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Topology optimization considering the effect of two-phase fluid

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Abstract

This study introduces a novel topology optimization method for transient two-phase fluid problem, which remains a challenging task despite advances in computing capabilities. The application of gradient-based optimizers to two-phase fluid systems is complicated because it requires modification of the governing equations to reflect changes in the interface between the two fluids and transient sensitivity analysis. To overcome these difficulties, this study develops a new topology optimization method for two-phase fluids that models the interface using the level-set approach and formulates transient sensitivity analysis.

Keywords: topology optimization, two-phase fluid, levelset

1 Introduction

The application of gradient-based optimizers to two-phase fluid systems is complicated because it requires modification of the governing equations to reflect changes in the interface between the two fluids and transient sensitivity analysis. To over-

come these difficulties, this study develops a new topology optimization method for two-phase fluids that models the interface using the level-set approach and formulates transient sensitivity analysis. All the material here is for the publication in [1].

From a topology optimization point of view, many researches exist for fluid related problem. The Darcy force whose magnitude is proportional to the fluid velocity can be added to the Navier-Stokes equation for topology optimization. With the large Darcy's force, the velocity becomes zero. Therefore, the parameterization of the Darcy's force and the optimal distribution of the Darcy's force are possible and the fluid topology optimization. Based on the concept of the novel Darcy's force, several innovative researches have been carried out (See [1] and the references therein). In addition, the transient fluid can be considered in topology optimization with the analytical sensitivity. As many relevant researches exist, it is impossible to cite all of them. In the relevant researches, the performance of fluid is mainly considered. Then, the conjugate heat transfer and fluid structure interaction is considered in topology optimization (It is also impossible to review and cite all relevant researches. See [1] and reference therein). With the help of the recent development on structural optimization, the complex heat conjugate problem is also researched such as natural convection, heat sink, RANS model and etc. The optimized results are analyzed from a theoretical point of view using the representative number, i.e. Reynolds number and Grashof number. The separated fluids between pseudo rigid domain are considered. In addition to these, researchers have developed a variety of strategies for simultaneously considering various physics and performances. However, to our best knowledge, the application of topology optimization for two-phase fluid is limited.

Many innovative approaches exist for the simulating steady-state or transient two-phase incompressible fluid flow with surface tension (See [1] and reference therein). One of the main difficulties for the numerical simulation of surface tension dominated flows is the representing the strong pressure discontinuity across the interface boundary between two fluids. In addition, the accurate representation of the interface boundary is important to describe the evolution of the interface boundary. Several moving mesh algorithms with moving mesh exist for the numerical simulation of two-phase fluid. However, this aspect becomes challenges in connection to the topology optimization with fixed mesh for two-phase fluid. To address this issue, therefore, this research employs the levelset-based modeling with the surface tension modeling. As it is complicate to consider and model complex interface with the levelset-based modeling, this research limits simpler boundary shapes in the considered optimization problems.

The remainder of this paper is organized as follows. Section 2 provides the mathematical formula pertaining to the two-phase analysis and the development of the sensitivity analysis. Several topology optimization examples for two-phase fluid are presented in Section 3. Section 4 presents the conclusions of the study and provides suggestions for future research.

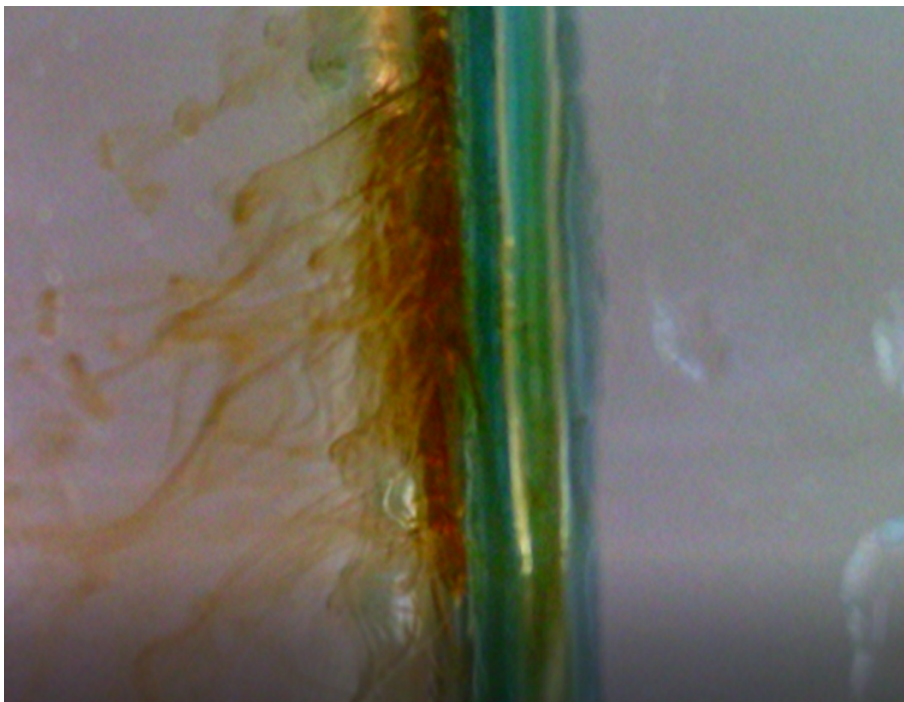
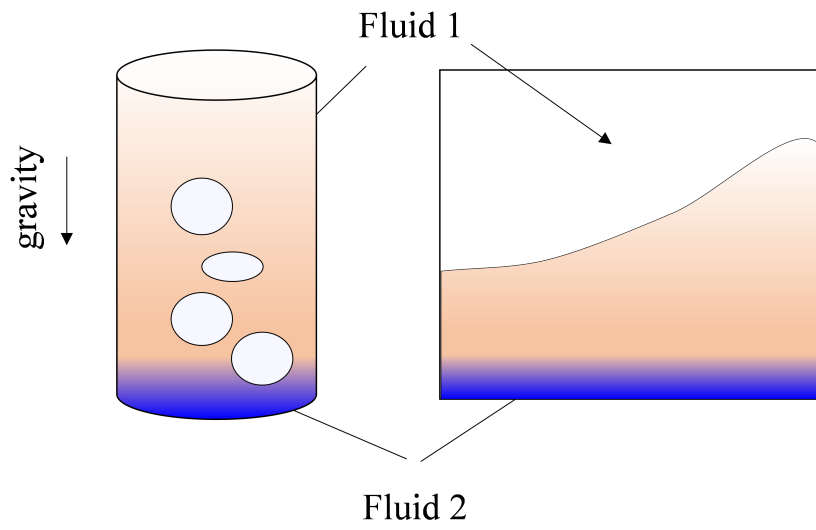


Figure 1: Example of a two phase fluid

2 Two-phase fluid simulation

Complying with the continuum condition, transient fluid flow is governed by the momentum conservation equation with the gravity force, $\rho\mathbf{g}$, and the surface tension force, \mathbf{F}_{st} .

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \mu \nabla \cdot (\nabla \mathbf{u} + \nabla \mathbf{u}^T) \cdot \mathbf{u} + \rho \mathbf{g} + \mathbf{F}_{st}, \mathbf{u} = \mathbf{u}(\mathbf{x}, t) \quad (1)$$

The velocities and pressure of fluid are denoted by \mathbf{u} and p , respectively. The spatial coordinator is denoted by \mathbf{x} and the time is denoted by t . The viscosity and the density are denoted by μ and ρ , respectively. The gravity constant in vector form is given as \mathbf{g} and the surface tension force which can be formulated as the body force is denoted by \mathbf{F}_{st} . This volumetric force is defined along the interface boundary Γ . In the simulation of two-phase fluid, the formulation and numerical simulation of this surface tension force make itself different to the simulation of one-phase fluid. The mass conservation equation is defined as follows:

$$\frac{\partial \rho}{\partial x} + \cdot(\rho \mathbf{u}) = 0 \quad (2)$$

In an incompressible homogeneous fluid, the above equation becomes $\nabla \cdot \mathbf{u} = 0$. The above two equations are subject to initial conditions $\mathbf{u}|_{t=0} = \mathbf{u}_0$ where the initial velocity is denoted by \mathbf{u}_0 . The boundary conditions are set as follows:

$$\begin{aligned} \text{No-slip b.c. : } & \mathbf{u} = \mathbf{0} \text{ on } \Gamma_{u^0} \\ \text{Inflow/outflow b.c. : } & \mathbf{u} = \mathbf{u}^* \text{ on } \Gamma_{u^*} \\ \text{Pressure b.c. : } & [-p\mathbf{I} + \mu(\nabla \mathbf{u} + \nabla \mathbf{u}^T)] \cdot \mathbf{n} = p_p \mathbf{n} \text{ on } \Gamma_{p^*} \end{aligned} \quad (3)$$

The no-slip boundary condition is defined along Γ_{u^0} , and the inflow/outflow boundary condition is defined as \mathbf{u}^* along the Γ_{u^*} . Along Γ_{p^*} , the pressure boundary condition (Neumann boundary condition) is imposed. The normal vector along the boundary is denoted by \mathbf{n} .

In order to take it to account the evolution of two-phase fluid within fixed Eulerian mesh and the interface boundary, the levelset-based approach can be employed. By employing the continuous function ϕ , the domains of two fluids as well as the interface boundary can be determined as follows:

$$\phi(\mathbf{x}, t) = \begin{cases} 0 & \text{if } \mathbf{x} \in \text{Fluid 1} \\ \text{Otherwise} & \text{if } \mathbf{x} \in \text{Interface boundary } \Gamma \\ 1 & \text{if } \mathbf{x} \in \text{Fluid 2} \end{cases} \quad (4)$$

where the interface boundary condition is defined by Γ that is actually defined inside the analysis domain. The original levelset variable determines the kind of fluid and the interaction interface boundary which is the VOF (Volume of Fluid) as follows:

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0 \quad (5)$$

Often the above levelset equation requires the reinitialization that resets the levelset function to be a signed distance function. To improve the convergence and impose the regulation, it is known that the re-initialization process is necessary. In the present study, the following modification on the levelset function is made with the reinitialization factor.

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi - \gamma_L \nabla \cdot \left(\varepsilon \nabla \phi + \phi(1 - \phi) \frac{\nabla \phi}{|\nabla \phi|} \right) = 0 \quad (6)$$

where the parameter for interface thickness is denoted by ε and the re-initialization parameter is denoted by γ_L . With this approach, the breakup or coalescence of bubble and droplet can be simulated as the levelset value can be merged or separated.

The boundary condition is defined as follows:

$$\mathbf{n} \cdot \left(\mathbf{u} \phi - \gamma_L \varepsilon \nabla \phi + \gamma_L \phi(1 - \phi) \frac{\nabla \phi}{|\nabla \phi|} \right) = 0 \quad (7)$$

As we are dealing with flow with more than one fluid, the surface tension at the fluid interface has to be accounted for. The surface tension is modeled as a body force concentrated at the interface by employing the Continuum Surface Force. Being dealing with flow with more than one fluid, it is necessary to take the surface tension along the fluid interface into account. The calculation of the free surface profile between fluids is a key factor in the accurate simulation of two-phase fluid. The surface tension can be modelled as a body force defined along the interface by employing the Continuum Surface Force model. With the above levelset parameter, it is possible to define the surface tension force, \mathbf{F}_{st} , as follows:

$$\mathbf{F}_{st} = \nabla \cdot [\sigma(\mathbf{I} - \mathbf{n}_{int} \mathbf{n}_{int}^T) \delta_{int}] \quad (8)$$

where the normal direction vector of the interface boundary is $\mathbf{n}_{int} = \frac{\nabla \phi}{|\nabla \phi|}$ and δ is defined as $6 |\phi(1 - \phi)| |\nabla \phi|$. The surface tension coefficient is denoted by σ .

The density and the viscosity are then interpolated with respect to the normalized levelset variable, ϕ_n .

$$\phi_n = \min(\max(\phi, 0), 1) \quad (9)$$

$$\rho = \rho_1 + (\rho_2 - \rho_1) \phi_n \quad (10)$$

$$\mu = \mu_1 + (\mu_2 - \mu_1) \phi_n \quad (11)$$

where the density values of the first fluid and the second fluid are ρ_1 and ρ_2 , respectively. The viscosity values of the first fluid and the second fluid are μ_1 and μ_2 , respectively. The above equations are able to be solved with the finite element procedure. During the simulation, it is challenging to capture the pressure difference and track

the evolution of the interface boundary condition stably. As the fluid velocity is different depending on the kind of fluid, the CFL (Courant - Friedrichs - Lewy Number) number is one of the parameters in the simulation. In this study, the maximum CFL value is limited less than 0.5.

3 Topology optimization example

The method of moving asymptotes (MMA) algorithm is employed as an optimization algorithm.

An extension to topology optimization method for transient two-phase fluid problem is newly developed in the present study. Two-phase flow denotes a combination of two distinct phases. Some typical examples are the combination of gas-liquid or liquid-liquid which can be observed in many scientific and engineering applications. For an example, Figure 1 shows the figures of the concept of two-phase fluid which requires the complicated simulation of the two-phase fluid. As the intricate physical phenomenon between two fluids are considered. It is challenging to underline the involved physics and topologically optimize a structure for transient two-phase fluid despite increased computing capabilities. In addition, the application of a gradient based optimizer toward the two-phase fluid system is intricate as it may require the modification of the governing equations to reflect the change of the interface between two fluids and the transient sensitivity analysis. In this study, to cope with these difficulties, a new topology optimization method for two-phase fluid is presented. The interface between two fluids is modelled with the levelset based approach and the transient sensitivity analysis is formulated. Using the developed approach, it is possible to obtain the optimized layout in Figure 2.(For more detail, see [1])

4 Concluding remarks

The objective was to give an overview of the recently developed topology optimization scheme in [1]. An extension to topology optimization method for transient two-phase fluid problem is newly developed in the present study. Two-phase flow denotes a combination of two distinct phases. Some typical examples are the combination of gas-liquid or liquid-liquid which can be observed in many scientific and engineering applications. For an example, Figure 1 shows the figures of the concept of two-phase fluid which requires the complicated simulation of the two-phase fluid. As the intricate physical phenomenon between two fluids are considered. It is challenging to underline the involved physics and topologically optimize a structure for transient two-phase fluid despite increased computing capabilities. In addition, the application of a gradient based optimizer toward the two-phase fluid system is intricate as it may require the modification of the governing equations to reflect the change of the interface between two fluids and the transient sensitivity analysis. In this study, to cope with these difficulties, a new topology optimization method for two-phase fluid is pre-

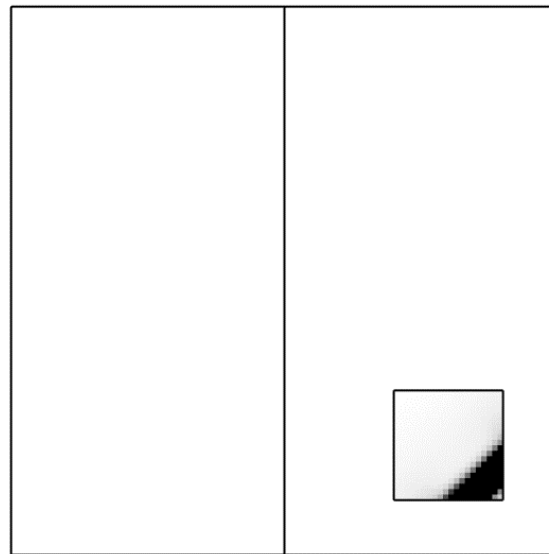
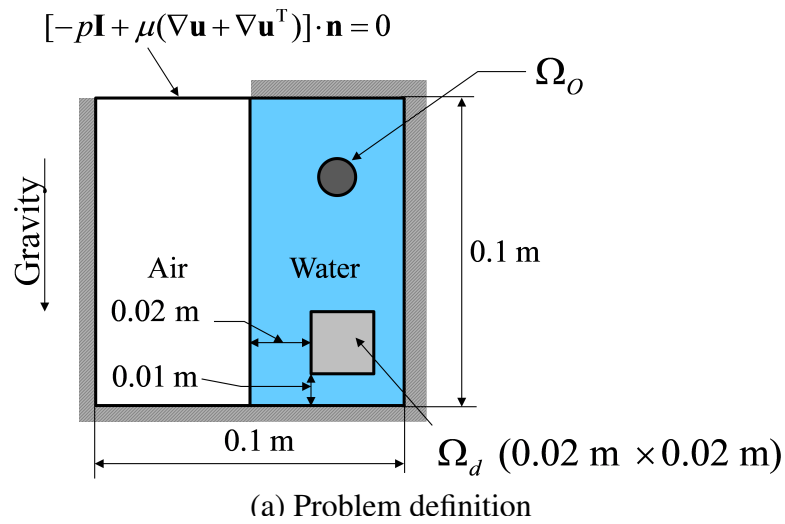


Figure 2: Optimization example

sented. The interface between two fluids is modelled with the levelset based approach and the transient sensitivity analysis is formulated. Using the developed approach, it is possible to obtain the optimized layout in Figure 2. (For more detail, see [1].)

References

- [1] G.H Yoon and M.K. Kim, "Topology optimization for transient two-phase fluid", in review.