} \tag{15}$$

3 Numerical simulation

Numerical model have been created in the Autodesk Robot Structural Analysis (ARSA) software. Numerical model consists of 903 finite elements, where 9 of them refer to the mast guys and they utilize the small-sag formulation of equilibrium state of the cable implemented in the numerical system ARSA. The rest of finite elements have been formulated as classical 2-node bar elements in 3D space and they create the mast shaft geometry. This numerical representation of the mast structure features height of 198.0 meters. Mast shaft members have been designed with S235J2 steel grade whereas mast guys have been modelled as single-strand steel rope 1x37 with a diameter of 32,0 mm that exhibits elastic moduli of 150 GPa and 1960 MPa of mean strength. Mast guys have been inclined by 45 degrees from the vertical orientation of mast shaft. Initial tension of this mast guys have been implemented through the pre-shortening of 11.0 cm, 22.0 cm and 31.0 cm for each attachment level, i.e. 60.0, 120.0 and 180.0 meters. A geometry overview of the entire structure and of the mast shaft have been shown in Figure 1a and Figure 1b, whereas cross-sectional members assignment has been shown in Table 1.



a) b)

Figure 1. Steel mast overall geometry (left – figure 1a) and geometry of mast shaft (right – figure 1b).

This structure has been subjected to the dynamic wind excitation in 10-minutes time interval shown in Figure 2. Structural responses in form of internal forces and nodal displacements have been recovered each 1.0 second of dynamic analysis. Initially performed deterministic approach to the design of the structure have been repeated for 11 different realizations of each uncertain design parameter under consideration so that the probabilistic approach could have been applied subsequently. This randomized parameters have been chosen of environmental and mechanical nature such as elastic moduli of mast shaft, elastic moduli of mast guys, thermal load and wind velocity as well.

Segment no.	Segment height	Legs section	Legs material	Bracing section	Bracing material
1 to 11	6.00 m	CHS 168.3x20	S235J2	CHS 63.5x8	S235J2
12 to 22	6.00 m	CHS 168.3x16	S235J2	CHS 63.5x6.3	S235J2
23 to 31	6.00 m	CHS 168.3x12	S235J2	CHS 63.5x5.6	S235J2
32 to 33	6.00 m	CHS 168.3x8	S235J2	CHS 63.5x3.2	S235J2

Table 1: Cross sections of structural members.

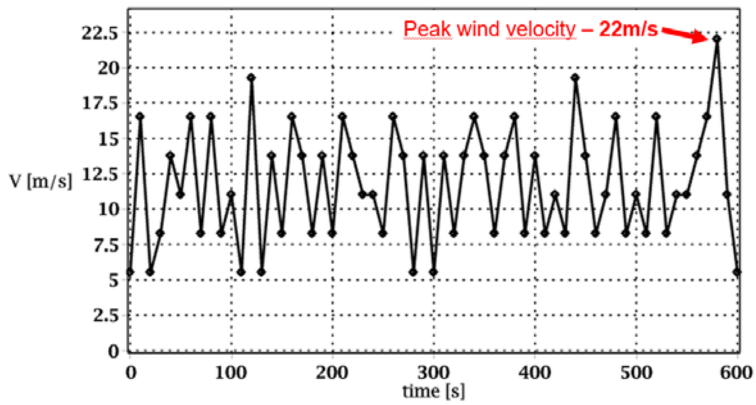


Figure 2. Dynamic wind spectrum applied to the structure.

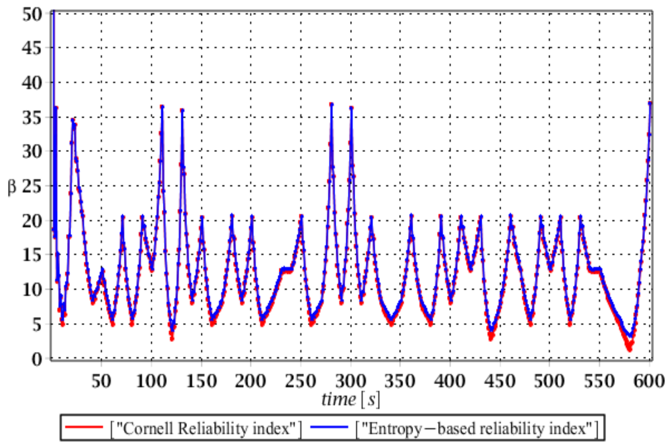


Figure 3. A comparison of the FORM and relative entropy-based reliability indices for the ULS while randomizing wind velocity.

Time fluctuations of both reliability indices have been been collected in Figs. 3-10, where the first four graphs correspond to the ULS analysis (cf. Figs. 3-6), and the remaining four (Figs. 7-10) – to the SLS verification. One compares in turn here an influence of uncertain wind velocity (Figs. 3 & 7), statistical scattering in guys elastic moduli (Figs. 4 & 8), random dispersion of the shaft members elastic moduli (Figs. 5 and 9) with an uncertainty in thermal conditions (Figs. 6 and 10, correspondingly). A very general observation is that both SLS and ULS time fluctuations have different extremes appearing in totally different time moments, even in case of the same input random parameters.

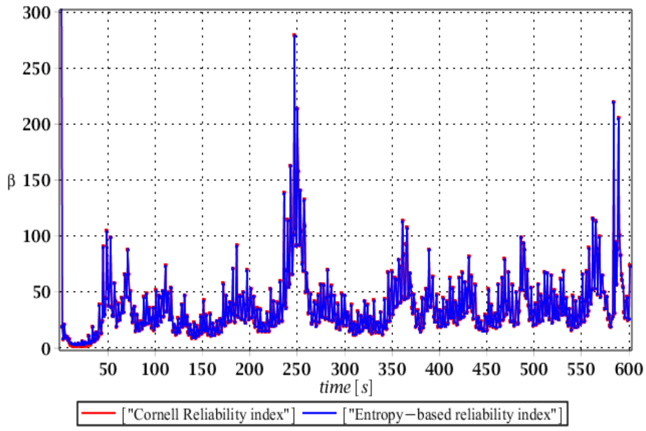


Figure 4. A comparison of the FORM and relative entropy-based reliability indices for the ULS while randomizing mast guys elastic moduli.

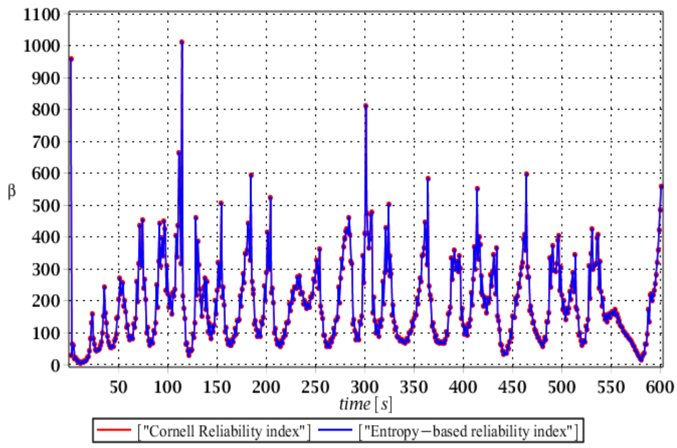


Figure 5. A comparison of the FORM and relative entropy-based reliability indices for the ULS while randomizing shaft members elastic moduli.

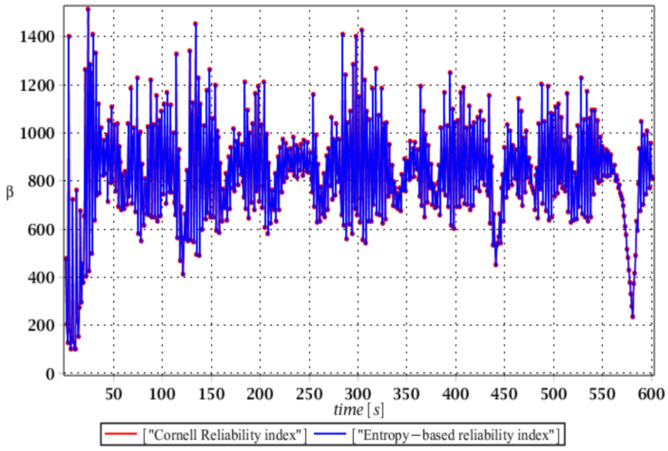


Figure 6. A comparison of the FORM and relative entropy-based reliability indices for the ULS while randomizing thermal load.

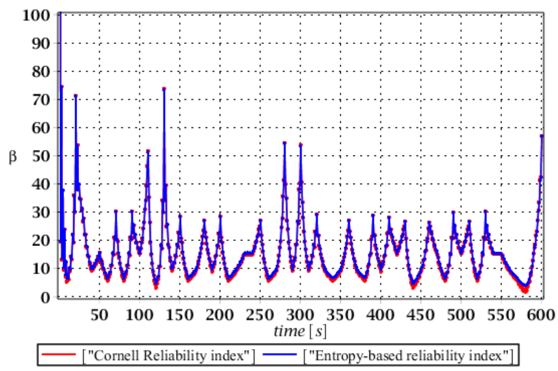


Figure 7. A comparison of the FORM and relative entropy-based reliability indices for the SLS while randomizing wind velocity.

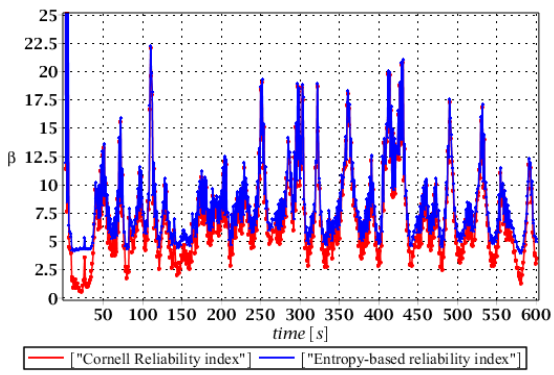


Figure 8. A comparison of the FORM and relative entropy-based reliability indices for the SLS while randomizing mast guys elastic moduli.

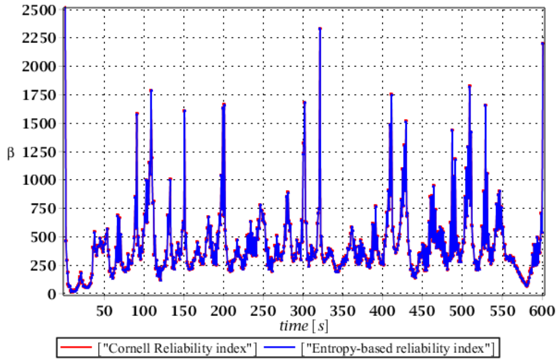


Figure 9. A comparison of the FORM and relative entropy-based reliability indices for the SLS while randomizing shaft members elastic moduli.

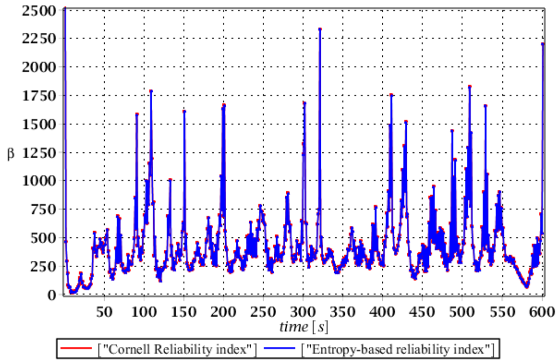


Figure 10. A comparison of the FORM and relative entropy-based reliability indices for the SLS while randomizing thermal load.

All the results contained in Figs. 3-4 show clearly that wind velocity uncertainty is the largest danger for the given mast safety – the reliability indices obtained with both FORM and relative entropy approaches reach minimum value within the given time interval. This remains true for the Ultimate Limit State (ULS), whereas the SLS indices reach minimum value while randomizing cable elastic modulus. Moreover, it is demonstrated that the two probabilistic methods return almost the same results for all uncertainty sources, which means that Bhattacharyya entropy apparatus can have general applicability in reliability assessment.

4 Concluding remarks

Reliability assessment of cables structures has been proposed in this work and discussed on the example of some steel mast using traditional First Order Reliability Method and some non-standard approach following one of the relatives entropy concepts. A coincidence of the FORM and relative entropy based reliability indices is the very promising result, especially in the context of application of higher order iterative stochastic perturbation technique, which minimizes simulation time as well as overall computer effort. Numerical methodology presented here may be employed in structural health monitoring of cable structures [11], whose precision and efficiency may be increased by additional non-destructive experiments.

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References

[1] H.M. Irvine, "Cable Structures". MIT Press, Cambridge, MA, 1981.

[2] M. Crusells-Girona, F.C. Fillipou, R.L. Taylor, "A mixed formulation for nonlinear analysis of cable structures", Computers & Structures, 186, 50-61, 2017. doi.org/10.1016/j.compstruc.2017.03.011.

[3] M. Kamiński, R. Bredow, "Uncertainty analysis for overhead powerlines by the generalized stochastic perturbation technique", ASCE Journal of Aerospace Engineering, 34(4), 04021024, 2012.

[4] J. Song, P. Wei, M. Valdebenito, M. Beer, "Adaptive reliability analysis for rare events evaluation with global imprecise line sampling", Computer Methods in Applied Mechanics and Engineering, 372, 113344, 2020. doi.org/10.1016/j.cma.2020.113344.

[5] C.A. Cornell, "Engineering Seismic Risk Analysis", Bulleting of Seismology Society of America, 58(5), 1583-1606, 1968.

[6] A. Bhattacharyya, "On a measure of divergence between two statistical populations defined by their probability distributions", Bulletin of Calcutta Mathematical Society, 35, 99-109, 1943.

[7] M. Kamiński, R. Błoński, "Analytical and numerical reliability analysis of certain Pratt steel truss", Applied Sciences, 12(6), 2901, 2022.

[8] R. Bredow, M. Kamiński, "Structural safety of the steel hall under dynamic excitation using the relative probabilistic entropy concept", Materials, 15(10), 3587, 2022.

[9] M. Kamiński, "Uncertainty propagation, entropy, and relative entropy in the homogenization of some particulate composites", International Journal for Numerical Methods in Engineering (in press). doi.org/10.1002/nme.7259.

[10] H.M. Hilber, T.J.R. Hughes, "Collocation, dissipation and ‘overshoot’ for time integration schemes in structural dynamics", Earthquake Engineering and Structural Dynamics, 6, 99-117, 1978.

[11] J. Wang, P.E. Stone, D. Coats, Y.J. Shin, R.A. Dougal, "Health monitoring of power cable via joint time-frequency reflectometry", IEEE Transactions on Instrumental Measurements, 60(3), 1047-1053, 2011. doi.org/10.1109/TIM.2010.2058730.