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An eight-dof truss finite element for FRCM reinforced rigid substrates subjected to shear tests

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Abstract

An 8-dof 1D finite element suited to study an FRCM reinforcing system is presented. The element is constituted by three trusses in parallel representing external matrix, central fiber and internal matrix layers, mutually exchanging tangential stresses by means of shear springs applied at the extremes of the element. Non-linearity is possible exclusively in upper/lower matrix layers and at the interface. A linear softening with residual tangential strength is assumed as constitutive law for the interface. Matrix is assumed elastic with linear softening in tension and infinite resistant in compression. The internal matrix layer is connected to the support by means of two elastic springs applied at the element nodes. The 8-dofs are represented by the nodal longitudinal displacements of the three layers plus those of the support. The present paper discusses the derivation of the element stiffness and how to deal with material non-linearities. An application of technical relevance is finally presented, relying into a masonry pillar reinforced with an FRCM package and subjected to a single lap shear experimental test.

Keywords: eight-dof, 1D-finite element, rigid support, FRCM, single lap shear tests

1 Introduction

FRCM is an innovative strengthening system for brittle substrates like masonry or concrete. FRCM is an acronym that stands for Fiber Reinforced Cementitious Matrix. It is an inorganic composite constituted by an internal fiber grid embedded into two external inorganic matrices made with mortar. The experimental characterization of such kind of strengthening system has recently received growing attention in the scientific community, and the specialized literature in the field is nowadays quite wide and comprehensive, see for the sake of example (not-exhaustive) [1]-[3]. It relies into tests conducted on the single materials, on coupons, single and double lap shear tests to evaluate the debonding resistance, and experimentation carried out on real scale structural elements. There are also numerical and analytical contributions, e.g. [4]-[6], aimed at better understanding and interpreting the actual experimental behavior, but still the literature appears jeopardized for what concerns the implementation of specialized finite elements able to describe the role played by the reinforcement in the strengthening of real scale structures and structural elements. This is mainly a consequence of the complexity of the mechanical behavior of such kind of retrofitting. In fact, when tensile stresses are transferred from the substrate to FRCM, several non-linearities arise in the different layers and at the interface between them. Usually, inelastic slippage is observed between matrix and fiber and cracks appear in the external matrix layer. A FE model, to be truly reliable, should be able to represent correctly the global and local behavior, intending with the latter the ability to identify location and amount of cracks in matrix layers and the progressive inelastic slippage at the interface. The present contribution is aimed at proposing a simple truss element with 8 degrees of freedom 8-dof, able to predict with the sufficient level of insight - when used in a standard FE commercial code- non only the global load displacement curves, but also position and extension of all the non-linearities developing during the deformation process. The new Finite Element proposed is a two-noded truss where the reinforcement is modelled with three in parallel layers (external matrix, central fiber and internal matrix) at constant longitudinal stress. Each layer is linked with the contiguous ones by means of shear springs lumped at the nodes, representing the interfaces. The initial and final node of the truss elements are characterized by four displacement variables, namely the longitudinal displacements of the three FRCM layers plus that of the substrate, evaluated in correspondence of each node. Non-linearity is assumed possible exclusively in matrix layers and the two interfaces between matrix and fiber grid. Matrix layers are assumed infinitely resistant in compression and elasto-fragile in tension. The stress-strain behavior is assumed in tension bilinear with finite fracture energy. For interfaces, a trilinear shear stress-slip constitutive relationship is adopted, exhibiting softening after the elastic phase and with possible residual constant tangential strength. Numerically, the non-linearity is dealt with a fully explicit iterative elastic strategy [6]-[10], where the stiffness is progressively reduced when the peak resistance is exceeded at the previous time step. The procedure is preliminary validated in this paper on a flat masonry pillar externally reinforced with FRCM and subjected to a standard single lap shear test. Reference is made to some experimental tests carried out at the University of Florence Ref. Good agreement between numerical prediction and experimental evidences is found, both considering the global and the local behavior of the specimens.

2 Methods

The FE proposed to model an FRCM strengthening system is depicted in Figure 1. As it is possible to notice, it is constituted by three trusses representing the upper and lower matrix layers and the central fiber grid, interacting one each other by means of shear springs representing the interfaces between matrix and fiber.

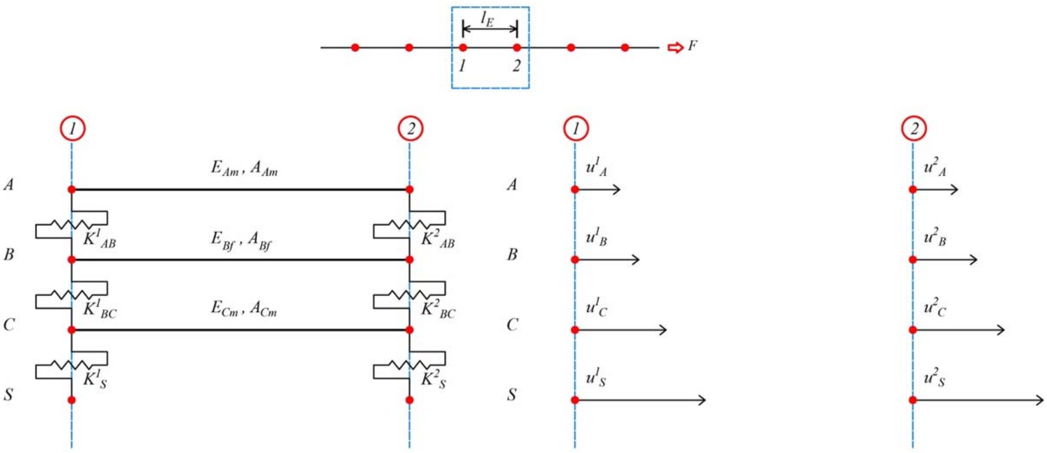

Figure 1: The new FE model proposed for FRCM reinforcement.

A further shear spring is located at the interface between internal matrix layer and support. Assuming that matrix, fiber and interfaces behave elastically (the discussion about the non-linearity of the materials is postponed to the following section), the stiffness matrix of the elements is the following:

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{12} & \mathbf{K}_{22} \end{bmatrix} \tag{1}$$

where:

$$\mathbf{K}_{11} = \begin{bmatrix} \frac{E_{MA}A_{MA}}{l_E} + K_{AB}^1 & -K_{AB}^1 & 0 & 0 \\ -K_{AB}^1 & \frac{E_{FB}A_{FB}}{l_E} + K_{AB}^1 + K_{BC}^1 & -K_{BC}^1 & 0 \\ 0 & -K_{BC}^1 & \frac{E_{MC}A_{MC}}{l_E} + K_{BC}^1 + K_S^1 & -K_S^1 \\ 0 & 0 & -K_S^1 & K_S^1 \end{bmatrix}$$

$$\mathbf{K}_{12} = \begin{bmatrix} -\frac{E_{MA}A_{MA}}{l_E} & 0 & 0 & 0 \\ 0 & -\frac{E_{FB}A_{FB}}{l_E} & 0 & 0 \\ 0 & 0 & -\frac{E_{MC}A_{MC}}{l_E} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

$$
\mathbf{K}_{22} = \begin{bmatrix} \frac{E_{MA}A_{MA}}{l_E} + K_{AB}^2 & -K_{AB}^2 & 0 & 0 \\ -K_{AB}^2 & \frac{E_{FB}A_{FB}}{l_E} + K_{AB}^2 + K_{BC}^2 & -K_{BC}^2 & 0 \\ 0 & -K_{BC}^2 & \frac{E_{MC}A_{MC}}{l_E} + K_{BC}^2 + K_S^2 & -K_S^2 \\ 0 & 0 & -K_S^2 & K_S^2 \end{bmatrix}
$$

In Eq. (2) the symbols have the following meaning:

- E_{MA}, E_{FB}, E_{MC} are upper matrix, fiber and lower matrix elastic moduli respectively;
- A_{MA}, A_{FB}, A_{MC} are upper matrix, fiber and lower matrix cross section area respectively;
- K_{AB}^i, K_{BC}^i, K_S^i are AB, BC and S interface stiffnesses lumped on node i ($i =$ 1,2);
- l_E is the element length.

The degrees of freedom of the element are the longitudinal displacements of layers A, B, C and S computed on nodes 1 and 2. The element proposed is therefore a linear truss element connecting nodes 1 and 2 to the support. The assemblage of the matrix stiffness into a FE code occurs in a standard way and it is not discussed here for the sake of brevity.

3 Material non-linearity

As far as the mechanical properties of the different components are concerned, fiber layer (B) is assumed elastic in agreement with experimental evidence, whereas non-linearities are assigned exclusively to matrix layers (A and C) and interfaces between matrix and fiber (interfaces between layers A-B, and between layers B-C). Matrix uniaxial stress-strain and fiber/matrix interface shear stress-slip constitutive relationships are depicted in Figure 2. For A-B and B-C interfaces a trilinear law with residual tangential strength is assumed; to introduce a residual resistance is beneficial to fit better experimental data, as shown by the improved results obtained in [3] with respect to those obtainable with the same model without it [4].

When dealing with non-linear mechanical properties of matrix and interfaces, an existing iteratively linear procedure is adopted. In particular, the non-linearity is accounted for through a sequence of linear problems, in agreement with some recent literature available [7]-[10]. With reference to Figure 3, the stiffness at the end of each iteration is progressively reduced and updated for the subsequent iteration, projecting the inadmissible stress state on the softening curve, if the elastic limit is exceeded.

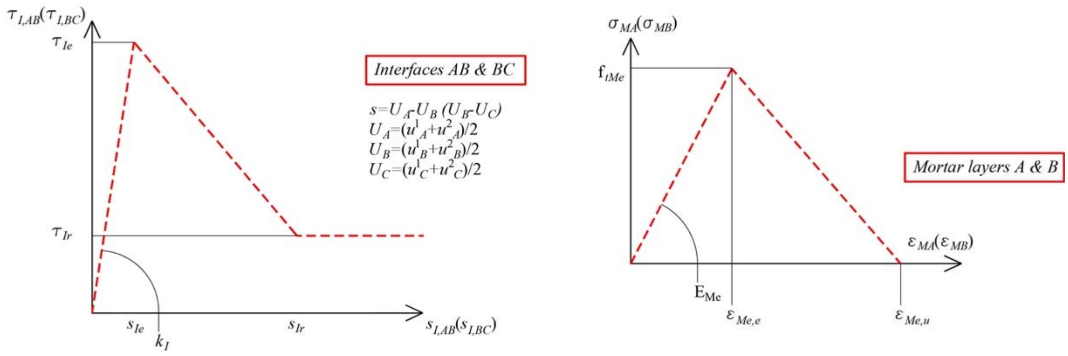

Figure 2: Constitutive relationships adopted for matrix layers A and C and for interfaces A-B and B-C.

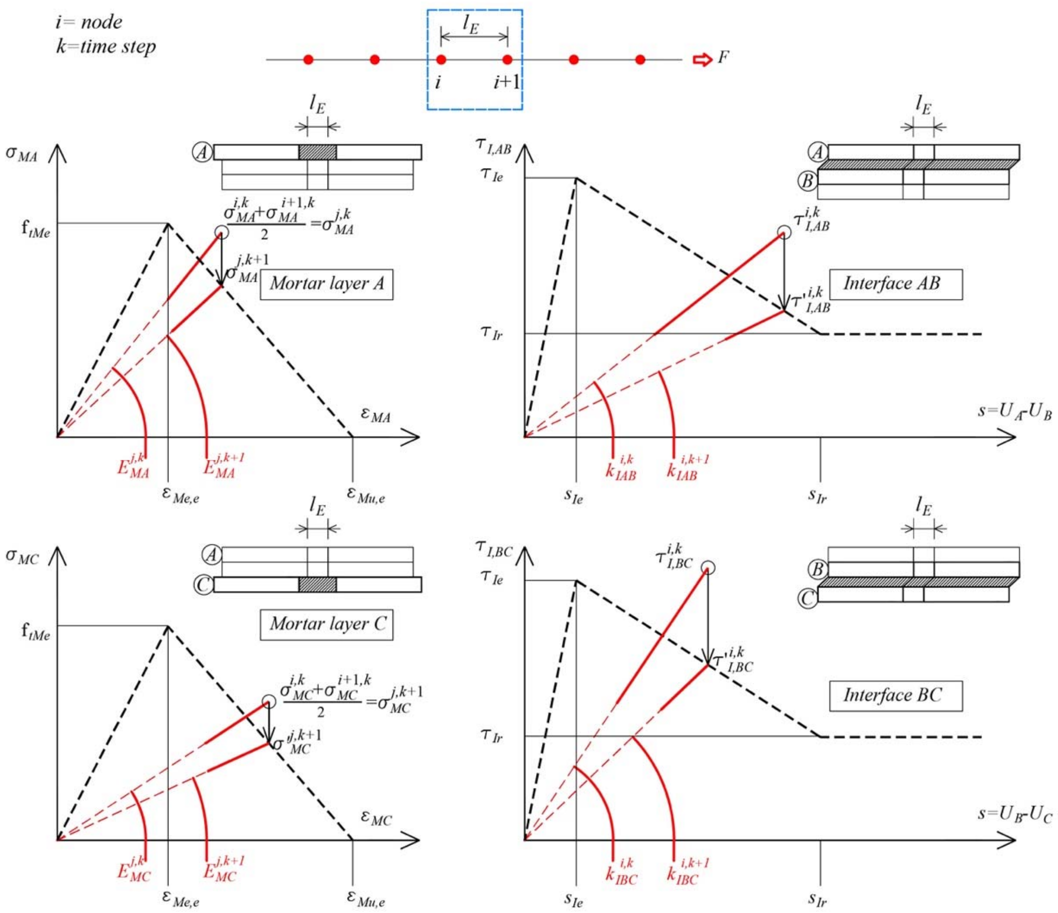

Figure 3: Algorithm to deal with material non-linearity, progressive degradation of element stiffness.

4 Validation

The FE numerical model is here preliminarily validated against some existing experimental data obtained by Rotunno and co-workers in [2]. The same data were then further analyzed in [5] by Grande et al. by means of a numerical model constituted by the assemblage of non-linear springs. Experimental data rely on some flat and curved masonry pillars reinforced with FRCM and subjected to standard single lap shear tests. Here only flat pillars are considered for the sake of conciseness. For the flat case, six replicates were tested. Each sample was obtained piling 5 masonry bricks of dimensions $250x120x65\ mm$ (standard Italian clay brick), interspersed with 4 mortar joints $10\ mm$ thick, then reinforcing one of the external faces with a PBO-FRCM strengthening system of dimensions $8x63x315\ mm$ (thickness, width and length).

The mechanical parameters used to perform the numerical simulations are those adopted in [4] and they are synoptically collected both in Table 1 (properties of matrix layers and fiber) and in Table 2 (properties of matrix/fiber interfaces).

As already mentioned, see Figure 2, for matrix layers, an elastic with linear softening constitutive law is assumed in tension, whereas the compression behavior is infinitely elastic. For matrix/fiber interfaces, a trilinear shear stress-slip relationship with non-zero residual strength is adopted. After the elastic phase, a linear softening is assumed before the activation of the residual strength and after the elastic limit is reached.

	External matrix layer	Fiber	Internal matrix layer
Elastic modulus	$E_{Me} = 5\ GPa$	$E_F = 241\ GPa$	$E_{Mi} = 5\ GPa$
Tensile strength	$f_{tMe} = 5.05\ MPa$	-	$f_{tMi} = 5.05\ MPa$
Strain	$\varepsilon_{Me,e} = f_{tMe}/E_{Me}$ $\varepsilon_{Me,u} = 2\varepsilon_{Me,e}$	-	$\varepsilon_{Mi,e} = f_{tMi}/E_{Mi}$

Table 1: Mechanical properties for upper matrix, fiber and lower matrix.

The symbols in Table 1 have the following meaning, see also Figure 2:

- E_{Me}, E_F, and E_{Mi} are upper matrix, fiber and lower matrix elastic moduli respectively;
- f_{tMe}, and f_{tMi} are upper and lower matrix layers tensile strengths, respectively;
- $\varepsilon_{Me,e}$ ($\varepsilon_{Me,u}$) and $\varepsilon_{Mi,e}$ ($\varepsilon_{Mi,u}$) are elastic limit (ultimate) strains of the upper and lower matrix layers respectively.

$\tau_{Ie}[MPa]$	$\tau_{Ir}[MPa]$	$s_{Ie}[mm]$	$s_{Ir}[mm]$	k_I	k_s
1.79	$\frac{\tau_{Ie}}{30}$	0.065	$3s_{Ie}$	$\frac{\tau_{Ie}}{s_{Ie}}$	$10k_I$

Table 2: Mechanical properties at the interfaces.

The symbols in Table 2 (Figure 2) have the following meaning:

- τ_{Ie} and τ_{Ir} are elastic and residual shear interface strengths, respectively;
- s_{Ie} and s_{Ir} are elastic and residual interface slips, respectively;
- k_I is the stiffness of the elastic phase;
- k_s is the elastic stiffness at the interface between FRCM and substrate.

A brief summary of the results obtained by means of the FE proposed is reported from Figure 4 to Figure 6. In particular, Figure 4 depicts the global force applied-displacement at the loaded edge curve obtained numerically (continuous curve); a comparison with experimental data is also reported (experimental curves are represented with a dashed line). As it can be seen, the agreement is quite satisfactory, considering also the scatter exhibited by experimental data. In Figure 5, the local bond behaviour at different meaningful time steps selected by the authors during the deformation process is investigated, focusing on the state of stress exhibited along the bond length by the different layers (A, B and C) and by the A-B and B-C interfaces.

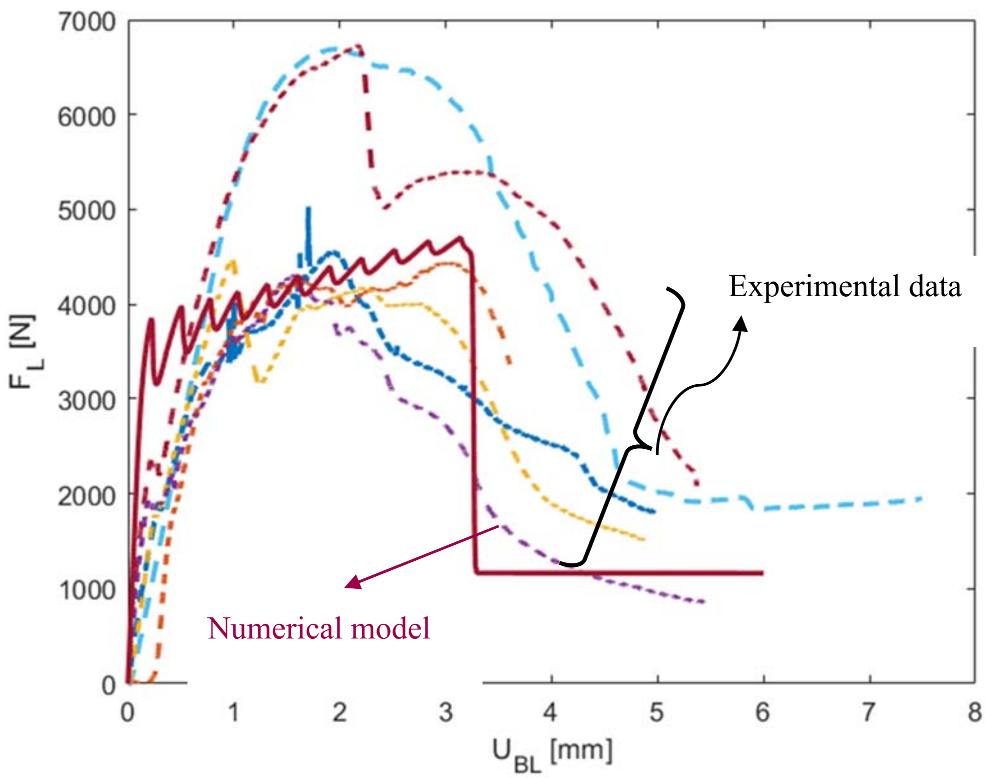

Figure 4: Global force-displacement curves at the loaded edge. Comparison between numerical prediction and experimental data.

The oscillatory behavior of $\tau_{I,AB}$, Figure 5-a, for instance, is interesting because it proves that the external matrix layer A cracks in several points. In particular, 7 cracks are visible at the end of the simulations. Furthermore, the longitudinal normal stress on layers A and C shows that the external layer (A) is always in tension, whereas the internal one passes from regions in compression to other in tension. Figure 6 completes the analyses reporting the local bond behavior in terms of slip and displacements of the different layers, along the bond length, again kept at meaningful instants. Among the others, the zigzagging shape of the slip at interface A-B along the bond length is worth noting. A sudden jump suggests in fact the formation of a crack in the external matrix layer, considering that the fiber is assumed elastic.

5 Conclusions

The paper presented a novel 1D 8-dof finite element suitable for studying the debonding behavior of inorganic matrix composite materials (FRCMs) subjected to single lap shear tests. In particular, it has been discussed how to derive the stiffness of the element and how to deal with the non-linearities of the constituent materials. The proposed element is constituted by three truss elements disposed in parallel. In particular, the first represents the external matrix, the second the central fiber and the third the internal matrix layer.

The three trusses exchange tangential stresses by means of shear springs applied to the ends of the element, reciprocally. The nonlinearities are concentrated at the external and internal matrix layers, and at the interface between them. The constitutive law assumed for the matrix exhibits linear softening in tension and infinite compressive strength; for the interfaces, and linear softening with residual shear strength was assumed.

The approach was validated against experimental data available in the literature and relying on a reinforced flat masonry pillar, reinforced with a PBO-FRCM package and subjected to standard shear tests. The model proved to be simple and robust, providing results consistent with the experimental one. Future developments envisage the possibility of using such element on more complex structures and in particular for a direct structural application on arches and vaults.

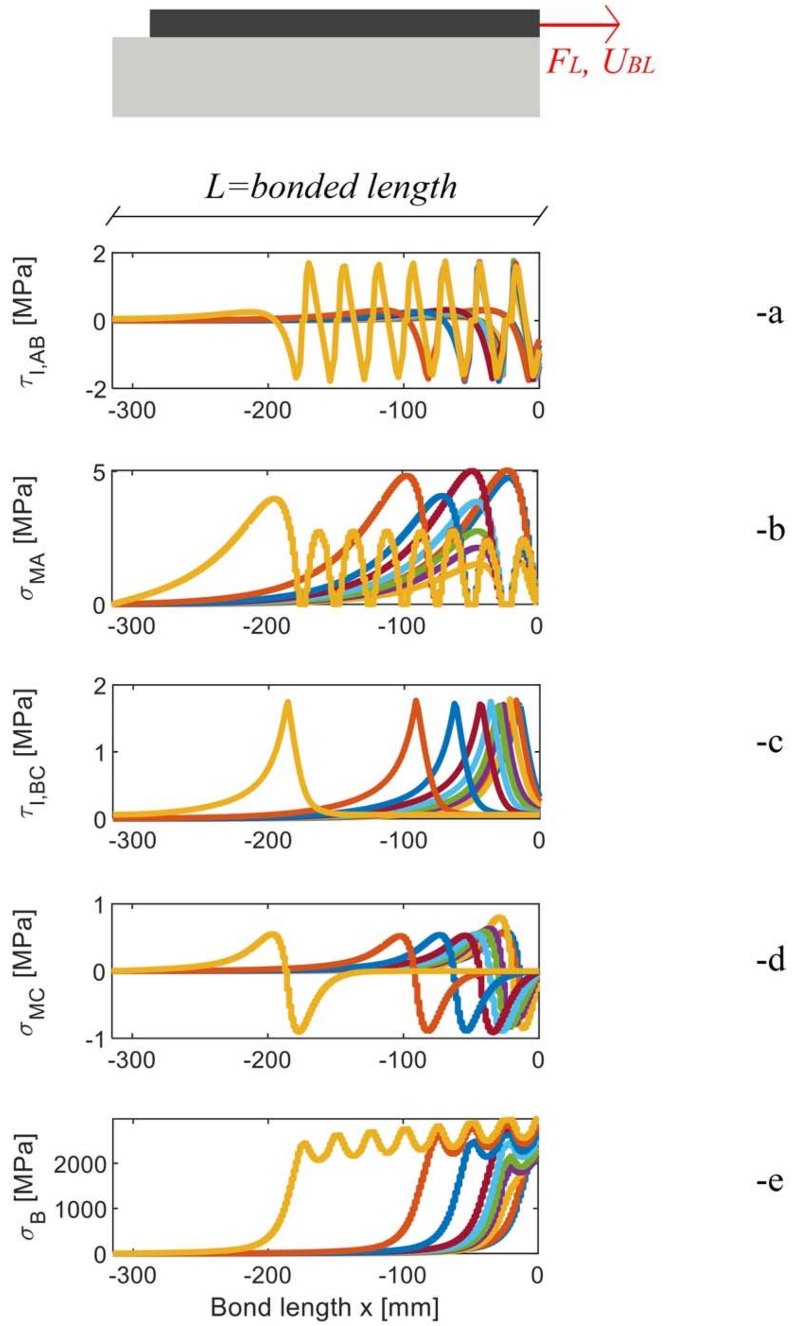

Figure 5: Local bond behaviour at different meaningful time steps. -a: shear stress at the interface A-B. -b: longitudinal normal stress on layer A. -c: shear stress at the interface B-C. -d: longitudinal normal stress on layer C. -e: tensile stress on fiber layer B.

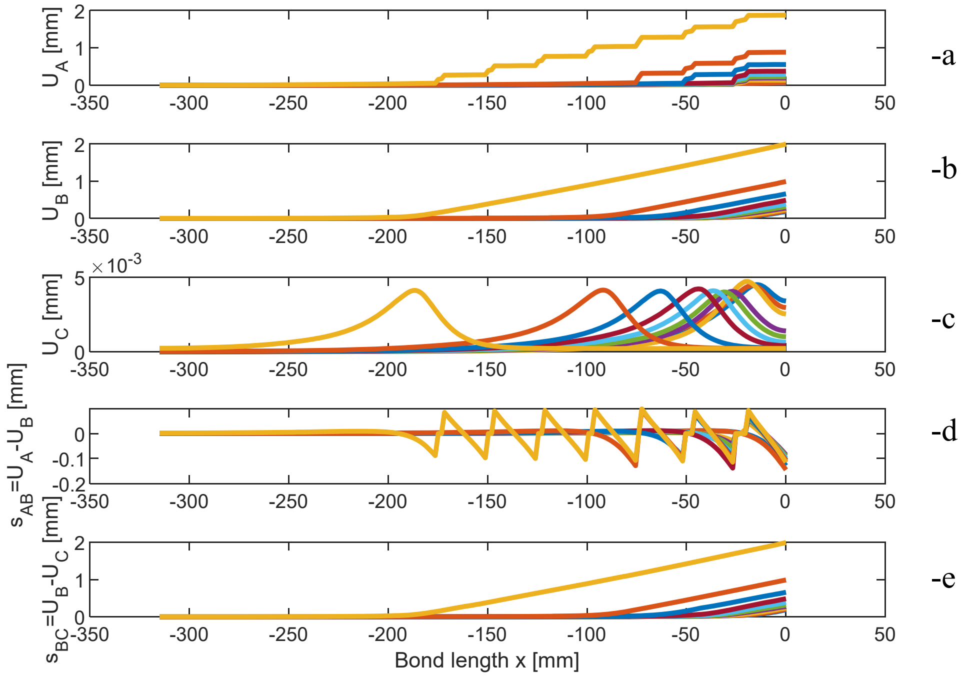

Figure 6: Local bond behaviour at different meaningful time steps. -a: longitudinal displacement of layer A. -b: longitudinal displacement of layer B. -c: longitudinal displacement of layer C. -d: slip at the interface A-B. -e: slip at the interface B-C.

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