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VSM-based modal detection for a tied-arch railway bridge beam using a passing vehicle

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Abstract

Based on the Vehicle-Bridge Interaction (VBI) theory, a moving instrumented vehicle is designed as a dynamic information scanner to detect the modal properties of a bridge. This idea is done using the vehicle scanning technique or indirect measurements proposed by Yang et al. in 2004 [1]. This study presents and implements the fundamental theory and conceptual ideas of the test vehicle used to detect the frequency messages of a tied-arch railway bridge. As a train connecting a test vehicle travels over the arch bridge at a constant speed, interesting study concerning the modal frequency detection of the VBI system will be conducted.

Keywords: damage detection, hanger, suspended beam, vehicle-bridge interaction, vehicle-scanning method, vibration

1 Introduction

An arch with curved form may create a gentle skyline and promote architectural aesthetics for modern infrastructure. A through arch bridge with arch-ribs above the deck is one of typical types in arch bridge design. The use of vertical hangers to suspend a flexible deck of an arch bridge is not only easier to build and design in constructional deployment but also effective to provide additional elastic supports for

the deck. Thus, a thin section is often considered as bridge deck for an arch bridge, in which the flexible deck is suspended by the arch ribs through hanger systems.

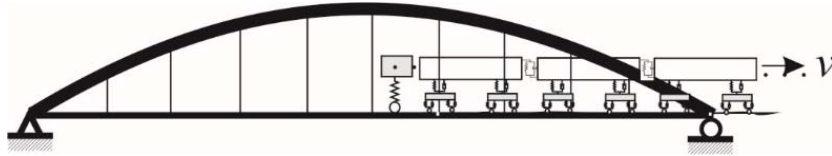


Fig. 1 Schematic of a tied-arch bridge and a train connected by a passing test vehicle

As shown in Fig. 1, the following structural characteristics are drawn:

1. A light-weight cross section of flexible decks not only saves structural materials but also reduces the self-weights transferred to the arch-ribs through hangers, which can offer an effective cost-down design in construction;
2. A stiff arch-rib system can offer enough structural strength to suspend the flexible deck through hangers; and
3. Deployment of suspended hangers to transfer the self-weight and traffic loads to the supporting arch-ribs.

With these features listed above, the compressive arch ribs, tensile hangers and bending decks constitute a definite carrying system for an arch bridge so that the mechanical performance of structural materials for each of components can be developed effectively. Such a design requirement of strong-arch and flexible-deck challenge structural engineers to carry out traffic-induced vibrations of arch-bridge structures since traffic load-induced broken hangers would result in severe damages/collapse accidents of the flexible deck-slabs from sudden loss or redistribution of stress in the hanger system. This particular issue has attracted bridge engineers' attention and interests to conduct the dynamic behaviors of the hanger system of arch bridges under traffics. According to the previous studies, the key factors leading to the hangers breaking can be attributed to: (1) accumulation of fatigue-damage by alternating-loads of traffics, (2) long-term traffic overloads induced fracture, (3) environmental corrosions and ductility loss of materials due to very cold temperature, (4) impact effects by road roughness or expansion joints due to heavy traffic loads. However, most of these studies are focused on the influence of structural materials or external loads on the hanger system. Few of them gave a satisfactory explanation why the vibration amplitudes of the short hangers at arch foot are amplified significantly in comparison with those hangers at middle span of the deck. Focusing on this topic, this paper starts from the analytical formulation of a parabolic arch under vertical loads to unveil how the arching action can affect the vibration modes of a suspended deck, from which the key parameters dominating the dynamic characteristics of vibration modes of the flexible deck are extracted.

For analytical demonstrations, the free vibration analysis of various types of through arch bridges considering different restraint supports of arch-ribs are explored. With the present investigations, the arching action on the arch-ribs of tied-arch bridges

plays a key role in affecting the dynamic forces of hanger system and the dynamic behaviors of deck-slabs under the action of moving train loads.

2. Problem formulation

2.1 Fundamental of arch structures

The primary objective of this paper is to investigate the dynamic effects of moving train loads on vibrations of tied-arch bridges by the FE approach, the arch rib is modeled as a parabolic arch and the deck as a uniform beam supported by hinged ends, as shown in the Fig 1. First, the arching action of a two-hinged parabolic arch under uniformly distributed vertical load is conducted analytically. Then the FE-based arch-beam element will be developed for dynamic response analysis of a single-span tied-arch bridge subject to moving trains. Finally, the free vibration analysis and train-induced vibrations of a tied-arch bridge are respectively carried out for evaluating the dynamic response of critical short hangers located at the arch foot of a tied-arch bridge under a train moving with resonant speeds.

2.2 Curved-beam based model for tied-arch bridges

The basic feature of an arch is that it can sustain the self-weight and external loads in a compressive manner, i.e., without introducing any tensile force on the structure. Such a structural form is particularly suitable for building materials that are strong in resisting compressive, rather than tensile forces, including stone and concrete. With mechanical properties similar to the arch structures, a vertical curved beam can transfer the gravity loads through the bending-tension coupling effect, by which the material can be used in a more efficient manner than a straight beam. However, the coupling effect has made the deformation behaviors of curved beam structures much more complicated than structures composed of straight beams.

Though some progress has been made in the past in the study of curved beam problems, the coupling behavior (bending-tension and bending-torsion) of curved beams remains a mathematical hindrance in the derivation of a consistent displacement (strain) field for the curved beam element aimed at avoiding the membrane locking problem. In the out-of-plane buckling analysis of curved beams, Yang and Kuo [2] have demonstrated that a curved beam can be simulated by a number of straight-beam elements through consideration of the equilibrium for structural joints connecting non-collinear members in the deformed configuration. As can be seen from the above review, there is an apparent lack of a simple and straightforward approach for formulating a curved beam element that is free of membrane locking in the elastic stiffness for linear analysis, and can duly take into account the effect of curvature on the geometric stiffness for buckling analysis. On the other hand, planar curved beams constitute a special class of structures in engineering applications, for which both the in-plane deformation and out-of-plane buckling behaviors have always been the concerns of structural designers. For the reasons stated above, a non-conventional structural approach will be proposed herein for deriving the planar curved beam element. For curved beams with small subtended

angles, the elastic stiffness matrix of the curved beam will be derived as the composition of two chordwise straight beam elements used to represent the curved beam. Based on the concept of rigid displacements, the geometric stiffness matrix of the curved beam will be derived by transforming the geometric stiffness matrix of the straight beam with identical nodal degrees from the rectangular coordinates to the curvilinear coordinates. To examine the applicability and accuracy of the curved beam element presented herein, four examples on linear static, buckling, and geometrically nonlinear analyses of curved beam structures will be studied.

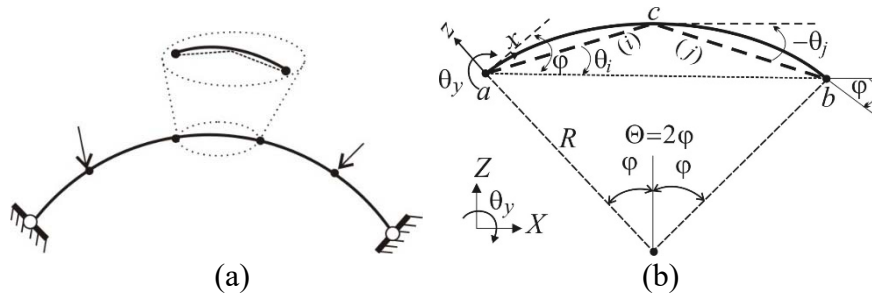


Fig. 2 Modeling of an arch structure.
(a) FE modelling. (b) Modeling of a curved arch element

2.2.1 Straight-beam based arch element

In this section, a simple, non-conventional *structural approach* will be presented for formulation of the planar curved beam element, which can be regarded primarily as extensions of the theories for the straight-beam element concerning derivation of the elastic stiffness matrix and geometric stiffness matrix. The following are the assumptions adopted for the planar curved beam:

- (1) The material is elastic and homogeneous;
- (2) The cross section of the curved beam is uniform and doubly symmetric;
- (3) Every cross section remains rigid, i.e., undistorted, during deformation;
- (4) The length and radius of the curved beam are large in comparison with the cross-sectional dimensions of the beam;
- (5) The shearing deformation on the curved beam is negligible; and
- (6) Only concentrated loadings are allowed to act at the two ends of the curved beam element.

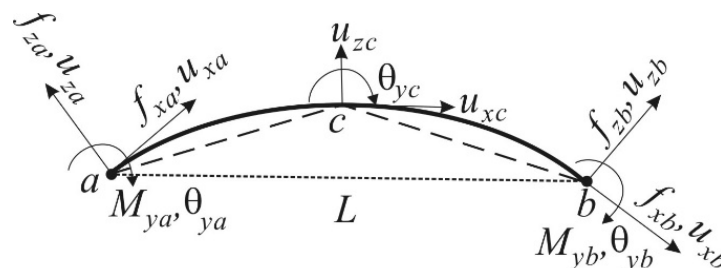


Fig. 3 Nodal forces of a curved beam element.

2.2.2 Elastic stiffness matrix of planar curved beam element

For analysis of a circular curved beam subjected to external loadings by the finite element method, it is reasonable to divide the curved beam into a number of curved beam elements, as shown in Fig. 2. To circumvent the problem associated with the selection of proper shape functions for treating the coupling effect of bending-extension deformations, each planar curved beam element with a *small* subtended angle ($= 2\varphi$) will be replaced by *two chordwise straight beam elements*, as schematically depicted in Fig. 2(b). The curved beam element of concern is shown in Fig. 2 with a radius R and subtended angle 2φ . The z -axis in Fig. 3 represents one of the principal axes of the cross section, and the x -axis is tangent to the curvilinear axis of the beam. Concerning derivation of the *elastic stiffness matrix* for describing the linear behavior, the curved beam element acb in Fig. 3 will be *approximated* by the *two chordwise straight beam elements* ac and cb , also named as beam elements i and j , respectively, which share a common *auxiliary node* c at the midpoint of the curved beam element. In addition, θ_i and θ_j represent the included angles of segments ac and cb , respectively, with respect to the chord ab of the curved element in the X - Z coordinate system shown in Fig. 2. The above procedure describe the derivation of the *elastic stiffness matrix* for the planar arch element with two nodes a and b based on the straight-beam element stiffness matrices is characterized by the fact that it is based purely on the consideration of structural geometry.

2.2.3 Free vibrations of a tied-arch bridges

A tied-arch bridge allows the elimination of horizontal forces at the abutments of arch-ribs through internal ties installed in the bridge decks so that it can be constructed with less robust base and prefabricated off-site. On the other hand, an arch bridge needs enough strong arch ribs to suspend the flexible thin-deck through tensile hangers. For numerical computations, the single-span arch-bridge is modeled as a beam-like deck suspended by an arch-rib through vertical hangers with uniform interval (d), in which the arch rib is made of CFST cross section, the concrete deck is simplified as a simple beam and the structural damping ratio of 2% is considered for the following example. With the curved beam element presented, Table 1 plots the fundamental modes of a tied arch bridge. As can be seen, the first mode of all the flexible deck is of anti-symmetrical shape due to the first anti-symmetrical mode induced by the longitudinal bending vibrations of the arch rib (see Table 1). In contrast, the second vibration mode of the deck is of symmetry and strengthened by the arching action on the arch-rib so that its corresponding frequency becomes higher than the first anti-symmetrical one, as explained in previous section.

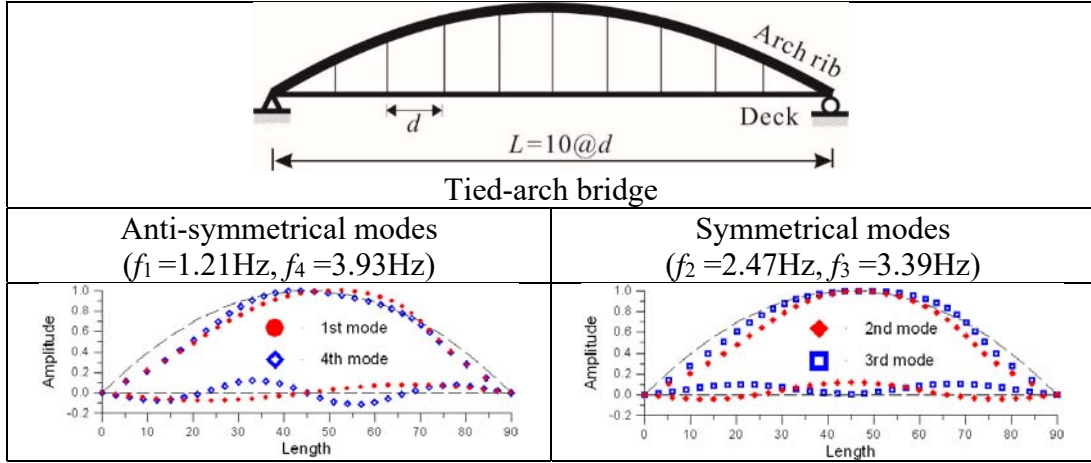


Table 1. Natural modes and frequencies of the tied-arch bridge

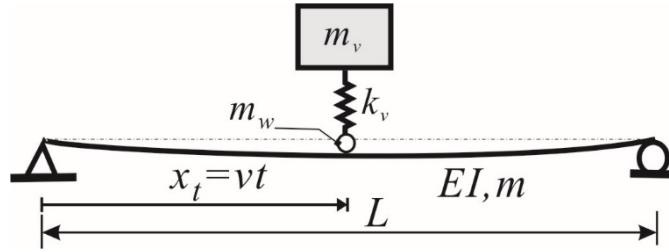


Fig. 4 A coupled model of a VBI system

3 Vehicle scanning technique

To clarify the key factors for VSM from the dynamic response of a test vehicle moving on a simple beam, as shown in Fig. 4, the damping and contact roughness of the VBI system would be neglected in the following formulation. Then the governing equations of Eqs. (1) and (2) are approximated as

$$m\ddot{u} + EIu'''' = -p_0\delta(x - vt) \quad 0 \leq t \leq L/v \quad (1)$$

$$m_v\ddot{u}_v + k_v u_v = k_v u(x_t, t)|_{x_t=vt} \quad (2)$$

Solving the differential equations with zero initial conditions to find the vertical displacement response (u_v) and then feedback it to the vehicle equation of Eq. (2), one can find the quasi-response of the moving vehicle on the beam. Next, one can relate vehicle's acceleration $\ddot{u}_v(t)$ to the contact-point acceleration response $\ddot{u}_c(t)$ using the following derivatives,

$$\ddot{u}_c(t) = \ddot{u}_v(t) + \frac{1}{\omega_v^2} \frac{d^2 \ddot{u}_v(t)}{dt^2} \quad (3)$$

To carry out the digital computations of fast Fourier transform (FFT) for the discrete acceleration data recorded by the vehicle, the term $d^2 \ddot{u}_v / dt^2$ in Eq. (3) can

be discretized using the central difference scheme. Then Eq. (3) is rewritten in an alternative form as [3]

$$\mathbf{F} [\ddot{u}_c] = \mathbf{F} \left[\ddot{u}_v(t) + \frac{\ddot{u}_v(t) - 2\ddot{u}_v(t - \Delta t) + \ddot{u}_v(t - 2\Delta t)}{(\omega_v \Delta t)^2} \right] \quad (4)$$

Here, $\mathbf{F} [\bullet]$ is denoted as Fourier transform. Clearly, since the vehicle acceleration (\ddot{u}_v) in Eq. (4) is made available, the spectral contact-point acceleration $\mathbf{F} [\ddot{u}_c]$ can be calculated using Eq. (4). Such a monitoring technique called vehicle scanning method (VSM) was proposed by Yang et al. [3].

4. Applications of VSM to tied-arch bridges

From an engineering viewpoint, a bridge is usually designed to provide sufficient structural strength and safety for vehicles moving over it. By this requirement, we can assume the moving vehicle-induced vertical inertial force acting on the bridge is negligible and the bridge is subjected to only sequential moving static forces of traffic loadings. By solving the equation of motion of the bridge analytically, the dynamic response of the vehicle excited by the feedback bridge response can be computed. This study calls such an approach as a two-stage technique and the decoupled vehicle-bridge system as quasi-VBI model. Obviously, the present two-stage technique is a simple and efficient way to analyze interaction dynamics of vehicle/bridge coupling system. For demonstration, the tied-arch bridge shown above is modeled by finite element method. The major components of the bridge modeled by finite elements are described as:

- (1) The bridge deck is modeled as a number of beam-column elements;
- (2) The arch is modeled by a number of beam-column elements with the axial and flexible rigidities;
- (3) The suspended hanger is represented as a two-force tensile bar element.

The vehicle-bridge interaction dynamic analysis will be carried out for the test vehicle moving on the bridge. Thus, we can detect the vertical vibration data of the bridge from the running test vehicle for identify the bridge frequencies using the contact-point response.

As indicated in Fig. 5, the present contact-point response analysis for VSM of vertical bridge frequencies is feasible using the test vehicle's response collected from the vibration data of the cable stayed bridge in cross winds.

5. Conclusions

Hangers are crucial load-bearing components of cable-supported bridges and provide elastic support to the suspended bridge deck. The dynamic behaviour of hangers has attracted the attention of many researchers during the past several years. Based on the Vehicle-Bridge Interaction (VBI) theory, a moving instrumented vehicle is designed as a dynamic information scanner to detect the damaged hangers of a suspended

simple beam. This article can be regraded as a preliminary study of modal detection of cable-supported arch bridge using VSM. A further study for detecting the damaged hangers (caused by fatigue or joint corrosions) of a cable-supported bridge will be carried out by applying the present VSM.

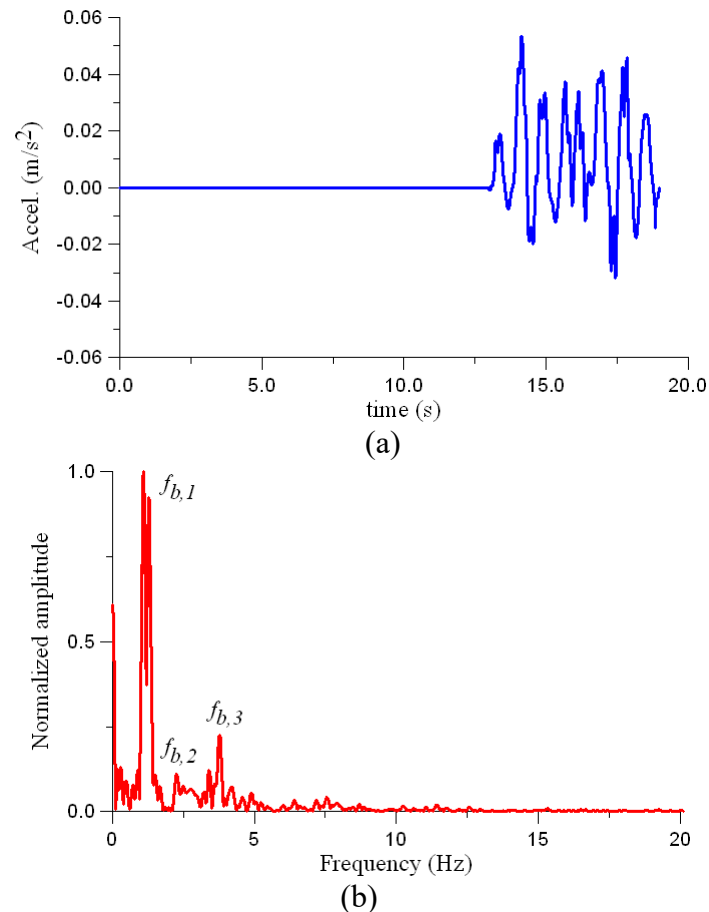


Fig. 5 Frequency detection of the tied-arch bridge
 (a) Time history of acceleration response. (b) response spectrum of the test vehicle

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