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# **Retrieving Bridge Surface Roughness from a Two-Axle Vehicle Response by Kalman Filter**

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# **Abstract**

The bridge surface roughness estimation plays a crucial role in the monitoring and maintenance of the bridge, vehicle suspension design and optimization, etc. However, the conventional methods via profilers cannot meet the actual needs due to its low efficiency and economy, the vehicle response-based techniques generally neglect the vehicle-bridge interaction (VBI) effect and therefore cause an inaccurate estimate. In this study, a new procedure for identifying the bridge surface roughness and vehicle states (responses) simultaneously based on the Kalman filter with unknown inputs (KF-UI) is proposed. Central to this study is the consideration of the vehicle-bridge interaction effect via deducting the bridge displacement from the estimated unknown input vector. The efficacy of the procedure is numerically validated and the robustness is also tested against different parameters, including vehicle speed, vehicle-bridge mass ratio, environmental noise, bridge damping.

**Keywords:** bridge, Kalman filter, road profile, vehicle scanning method

# **1 Introduction**

Road surface roughness is a decisive factor that affects the vibration of the vehiclebridge system. On one hand, as the main source of excitation for the ongoing vehicles of the bridge, it induces the vibration of the vehicles, especially the vertical vibration, which greatly affects the safety and riding comfort. On the other hand, the vibration of vehicles in turn acts on the bridge due to the vehicle-bridge interaction effect, which induces the harmful dynamic impact on the bridge and causes the continuous deterioration for the bridge surface.

The conventional road surface roughness measurement techniques have been developed in the past few decades. Typical techniques are the Longitudinal Profile Analyser (LPA) for the contact measurement, and LIDAR system and 3D image device for the non-contact measurement. Although these techniques show some advantages in measurement accuracy, they have more disadvantages in comparison, such as time-consuming, low efficiency, equipment expensive to purchase and operate specially, etc.

Recently, a new indirect monitoring approach for bridges based on the responses of moving vehicles was proposed by Yang et al.[1, 2]. This method only requires one or several sensors installed on the vehicles passing over the bridge and has the advantages of relatively low cost and high efficiency. Based on this idea, a large number of research results also emerged with regard to the identification of road surface roughness. Imine and Delanne[3]based on the Sliding Mode (SM) observation algorithm, using the vertical response of a full vehicle model with 16 degree of freedom identified the road surface roughness. Ngwangwa et al. [4] established a single-axle vehicle model and applied it with the Artificial Neural Network (ANN) algorithm to reconstruct the road roughness. Doumiati et al.[5] made use of the suspension deflection and body acceleration response of a single-axle vehicle model to measure the road surface roughness based on the Kalman Filter (KF).

In summary, it can be found that the roughness identification techniques based on the vehicle response mostly focus on the road pavement. Nevertheless, for the bridge roughness identification, due to the VBI effect, the displacement of the bridge is mixed with the roughness and creates an adverse effect on roughness identification. Therefore, the most of the existing methods neglect the VBI effect for the bridge surface roughness identification, which generally leads to inaccurate results. In other words, in order to obtain the accurate bridge surface roughness, the VBI effect must be considered.

In this paper, a new procedure for the estimation of bridge surface roughness and vehicle states based on the KF-UI algorithm, in which the bridge deflection is eliminated by using the contact residual of the two-axle test vehicle.

#### **2 Brief on Kalman filter with unknown inputs (KF-UI)**

As was stated, this section forms the backward phase of the numerical study. The KF-UI was proposed as an improved version of the Kalman filter by JN Yang et al. [6]. The following is a brief description of the KF-UI algorithm.

For a linear system with unknown inputs, the equation of motion is  $\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{\eta}^* \mathbf{f}^*(t) + \mathbf{\eta} \mathbf{f}(t)$  (1) where M, C, K denote the mass, damping and stiffness matrices of the system,

respectively;  $\ddot{x}(t)$ , $\dot{x}(t)$ , $x(t)$  the acceleration, velocity and displacement responses;

 ${\bf f}^*(t)$  and  ${\bf f}(t)$  the unknown and known inputs vectors, respectively; and  ${\bf \eta}^*$  and  ${\bf \eta}$  the corresponding influence matrices for  $f^*(t)$  and  $f(t)$ . By defining the state vector as  $\mathbf{Z}(t) = {\dot{\mathbf{x}}^T, \mathbf{x}^T}^T$ , Equation (1) can be transformed into the discrete-time state space as:

$$
\mathbf{Z}_{k+1} = \mathbf{A}_k \mathbf{Z}_k + \mathbf{B}_k \mathbf{f}_k + \mathbf{B}_k^* \mathbf{f}_k^* + \mathbf{w}_k \tag{2}
$$

where  $A_k$  is the state transition matrix, and  $B_k$  and  $B_k^*$  the influence matrices of the known and unknown inputs  $f_k$  and  $f_k^*$ , respectively;  $w_k$  is the noise vector caused by system uncertainty, which has a zero mean and a covariance matrix as  $\mathbf{Q}_k$ , i.e.  $E[\mathbf{w}_k] = 0$  and  $E[\mathbf{w}_k \ \mathbf{w}_k]^{\mathrm{T}}] = \mathbf{Q}_k$ .

The discrete measurement equation of the system can be expressed as:

$$
\mathbf{y}_{k+1} = \mathbf{C}_{k+1} \mathbf{Z}_{k+1} + \mathbf{D}_{k+1} \mathbf{f}_{k+1} + \mathbf{D}_{k+1}^* \mathbf{f}_{k+1}^* + \mathbf{v}_{k+1}
$$
(3)

where  $y_{k+1} = y(t)|_{t=(k+1)T_s}$  denotes the measurement vector,  $C_{k+1}$  the measurement matrix,  $D_{k+1}$  and  $D_{k+1}^*$  the influence matrices of the known input  $f_{k+1}$  and unknown input  $f_{k+1}^*$ , respectively, on the measurement vector  $y_{k+1}$ ; and  $v_{k+1}$  the measurement noise vector, assumed to be a Gaussian white noise with zero mean and a covariance matrix as  $\mathbf{R}_k$ , i.e.,  $E[\mathbf{v}_k] = 0$  and  $E[\mathbf{v}_k, \mathbf{v}_k^{\mathrm{T}}] = \mathbf{R}_k$ .

Let  $\hat{\mathbf{Z}}_{k+1|k+1}$  and  $\hat{\mathbf{f}}_{k+1|k+1}^*$  be the improved estimates of the state vector  $\mathbf{Z}_{k+1}$  and unknown input vector  $f_{k+1}^*$ , respectively. Based on the KF-UI algorithm, they can be obtained recursively by the following five steps:

Step 1: Assign the initial estimates for the state vector  $\hat{\mathbf{Z}}_{0|0}$ , unknown input force  $\hat{\mathbf{f}}_{0|0}^*$ and error covariance matrix  $P_{Z,0|0}$ :

$$
\hat{\mathbf{Z}}_{0|0} = E[\mathbf{Z}_0]
$$
 (4a)

$$
\hat{\mathbf{f}}_{0|0}^* = E[\mathbf{f}_0^*]
$$
 (4b)

$$
\mathbf{P}_{\mathbf{Z},0|0} = E\left[\left(\mathbf{Z}_0 - \hat{\mathbf{Z}}_{0|0}\right)\left(\mathbf{Z}_0 - \hat{\mathbf{Z}}_{0|0}\right)^T\right]
$$
(4c)

Step 2: Calculate the predicted state vector  $\hat{\mathbf{Z}}_{k+1|k}$  and associated error covariance matrix  $P_{Z,k+1|k}$ :

$$
\hat{\mathbf{Z}}_{k+1|k} = \mathbf{A}_k \hat{\mathbf{Z}}_{k|k} + \mathbf{B}_k \mathbf{f}_{k|k} + \mathbf{B}_k^* \hat{\mathbf{f}}_{k|k}^* \tag{5a}
$$

$$
\mathbf{P}_{\mathbf{Z},k+1|k} = \mathbf{A}_k \mathbf{P}_{\mathbf{Z},k|k} \mathbf{A}_k^{\mathrm{T}} + \mathbf{Q}_{k+1}
$$
(5b)

Step 3: Calculate the Kalman gain matrix  $K_{Z_{k+1}}$ :

$$
\mathbf{K}_{\mathbf{Z},k+1} = \mathbf{P}_{\mathbf{Z},k+1|k} \mathbf{C}_{k+1}^{\mathrm{T}} \left[ \mathbf{C}_{k+1} \mathbf{P}_{\mathbf{Z},k+1|k} \mathbf{C}_{k+1}^{\mathrm{T}} + \mathbf{R}_{k+1} \right]^{-1}
$$
(6)

Step 4: Calculate the error covariance matrix  $S_{k+1}$  and unknown input vector  $\hat{\mathbf{f}}_{k+1|k+1}^*$ :

$$
\mathbf{S}_{k+1} = \left[ \mathbf{D}_{k+1}^{*T} \mathbf{R}_{k+1}^{-1} \left( \mathbf{I} - \mathbf{C}_{k+1} \mathbf{K}_{k+1} \right) \mathbf{D}_{k+1}^{*} \right]^{-1} \tag{7a}
$$

 $\hat{\mathbf{f}}_{k+1|k+1}^{*} = \mathbf{S}_{k+1} \mathbf{D}_{k+1}^{*T} \mathbf{R}_{k+1}^{-1} (\mathbf{I} - \mathbf{C}_{k+1} \mathbf{K}_{\mathbf{Z},k+1}) (\mathbf{y}_{k+1} - \mathbf{C}_{k+1} \hat{\mathbf{Z}}_{k+1|k} - \mathbf{D}_{k+1} \mathbf{f}_{k+1|k+1})$  (7b) Step 5: Calculate the improved state vector  $\hat{\mathbb{Z}}_{k+1|k+1}$  and associated error covariance matrix  $P_{Z,k+1|k+1}$ :

$$
\hat{\mathbf{Z}}_{k+1|k+1} = \hat{\mathbf{Z}}_{k+1|k} + \mathbf{K}_{\mathbf{Z},k+1} [\mathbf{y}_{k+1} - \mathbf{C}_{k+1} \hat{\mathbf{Z}}_{k+1|k} - \mathbf{D}_{k+1} \mathbf{f}_{k+1} - \mathbf{D}_{k+1}^* \hat{\mathbf{f}}_{k+1|k+1}^*] \tag{8a}
$$

 $P_{Z,k+1|k+1} = (I + K_{Z,k+1}D_{k+1}^*S_{k+1}D_{k+1}^*R_{k+1}^{-1}C_{k+1})(I - K_{Z,k+1}C_{k+1})P_{Z,k+1|k}$  (8b) The above procedure can be represented by the flowchart given in Figure 1.



Figure 1. Flowchart of the KF-UI algorithm

## **3 Formulation of the Problem of Concern**

#### **3.1 Vehicle-bridge interaction (VBI) model**



Figure 2 Vehicle-bridge interaction model with roughness

Consider a two-axle test vehicle moving over a simply supported bridge, as shown in Fig. 2. The vehicle (body) is simulated as a rigid beam of mass  $m<sub>v</sub>$  and moment of inertia  $I_v$ , and supported by two springs spaced at *d* and of stiffnesses  $k_1$  and  $k_2$ . The vehicle is *asymmetric* in that its center of gravity *C* is unequally spaced from the front axle A<sub>1</sub> and rear axle A<sub>2</sub>, i.e., with distances  $d_1$  and  $d_2$ , respectively.  $y_1$  and  $y_2$  are responses of the front and rear axles, which can be measured in practice. The equation of motion for the test vehicle can be written with respect to the two axles, i.e.,

$$
m_{v}J_{v}\ddot{y}_{1}(t) + k_{1}(J_{v} + d_{1}^{2}m_{v})\{y_{1}(t) - [u_{1}(t) + r(x)|_{x=vt}]\}
$$
  
+
$$
k_{2}(J_{v} - d_{1}d_{2}m_{v})\{y_{2}(t) - [u_{2}(t) + r(x-d)|_{x=vt}]\} = 0
$$
 (9a)

$$
m_{v}J_{v}\ddot{y}_{2}(t) + k_{1}(J_{v} - d_{1}d_{2}m_{v})\{y_{1}(t) - [u_{1}(t) + r(x)|_{x=vt}]\}\
$$

$$
+k_2(J_v + d_2^2 m_v) \{y_2(t) - [u_2(t) + r(x - d)]_{x = vt}]\} = 0
$$
 (9b)

which can be expressed in matrix form as

$$
\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{\eta}^* \mathbf{f}^*(t) \tag{10}
$$

where

$$
\mathbf{x} = [y_1 \quad y_2]^\mathrm{T}, \qquad \qquad \mathbf{M} = \begin{bmatrix} m_v & 0 \\ 0 & m_v \end{bmatrix} \tag{11a,b}
$$

$$
\mathbf{K} = \begin{bmatrix} k_1 \frac{(J_v + d_1^2 m_v)}{J_v} & k_2 \frac{(J_v - d_1 d_2 m_v)}{J_v} \\ k_1 \frac{(J_v - d_1 d_2 m_v)}{I_v} & k_2 \frac{(J_v + d_2^2 m_v)}{I_v} \end{bmatrix}
$$
(11c)

$$
\mathbf{n}^* = \begin{bmatrix} k_1 & \frac{J_v}{J_v} & \frac{J_v}{J_v} \\ k_1 \frac{(J_v + d_1^2 m_v)}{J_v} & k_2 \frac{(J_v - d_1 d_2 m_v)}{J_v} \\ k_1 \frac{(J_v - d_1 d_2 m_v)}{J_v} & k_2 \frac{(J_v + d_2^2 m_v)}{J_v} \end{bmatrix}
$$
(11d)

$$
\begin{bmatrix} x_1 & 0 & x_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) + r(x) \vert_{x = vt} \\ u_2(t) + r(x - d) \vert_{x = vt} \end{bmatrix} \tag{11e}
$$

As indicated by Equation (11e), the unknown input to the test vehicle consists of two parts, i.e., the contact displacement u and surface profile r. In order to retrieve the surface profile  $r$ , a two-step procedure is proposed herein. The first is to estimate the unknown input  $u + r$  by the KF-UI algorithm, and the second is to calculate the contact displacement  $u$  (which should be roughness profile free) and then deduct it from the unknown input  $u + r$  for retrieving the bridge surface profile r.

#### **3.2 Step 1: Using KF-UI algorithm to estimate the unknown inputs**

As indicated in Section 2, the state and measurement equations are required for estimate of the unknown input. The entire procedure is outlined as follows.

#### **3.2.1 State-space equation for test vehicle moving over bridge**

Let  $\mathbf{Z}(t)$  denote the vehicle state vector:  $\mathbf{Z}(t) = [\dot{y}_1(t) \quad \dot{y}_2(t) \quad y_1(t) \quad y_2(t)]^{\mathrm{T}}$  (12) One can transform Eq. (10) into the state space as  $\dot{\mathbf{Z}}(t) = \mathbf{A}_c \mathbf{Z}(t) + \mathbf{B}_c^* \mathbf{f}^*(t)$ (13)

where  $A_c$  and  $B_c^*$  can be calculated as

$$
\mathbf{A}_{c} = \begin{bmatrix} 0 & 0 & -\frac{k_{1}(J_{v} + d_{1}^{2}m_{v})}{m_{v}J_{v}} & -\frac{k_{2}(J_{v} - d_{1}d_{2}m_{v})}{m_{v}J_{v}} \\ 0 & 0 & -\frac{k_{1}(J_{v} - d_{1}d_{2}m_{v})}{m_{v}J_{v}} & -\frac{k_{2}(J_{v} + d_{2}^{2}m_{v})}{m_{v}J_{v}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & m_{v}J_{v} & m_{v}J_{v} \end{bmatrix}
$$
(14a)  

$$
\mathbf{B}_{c}^{*} = \begin{bmatrix} \frac{k_{1}(J_{v} + d_{1}^{2}m_{v})}{m_{v}J_{v}} & \frac{k_{2}(J_{v} - d_{1}d_{2}m_{v})}{m_{v}J_{v}} \\ \frac{k_{1}(J_{v} - d_{1}d_{2}m_{v})}{m_{v}J_{v}} & \frac{k_{2}(J_{v} + d_{2}^{2}m_{v})}{m_{v}J_{v}} \\ 0 & 0 & 0 \end{bmatrix}
$$
(14b)

Considering that the vehicle accelerations recorded are discrete in nature, one can discretize the state-space equation as follows:

$$
\overline{\mathbf{Z}}_{k+1} = (\mathbf{I} + T_s \mathbf{A}_c) \mathbf{Z}_k + T_s \mathbf{B}_c^* \mathbf{f}_k^* = \mathbf{A} \mathbf{Z}_k + \mathbf{B}^* \mathbf{f}_k^* \tag{15}
$$

*3.2.2 Measurement equation for test vehicle moving over bridge*  Let  $y_k$  denote the measurement vector for the test vehicle:

$$
\mathbf{y}_k = \{ \ddot{y}_1(k) \quad y_1(k) \quad \ddot{y}_2(k) \quad y_2(k) \}^{\text{T}}
$$
\nThe measurement equation for the vehicle can be derived as\n
$$
\mathbf{y}_k = \begin{cases} 16 & \text{if } k = 1000; \text{if } k = 1
$$

$$
\mathbf{y}_k = \mathbf{C}_k \mathbf{Z}_k + \mathbf{D}_k^* \mathbf{f}_k^* \tag{17}
$$

where  $C_k$  and  $D_k^*$  are given as

$$
\mathbf{C}_{k} = \begin{bmatrix} 0 & 0 & -\frac{k_{1}(J_{v} + d_{1}^{2}m_{v})}{m_{v}J_{v}} & -\frac{k_{2}(J_{v} - d_{1}d_{2}m_{v})}{m_{v}J_{v}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{k_{1}(J_{v} - d_{1}d_{2}m_{v})}{m_{v}J_{v}} & -\frac{k_{2}(J_{v} + d_{2}^{2}m_{v})}{m_{v}J_{v}} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & m_{v}J_{v} & m_{v}J_{v} \end{bmatrix}
$$
(18a)  

$$
\mathbf{D}_{k}^{*} = \begin{bmatrix} \frac{k_{1}(J_{v} + d_{1}^{2}m_{v})}{m_{v}J_{v}} & \frac{k_{2}(J_{v} - d_{1}d_{2}m_{v})}{m_{v}J_{v}} \\ \frac{k_{1}(J_{v} - d_{1}d_{2}m_{v})}{m_{v}J_{v}} & \frac{k_{2}(J_{v} + d_{2}^{2}m_{v})}{m_{v}J_{v}} \\ 0 & 0 & 0 \end{bmatrix}
$$
(18b)

#### **3.3 Step 2: Calculation of bridge displacements at contact points**

The contact displacements  $u_1(k)$  or  $u_2(k)$  obtained should be roughness-free, such that it can be deducted from the unknown input  $f_k^*$  to yield the bridge surface profile. Note that the bridge displacements used are those calculated from the improved (noise-reduced) vehicle state vector  $\hat{\mathbf{Z}}_k$  generated by the KF-UI algorithm.

The residual response  $\Delta u(t)$  in discrete form of the two contact points at the same location  $x$  of the bridge:

$$
\Delta u(k) = u_1(k) - u_2 \left(k + \frac{d}{vT_s}\right) = \frac{m_v d_2^2 + J_v}{k_1 d^2} \ddot{y}_1(k) - \frac{m_v d_1 d_2 - J_v}{k_2 d^2} \ddot{y}_1\left(k + \frac{d}{vT_s}\right) + \frac{m_v d_1 d_2 - J_v}{k_1 d^2} \ddot{y}_2(k) - \frac{m_v d_1^2 + J_v}{k_2 d^2} \ddot{y}_2\left(k + \frac{d}{vT_s}\right) + y_1(k) - y_2\left(k + \frac{d}{vT_s}\right) \tag{19}
$$

With this, the cumulated contact residuals,  $\sum_{i=1}^{k} \Delta u(k)$ , that is roughness-free, can be calculated as well. For the present purposes, one assumes a priori that there exists a correlation between  $\sum_{i=1}^{k} \Delta u(k)$  and  $u(k)$ , i.e.,

Then one obtains approximate expressions for the coefficients  $\varphi_k$  and  $\lambda_k$ , i.e.,  $\varphi_k^s$ and  $\lambda_k^s$ , as follows:

$$
\varphi_k \approx \varphi_k^s = \frac{\sum_{i=1}^k [u_1^s(k) - u_2^s(k + d/vT_s)]}{u_1^s(k)}
$$
(20a)

$$
\lambda_k \approx \lambda_k^s = \frac{\sum_{i=1}^k [u_1^s(k) - u_2^s(k + d/vT_s)]}{u_2^s(k)}
$$
(20b)

Obviously, to retrieve the contact displacements  $u(k)$ , the correlation between  $\sum_{i=1}^{k} \Delta u(k)$  and  $u(k)$ , i.e., the coefficients  $\varphi_k$  and  $\lambda_k$ , should be determined first, as will be explained in the following. It is interesting to note that the flexural rigidity EI is the only property of the bridge involved in Equation (20) for calculating the contact displacements, and that it will be cancelled out since it appears both in the numerator and denominator. As a consequence, the coefficients  $\varphi_k$  and  $\lambda_k$  depend only on the vehicle's parameters and location on the bridge, all of which are known during the test. In other words, the coefficients  $\varphi_k$  and  $\lambda_k$  can be readily made available for each test vehicle. Finally, the surface profile  $r(k)$  of the bridge can be recovered by deducting  $u(k)$  from the estimated input  $f_k^*$ .

#### **3.4 Flowchart for retrieval of bridge profile**

A summary of the proposed technique was given in the flowchart of Figure 3. Firstly, through the KF-UI algorithm, both the unknown input vector  $f_k^*$  and the improved (noise-reduced) vehicle responses  $\hat{\mathbf{Z}}_k$  can be estimated, and the latter are used to calculate the cumulative contact residual  $\sum_{i=1}^{k} \Delta u(k)$ . Secondly, the contact (bridge) displacements  $u(k)$  are estimated by using  $\sum_{i=1}^{k} \Delta u(k)$  (which are roughness free) and the coefficients  $\varphi_k^s$  and  $\lambda_k^s$ , as given in Equation (20). Finally, by deducting the contact displacements  $u(k)$  from the estimated  $f_k^*$ , one obtains the bridge surface profile  $r(k)$ .



Figure 3. Flowchart of the proposed technique

## **4 Results**

Roughness is a source for vehicles' vibration. In the ISO 8608 standard, the road surface profile  $r(x)$  is expressed as the superposition of a series of trigonometric functions via the power spectrum density (PSD) function as:

$$
r(x) = \sum_{N} \sqrt{2G(n_i)\Delta n} \cos(2\pi n_i x + \theta_i)
$$
 (21)

For the two-axle vehicle passing a bridge, it can be modeled by the VBI element. Environmental noise may pollute the data collected by the moving test vehicle and therefore reduce the measurement accuracy of the vehicle scanning method for bridges. To simulate such an effect, Gaussian white noise will be superimposed on the calculated vehicle responses, i.e.,

$$
y_k^p = y_k + E_p N_s \sigma_{y_k} \tag{22}
$$

where  $y_k$  and  $y_k^p$  denote the original and polluted responses of the test vehicle, respectively,  $N_s$  the standard normal distribution,  $\sigma_{y_k}$  the standard deviation of  $y_k$ , and  $E_p$  the noise level.

The properties of the test vehicle are: mass  $m_v = 2500$  kg, moment of inertia  $J_v =$ 2300 kg ⋅ m<sup>2</sup>, axle distances to center of gravity  $d_1 = 1.7$  m and  $d_2 = 1.3$  m, and axle suspension stiffness  $k_1 = 230 \text{ kN} \cdot \text{m}^{-1}$ ,  $k_2 = 180 \text{ kN} \cdot \text{m}^{-1}$ . Vehicle speed is set at  $v = 2$  m/s and time step is 0.001s. Meanwhile, the following initial values are adopted for the KF-UI:  $\hat{\mathbf{Z}}_{0|0} = [0 \ 0 \ 0 \ 0]^{T}$   $\hat{\mathbf{f}}_{0|0}^{*} = [0 \ 0 \ 0 \ 0]^{T}$ ,  $\mathbf{P}_{\mathbf{Z},0|0} =$ diag[1 1 10<sup>6</sup> 10<sup>6</sup>],  $Q = 10^{-8}I_4$ , and  $R = 10^{-3}I_4$ , where  $I_4$  denotes the (4×4) identity matrix. In addition, a noise of  $E_n = 2\%$  is added to the calculated vehicle response through Equation (37) to simulate the noise-contaminated effect.

Young's modulus	Ε	GPa	27.5
Moment of inertia		m <sub>4</sub>	0.2
Mass per unit length	m	$kg·m-1$	2,000
Span length		m	30
Beam element length	le	m	1.0
First modal damping ratio	$\xi_1$	$\frac{0}{0}$	3.0
Second modal damping ratio	ξ2	$\frac{0}{0}$	3.0

Table 2: Physical properties of the bridge.

In this section, the bridge surface profile will be retrieved by use of the proposed two-step technique and the result will be compared with the original (assumed) input. The surface profiles of Classes A and C roughness for the simple beam retrieved by the KF-UI algorithm have been plotted in Figures 4 and 5, respectively, along with the original (assumed) ones (generated by the PSD). In the figures, parts (a) and (b) denote the results in spatial and frequency domains, respectively. It is confirmed that regardless of the varying class of roughness, the retrieved profile is in good agreement

with the original one in both the spatial and frequency domains. This is an indication of the reliability of the proposed technique for retrieving the bridge surface profile.



Figure 5. Retrieved and original surface profile (Class C roughness)

## **4 Conclusions and Contributions**

In this study, a two-step technique is proposed for retrieving the bridge surface profile in a noisy environment from the responses recorded of a two-axle test vehicle moving over the bridge. Firstly, the KF-UI algorithm is employed to estimate of the improved (noise-reduced) vehicle states and the unknown inputs consisting of surface profile and contact displacement; Secondly, the contact displacements are calculated from the cumulated contact residuals (that is roughness-free) using the improved vehicle states, and then deducted from the unknown input for retrieving the surface profile. The estimated vehicle states and bridge surface profile by the two-step technique agree well with the original ones, which verifies the feasibility of the proposed procedure. The coefficients  $\varphi_k^s$  and  $\lambda_k^s$  used to define the correlation between the cumulated contact residuals  $\sum_{i=1}^{k} \Delta u(k)$  and the contact displacement  $u(k)$  of two axles are reliable, which can be accurately estimated without prior knowledge of the bridge dynamic properties.

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