



Proceedings of the Seventeenth International Conference on
Civil, Structural and Environmental Engineering Computing
Edited by: P. Iványi, J. Kruis and B.H.V. Topping
Civil-Comp Conferences, Volume 6, Paper 2.3
Civil-Comp Press, Edinburgh, United Kingdom, 2023
doi: 10.4203/ccc.6.2.3
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Moving Force Identification based on Dictionary Learning

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Abstract

This paper presents a moving force identification (MFI) approach based on dictionary learning (DL) with double sparsity. Firstly, the MFI equation is established, and a sparse dictionary model is designed. Then the sparse K-singular-value-decomposition (K-SVD) algorithm is employed for dictionary learning. Finally, the moving forces is identified via force dictionary and double sparse codes. Numerical example of a simply supported beam subjected to a moving force is used to verify the effectiveness of the proposed approach.

Keywords: moving force identification, dictionary learning, double sparsity

1 Introduction

As one of the main loads of bridge, moving force places an important role in the bridge construction and maintenance [1-2]. However, it is normally difficult to obtain the moving force information through direct measurement [3-4]. Therefore, estimating the moving force through the bridge dynamic response induced by the moving force has been a hot topic in the field of bridge engineering [5]. Generally, such methods can be classified into two categories, i.e., frequency domain methods and time domain methods [6].

Time domain moving force identification (MFI) method is widely investigated due to its clear concept and high precision [3-4]. Generally force-response equations related to the moving forces and bridge responses are developed and solved to obtain the moving force. Time discretization is required to form the force-response equation

as the dynamic response is measured at sampling points for a real vibration test. The size of the force-response equation is determined by the sampling time interval. Small sampling time interval lead to high identification accuracy at the cost of high computation resource, while large sampling time interval lead to low identification accuracy with high computational efficiency [7-8].

In order to improve the identification accuracy efficiently, basis functions are adopted to simulate the moving force. Thus the problem of solving force-response equation can be changed into the problem of coefficients identification. Force dictionary is a typical basis function used in moving force identification under sparse restriction, which is meaningful in improving the identification accuracy and efficiency [9-10]. As a matter of fact, the form of the force dictionary greatly affects the effectiveness of the dictionary based moving force identification method [11-12]. However, it is a headache to design a fixed force dictionary matching the moving force well in advance for the lack of in-depth understanding of moving force [13-15].

In view of this, this paper presents a moving force identification approach based on dictionary learning with double sparsity, in which the force dictionary is determined according to the measured dynamic response induced by moving force.

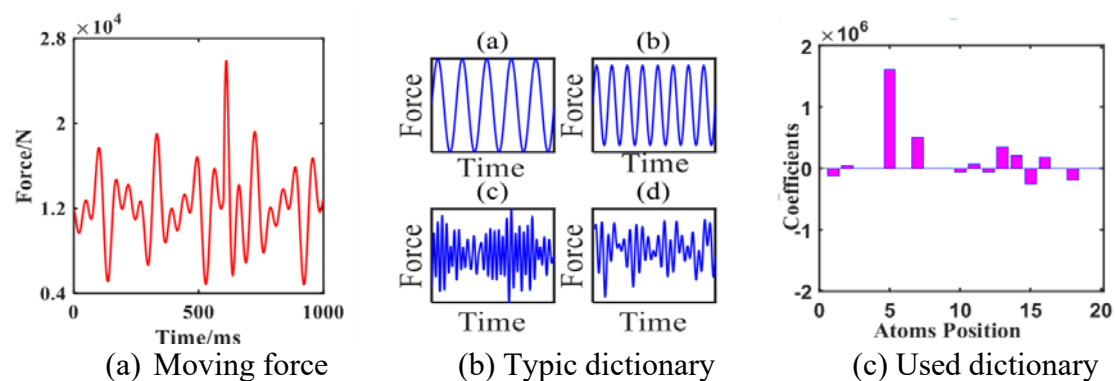


Figure 1: MFI based on force dictionary

2 Methods

When the force dictionary is employed, the moving force identification problem can be expressed as

$$ADs = r \quad (1)$$

in which A is the system matrix, D is the force dictionary, r is the dynamic response, and s is the sparse code to be determined.

It should be noted that the atoms in the force dictionary are predetermined for the normally dictionary based moving force identification method, which may not well match the real moving force. In this section, the concept of dictionary learning is introduced to avoid using the fixed force dictionary.

According to the dictionary learning, D is further expressed as [16]

$$D = BC \quad (2)$$

in which B is the base force dictionary, C is the sparse code of force dictionary D over the base force dictionary B .

Thus the moving force identification problem is transformed into the following optimization problem

$$(C, s) = \underset{(C, s)}{\operatorname{argmin}} \|r - ABCs\|_2^2 \quad \text{subject to } \forall i, \|s_i\|_0 \leq T_1, \text{ and } \|c_i\|_0 \leq T_2 \quad (3)$$

in which T_1 is the target sparsity of s , and T_2 is the target sparsity of C , respectively.

The main procedures of the proposed moving force identification method based on dictionary learning with double sparsity are as follows:

- (1) Establish the moving force identification equation, and design a sparse dictionary model.
 - (2) Employ the sparse K-singular-value-decomposition (K-SVD) algorithm for dictionary learning.
 - (3) Identify the moving force via force dictionary and double sparse codes.
- The details of the process are shown in Figure 2.

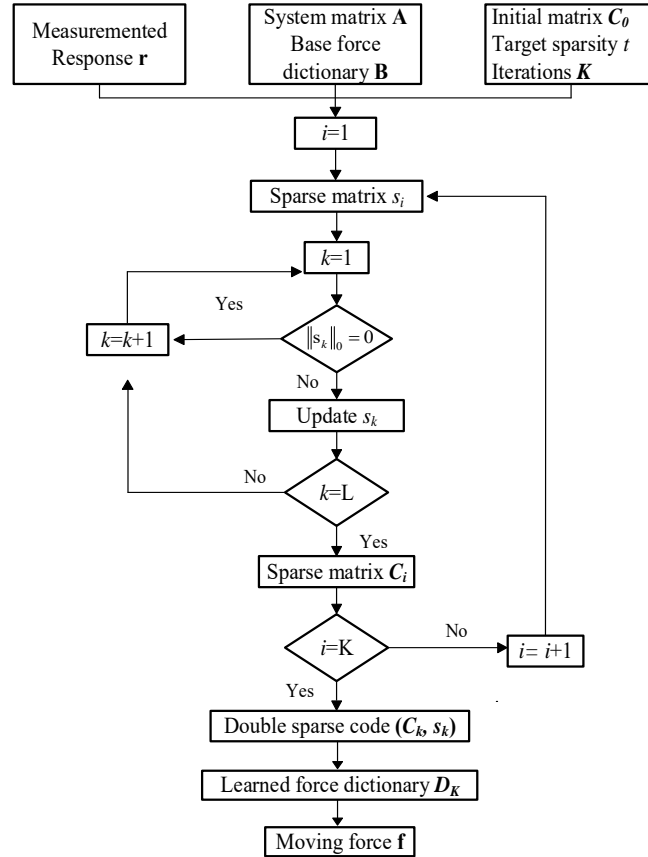


Figure 2: MFI procedure

3 Numerical example

A simply supported beam is simulated to verify the effectiveness of the proposed moving force identification method.

As shown in Figure 3, a simply supported beam is subjected to a moving force $f_2 = 8 \times [1 + 0.32 \cos(17.4\pi t + \frac{1}{4}\pi) - 0.24 \cos(26.7\pi t + \frac{1}{2}\pi) + 0.16 \cos(32.1\pi t + \frac{3}{4}\pi)]$ kN . The parameters of the beam are: span $l = 16m$, Young's modulus $E = 7 \times 10^{10} N \cdot m^2$, density $\rho = 2500 \text{kg} \cdot \text{m}^{-3}$, and cross section $A = 4m \times 1m$. Strain responses at 1/4 span and acceleration response at 1/2 span with 5% noise are used as to identify the moving force. The target double sparsity in this simulation is set to be $t_1 = t_2 = 11$.

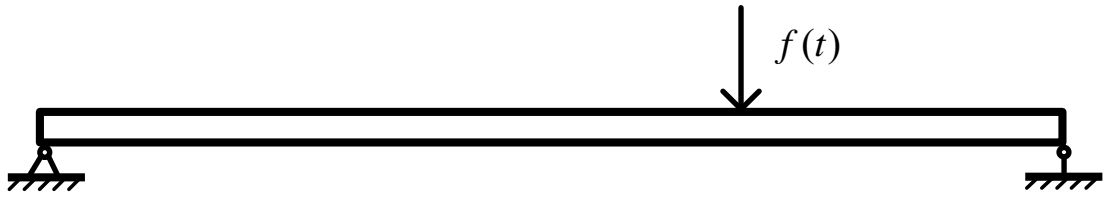


Figure 3: A simply supported beam subjected to a moving force

The base force dictionary \mathbf{B} is assumed to be

$$\mathbf{B} = \begin{Bmatrix} 1 & \cos(\pi \cdot \Delta t) & \sin(\pi \cdot \Delta t) & \cdots & \cos(n\pi \cdot \Delta t) & \sin(n\pi \cdot \Delta t) \\ 1 & \cos(\pi \cdot 2\Delta t) & \sin(\pi \cdot 2\Delta t) & \cdots & \cos(n\pi \cdot 2\Delta t) & \sin(n\pi \cdot 2\Delta t) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & \cos(\pi \cdot m\Delta t) & \sin(\pi \cdot m\Delta t) & \cdots & \cos(n\pi \cdot m\Delta t) & \sin(n\pi \cdot m\Delta t) \end{Bmatrix} \quad (1)$$

where $m = 999$ is the sampling point number, and $n = 60$ is the highest series for triangular functions.

This base force dictionary is widely used in the fixed dictionary based moving force identification methods. It should be noted that the headache of designing a fixed force dictionary matching the force well in advance is not required due to the introducing of dictionary learning.

The moving force is identified by following the procedures presented in Figure 2, and the results before and after learning are compared in Figure 4, which verifies that proposed approach performance better than the traditional fixed dictionary based approach. This is because the force dictionary after learning according to the measured dynamic response induced by moving force can match the moving force well.

Given that the proposed approach relies on response data, it is crucial for controlling the algorithm's bootstrapping process. Therefore, relative error of the response is determined by comparing the reconstructed response to the measured response, where reconstructed response is calculated via the identified moving force.

Typically, the relative error decreases as iterations progress. The iteration process halts once a satisfactory level of convergence is attained. Figure 5 indicates in this numerical example the relative error of the response reached a convergence after 8 iterations at a level of 0.85%. The identified moving force at iteration 8 is thereby selected as final output.

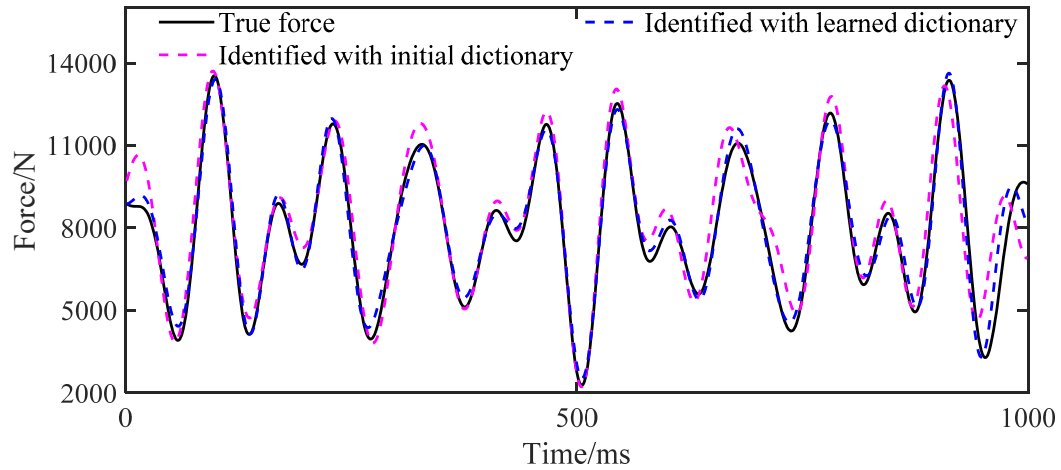


Figure 4: MFI results

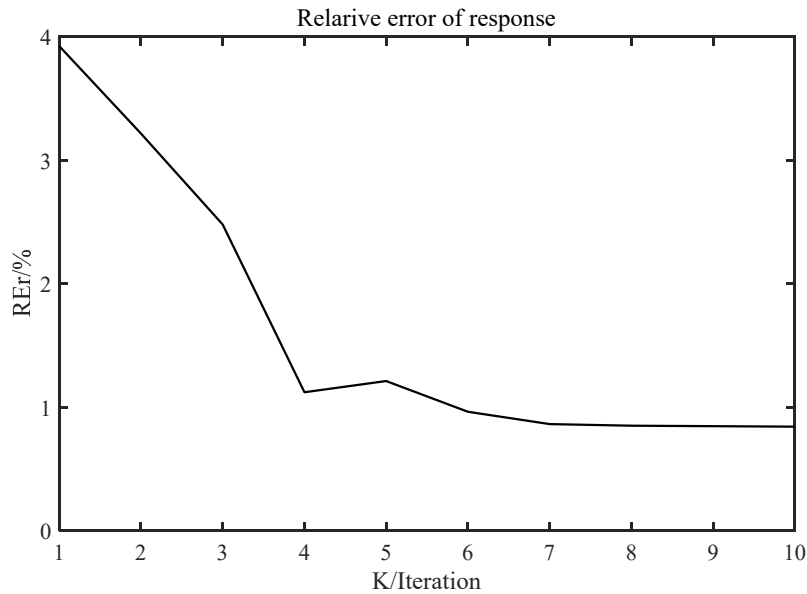


Figure 5: The relative error of responses of iteration steps

Besides, the relative error of moving force is also defined to verify the accuracy of proposed approach, it can be observed in Figure 6 that the final identification result has a relative error of 3.76% after 8 iterations, which is an apparent improvement compared to the initial result at 7.74% before learning process.

In this study, two atoms of initial force dictionary, No.31 and No. 84, are drawn in in Figure 7(a) and Figure 7(b). Additionally, the corresponding learned atoms are presented in Figure 7(c) and Figure 7(d) for comparative purposes. Notably, upon learning, the updated atoms exhibit enhanced suitability for representing genuine moving forces. Theoretical foundations of the proposed method affirm that, moving forces demonstrate a higher degree of sparsity with the learned dictionary with updated atoms, in relation to the initial force dictionary. Consequently, the identification results achieved a satisfying level of accuracy.

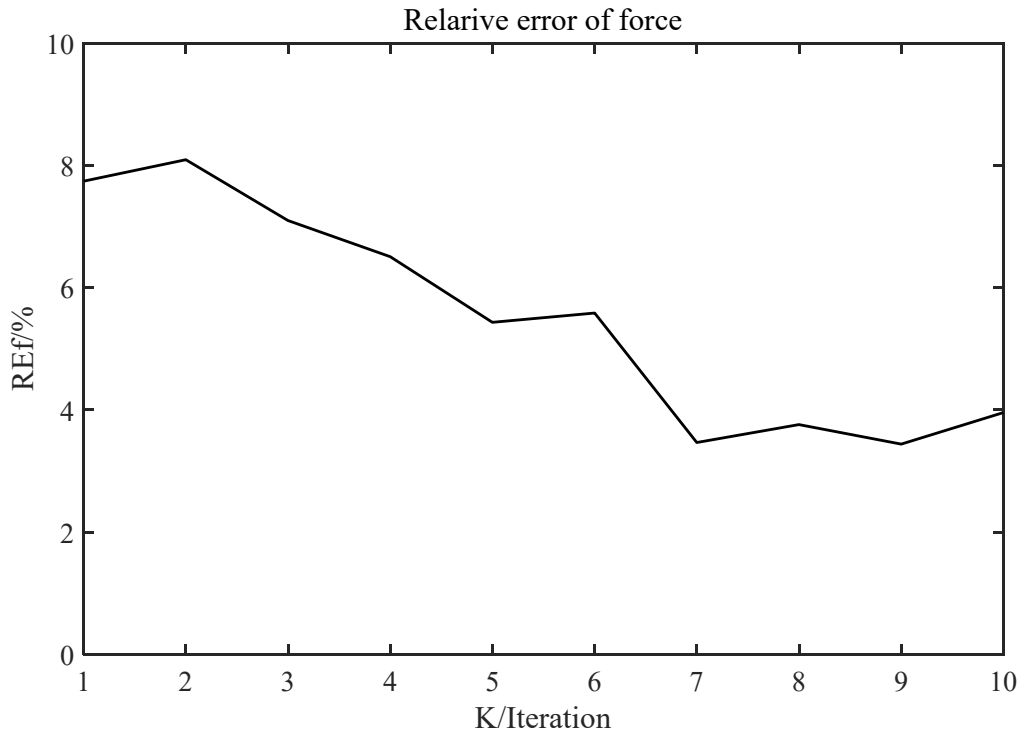


Figure 6: the relative error of moving forces of iteration steps

4 Conclusions and Contributions

In order to enhance the performance of the dictionary based moving force identification, this study presented a moving force identification approach based on dictionary learning with double sparsity. Numerical example of a simply supported beam subjected to a moving force verified that proposed approach could obtain better results than the traditional fixed dictionary based approach.

Acknowledgements

The paper is supported by Fundamental Research Funds for the Central Universities (JZ2019HGPA0101), and Science and Technology Program of Anhui Transportation Holding Group Co., LTD (JKKJ-2021-01).

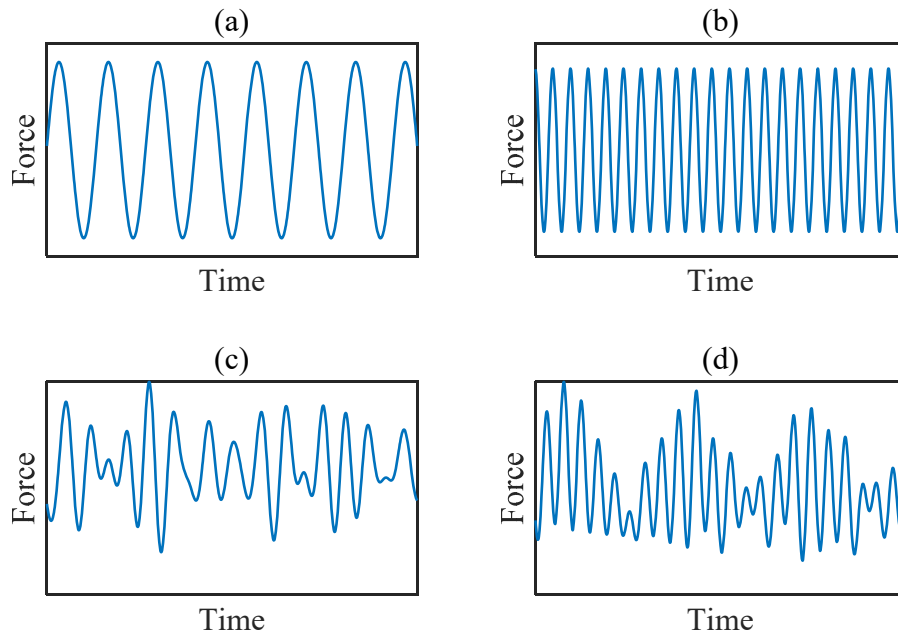


Figure 7: the comparison of atoms No.31 and No.84 before and after learning

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