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On Filtering Techniques for Topology Optimisation based on B-Spline Entities

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Abstract

This work examines the problem of oscillating boundaries in density-based topology optimisation algorithms based on B-spline hyper-surfaces. We compare recent developments in the different approaches proposed in the literature and attempt to find a relatively easy and robust method to obtain smooth optimal solutions by minimising the number of design variables and the associated computational effort.

Keywords: topology optimisation, NURBS, B-spline, Shepard, filters, surface fitting

1 Introduction

The finite element method (FEM) and computer-aided design (CAD) software had a huge impact on multiple engineering fields and allowed for the developpement of different tools for the deisgn of optimal mechanical structures. On the one-hand, the FEM allows to compute the mechanical response of a structure, by solving a finite element model defined on a computational mesh approximating the prescribed domain of the structure. On the other hand, CAD provides powerful tools to describe the geometry and topology of such a domain, among which, the explicit representation of geometrical features based on Non-Uniform Rational Basis Spline (NURBS) entities, which are the most used entities in CAD. NURBS entities have been introduced to overcome certain limitation of Basis Spline (B-spline) and Bézier entities, such as, the exact representation of quadratic curves and surfaces, [1].

Topology optimisation (TO) has benefitted greatly from both FEM and CAD, and turned into a widespread research field which allowed for the developement of different TO methods. The most popular ones are density-based methods [2, 3] and the level-set method (LSM) [4, 5]. They allow to determine the optimal distribution of a material, within a precribed domain, to minimise a given objectif function while satisfying a set of design requirements, and have widely proven to be an efficient tool for the preleminary design of structures in several industrial sectors.

Probably the most popular density-based method is the one making use of the solid isotropic material with penalisation (SIMP) approach to penalise the elasticity matrix in the framework of structural analysis. In the context of density-based algorithms, the topology of the structure is described through a pseudo-density field $\rho(x) \in [0, 1]$ which directly affects the stiffness tensor of the finite element model. This facilitates the implementation of the method for descent algorithms. Nevertheless, other issues must be carefully studied when working with density-based TO algorithms, such as the checkerboard effect, mesh dependency of the optimised topology, local minima, etc. [6] . These problems are mainly due to the dependency of the pseudo-density field $\rho(x)$ on the finite elements mesh, and the last 20 years have seen the development of different approaches to tackle these issues, such as the use of smooth filters [7, 8], relaxation of the optimisation problem [9, 3], or the post-processing of the optimal solution[10, 11].

Among the most recent developments in the field of density-based TO algorithms, one can find the NURBS-density-based TO method [12, 13, 14, 15, 16, 17]. This approach can be seen as a hybrid method combining the main features of density-based TO algorithm and the LSM: its main advantage consists of separating the pseudo-density field from the finite elements mesh, which is now, described by a purely geometric entity. More precisely, a NURBS entity of dimension $D + 1$ is used as a topological descriptor for a TO problem of dimension D. The NURBS formalism allows some nice features such as the local support and the convex hull property [1], and in analogy with the LSM, NURBS entities allow access to the topology boundary during iterations and are CAD-compatible. Thus, a theoretical/numerical framework for TO based on NURBS entities has been developped and applied to different mechanical problems with varying complexities both in 2D and 3D [13, 18, 12].

Of course, NURBS-density-based TO algorithm comes at a price which is an increase of the design variables when NURBS hyper-surfaces are used as topological descriptor. Indeed, a NURBS surface is defined as the weighted sum of a tensor product of the NURBS basis functions at each control point (CP). Thus, the design variables vector contains both the density value at CPs and the associated weights. This can be computationally expensive when considering 3D problems or multiple optimisation parameters. An easy solution is to take a constant unit weight, which corresponds of using B-splines entities instead of NURBS ones, this approach however yields the well-known problem of oscillating boundaries [14].

To this purpose, this paper investigates the problem of oscillating boundaries when using B-spline entities as topological descriptor and its possible solutions. Specifically, this paper aims to provide an answer to the following research question: is it possible to use B-spline entities to describe the topology of the continuum and find a way to reduce/avoid wavy boundaries, or are NURBS entities the best choice in terms of topological descriptor? If we can find a solution to reduce/eliminate the boundary oscillations, B-spline entities would be the best choice as they represent the best compromise between saving design variables (thus reducing the computational effort) and accuracy of results. In this context, three different approaches are proposed in the literature which are all based on filtering. The first approach consists in reformulating the optimisation problem in a way to ensure a non-oscillatory density field through the use of linear filters such as Shepard's filter [19]. The second one is to apply the filter on the gradient of the merit function, which the authors in [17] apply to obtain smooth boundaries in the context of multi-resolution topology optimisation (MTOP) . Finally, the third approach consists in performing a dedicated post-processing of the optimised solution to filter out noise from the boundaries through image-processing techniques [20]. The remainder of the paper is as follows. Section 2 briefly recalls the fundamentals of NURBS hyper-surfaces and states the TO problem. Section 3 introduces two well-known linear filtering techniques and their adaptation to density-based TO algorithms. Section 4 presents filtering methods to smooth the gradient of the response functions involved in the problem formulation. Section 5 recalls the fundamental concepts at the basis of the surface fitting method presented in [21, 22, 23], which is adapted in this paper to approximate the optimised topology by filtering out the boundary oscillations. Section 6 gives some preliminary numerical results, whilst section 7 draws some concluding remarks and prospects.

2 Problem formulation with non-uniform rational basis spline hyper-surfaces

A detailed description of the mathematical background of the NURBS-based SIMP method is available in [13, 12]. Let us just recall that a NURBS hyper-surface is a polynomial-based function, defined as $\mathbf{h} : \mathbb{R}^N \to \mathbb{R}^D$, where N and D are respectively the dimensions of the parametric space, and the co-domain. The formula of a NURBS hyper-surface reads :

$$\mathbf{h}(\xi_1, \ldots, \xi_N) := \sum_{i_1=0}^{n_1} \cdots \sum_{i_N=0}^{n_N} R_{i_1 \ldots i_N}(x_1, \ldots, x_N) \boldsymbol{\xi}_{i_1,\ldots,i_N}, \tag{1}$$

where

$$R_{i_1 \ldots i_N}(x_1, \ldots, x_N) = \frac{\omega_{i_1,\ldots,i_N} \prod_{k=1}^{N} N_{i_k,p_k}(x_k)}{\sum_{j_1=0}^{n_1} \cdots \sum_{j_N=0}^{n_N} \left[\omega_{j_1,\ldots,j_N} \prod_{k=1}^{N} N_{j_k,p_k}(x_k)\right]} \tag{2}$$

are the piece-wise rational basis functions.$N_{i_k,p_k}(x_k)$ are the standard Bernstein's polynomials calculated recursively as discussed in [1], $x_k \in [0,1]$ is the kth dimensionless coordinate, and $\boldsymbol{\xi}_{i_1,\dots,i_N} \in \mathbb{R}^D$ is the vector containing the coordinates of the generic CP. Finally, (n_j+1) and p_j are respectively the number of CPs and the degree of the basis functions along the parametric direction x_j.

We consider the classical problem of minimising the compliance of a $2D$ structure. Let $D = \{(X_1, X_2) \in \mathbb{R}^2 | X_1 \in [0, a_1], X_2 \in [0, a_2]\}$ be a compact subset defined in the Cartesian orthogonal frame $O(X_1, X_2)$, where a_1, and a_2 are the reference lengths of the domain. We seek the optimal distribution of a given isotropic heterogeneous material, with a prescribed volume V, in the design domain D in order to minimise the compliance of the structure. The equilibrium problem for a linear elastic structure reads:

$$\begin{bmatrix} \mathbf{K} & \mathbf{K_{BC}} \\ \mathbf{K_{BC}^T} & \tilde{\mathbf{K}} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{u_{BC}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{r} \end{Bmatrix}, \tag{3}$$

where $\mathbf{u}$ and $\mathbf{u_{BC}}$ are the unknown and imposed vectors of generalised displacements. $\mathbf{f}$ is the vector of applied nodal forces and $\mathbf{r}$ contains the nodal reactions on nodes where Dirichlet's boundary conditions are imposed. $\mathbf{K}, \mathbf{K_{BC}}$ and $\tilde{\mathbf{K}}$ are the stiffness matrices of the FE model after applying boundary conditions. A pseudo-density field ρ is used to represent the distribution of the material, such that $\rho(X_1, X_2) = 0$ means absence of material, whilst $\rho(X_1, X_2) = 1$ implies a completely dense base material. For a $2D$ problem, a $3D$ NURBS surface is used, whose third coordinate is the pseudo-density field that reads:

$$\rho(x_1, x_2) = \sum_{\tau=0}^{n_{\text{CP}}} R_\tau(x_1, x_2)\rho_\tau \in [0,1], \tag{4}$$

where $n_{\text{cp}} = (n_1+1)(n_2+1)$ is the number of CPs and τ is the linear index defined by :

$$\tau := 1 + i_1 + i_2(n_1+1), \quad \forall i_1 = 0, \dots, n_1, \quad i_2 = 0, \dots, n_2.$$

In Eq. (4), the dimensionless coordinate is defined as $x_j = \dfrac{X_j}{a_j}$. In the context of the SIMP approach, the pseudo-density field is used to penalise the element stiffness matrix and area as follows:

$$\mathbf{M} = \sum_{e=1}^{N_e} \mathbf{L}_{eM}^{\text{T}} \rho_e^\alpha \mathbf{K}_{e0} \mathbf{L}_{eM},\ A = \sum_{e=1}^{N_e} \rho_e A_e, \tag{5}$$

where ρ_e is the pseudo-density computed at the centroid of the generic element, A_e is the element area, $\mathbf{K}_{e0}$ is the generic stiffness matrix appearing in Eq. (3) and $\mathbf{L}_{eM}$ the connectivity matrix associated with matrix $\mathbf{M} = \mathbf{K}, \tilde{\mathbf{K}}, \mathbf{K}_{\text{BC}}$. The generalised compliance of the structure is defined as [24]:

$$C(\tilde{\rho}(\tilde{\boldsymbol{\xi}}_\mathbf{1}, \tilde{\boldsymbol{\xi}}_\mathbf{2})) = \mathbf{f}^T\mathbf{u} - \mathbf{u}_{BC}^T\mathbf{r}, \tag{6}$$

where $\tilde{\boldsymbol{\xi}}_{\mathbf{1}}^{\mathbf{T}} = \{\rho_0, \rho_1, \dots \rho_{n_{\text{CP}}}\} \in [\rho_{\text{LB}}, \rho_{\text{UB}}]^{n_{\text{CP}}}$ and $\tilde{\boldsymbol{\xi}}_{\mathbf{2}}^{\mathbf{T}} = \{\omega_0, \dots, \omega_{n_{\text{CP}}}\} \in [\omega_{\text{LB}}, \omega_{\text{UB}}]^{n_{\text{CP}}}$ are the design variables vectors collecting the CPs densities and weights, respectively, and $\tilde{\rho}$ is the mapping defined by :

$$\tilde{\rho} \,:\, (\tilde{\boldsymbol{\xi}}_{\mathbf{1}}, \tilde{\boldsymbol{\xi}}_{\mathbf{2}}) \in [\rho_{\text{LB}}, \rho_{\text{UB}}]^{n_{\text{CP}}} \times [\omega_{\text{LB}}, \omega_{\text{UB}}]^{n_{\text{CP}}} \mapsto \rho \in \mathcal{L}\big(D, [\rho_{\text{LB}}, \rho_{\text{UB}}]\big).$$

Thus, TO problem of compliance minimisation under area constraint reads:

$$\min_{\tilde{\boldsymbol{\xi}}_{\mathbf{1}}, \tilde{\boldsymbol{\xi}}_{\mathbf{2}}} \frac{C(\tilde{\rho}(\tilde{\boldsymbol{\xi}}_{\mathbf{1}}, \tilde{\boldsymbol{\xi}}_{\mathbf{2}}))}{\|C_{\text{ref}}\|},$$
$$\text{subject to:}$$
$$\begin{cases} \dfrac{A(\rho_e)}{A_{\text{ref}}} \leq \gamma, \\ \tilde{\boldsymbol{\xi}}_{\mathbf{1}} \in [\rho_{\text{LB}}, \rho_{\text{UB}}]^{n_{\text{CP}}} \,,\; \tilde{\boldsymbol{\xi}}_{\mathbf{2}} \in [\omega_{\text{LB}}, \omega_{\text{UB}}]^{n_{\text{CP}}} \,, \end{cases}$$

where $\rho_{\text{LB}} = 10^{-3}$, $\rho_{\text{UB}} = 1$, $\omega_{\text{LB}} = 0.5$ and $\omega_{\text{UB}} = 10$ are the bounds on the design variables, whilst C_{ref} and A_{ref} are the reference values for compliance and area, respectively, and γ is the area fraction defined by the user. The formal expression of the gradient of the generalised compliance and of the area can be found in [24].

3 Filtering of the density field

The first approach for handling the wavy boundary problem is to apply a linear filter to the density distribution function. The idea is to smooth the value of the control point ρ_τ based on the values of its neighboring control points ρ_i, $i \in \Omega_\tau$, where $\Omega_\tau \subset \{1, \dots, n_{\text{CP}}\}$. The quality of the filter highly depends on the choice of the neighborhood Ω_τ; a common choice is based on the distance between the considered CP and those falling in Ω_τ as is the case for Shepard's filter used in [19], where

$$\Omega_\tau = \{\rho_i \,|\, d(\rho_i, \rho_\tau) < r, \, r > 0\}.$$

Another choice is based on windows, where a constant number n_w of CPs is taken around ρ_τ; this choice of neighborhood points is characteristic of least-squares filters, such as Savitsky-Golay filters [25]. Thus the filtered pseudo-density field described through a B-spline surface can be written as

$$\begin{aligned} \hat{\rho}(x_1, x_2) &= \sum_{\tau=0}^{n_{\text{CP}}} R_\tau(x_1, x_2)\hat{\rho}_\tau \,\in [\rho_{\text{LB}}, \rho_{\text{UB}}], \\ &= \sum_{\tau=0}^{n_{\text{CP}}} \sum_{i \in \Omega_\tau} R_\tau(x_1, x_2)\psi(\rho_i)\rho_i, \end{aligned} \tag{7}$$

where $\psi(\rho_i) \in \mathbb{R}$ is a general filtering weight function.

4 Smoothing of gradient

The second approach is the filtering of the sensitivity data $\frac{\partial C}{\partial \xi_\tau}$. Smoothing the sensitivity data is a widely known practice which allows to avoid numerical instabilities in optimisation algorithms. However, there is no general rule for an efficient smoothing of the sensitivity data which ensures good convergence, since the smoothing filters parameters are often case-dependent. Here we use a traditional linear filter as discussed in [17], where the smooth sensitivity is defined as

$$\overline{\frac{\partial C}{\partial \xi_\tau}} = \frac{1}{\xi_\tau \sum_{i \in \Omega_\tau} H_{i\tau}} \sum_{i \in \Omega_\tau} H_{i\tau} \xi_i \frac{\partial C}{\partial \xi_i}, \tag{8}$$

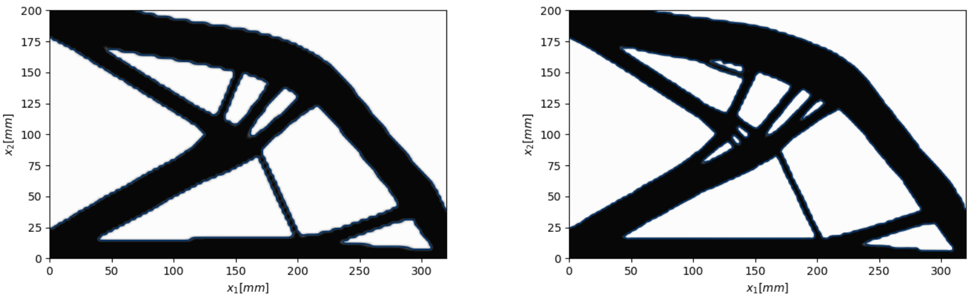
where $H_{i\tau} = \max(0, r_{\min} - d(\rho_i, \rho_\tau))$, and $r_{\min}$ is the filter radius.

5 Post-processing of the boundary

The third and final approach is the post-processing of the computed optimal density distribution to get rid of oscillating boundary. This can be done by traditional methods of boundary filtering, or more sophisticated approaches like smooth surface fitting [22] where NURBS entities are used to fit complex shape with noisy boundary by employing an original formulation involving the second-order partial derivative of the NURBS entity to avoid oscillating boundary. Other surface reconstruction techniques, well known in the field of meshing, such as the screened Poisson surface reconstruction [23], can be used to reconstruct a smooth version of the tessellation. Unlike the above approaches, the post-processing of the boundary is not case-dependent and can be done very efficiently and often produces results that have a similar performance to the computed optimal ”wavy” structure.

6 Preliminary results

For our numerical results, we consider the standard benchmark of a cantilever plater of height $H = 200\ mm$, width $W = 320\ mm$, thickness $t = 2\ mm$. The Young's modulus of the isotropic linear elastic material is $E = 1MPa$, and Poisson ratio is $\nu = 0.3$. The FE static analysis is solved on a mesh of 160×100 plane elements (4 nodes and 2 degrees of freedom per node) with a zero Dirichlet boundary condition along the left edge $X_1 = 0$ and is subject to an applied force $\mathbf{f^T} = (0, -1)$ on the bottom right corner $X = (W, 0)$. The pseudo-density field is described by B-spline surface of degrees $p = q = 2$ and with a number of CPs along each parametric direction set as $(n_1 + 1) = 140$, and $(n_2 + 1) = 85$. All the simulations presented in this section have been carried out by using the globally-convergent method of moving asymptotes [26] with the default value of the parameters tuning its behaviour.



$C = 57.12\,\mathrm{N}$ $\qquad$ $C = 55.77\,\mathrm{N}$

Figure 1: Optimised densities using B-spline entities (left), and NURBS entities (right).

6.1 Unfiltered optimised topologies

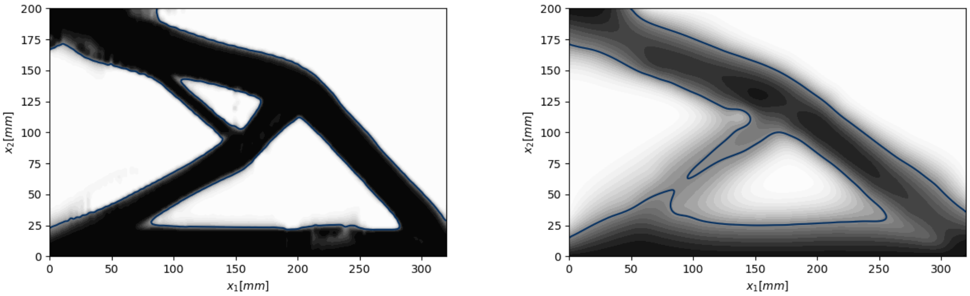
Figure 1 shows the computed optimal densities using both B-Spline and NURBS entities. As stated earlier, the problem of oscillating boundary is exagerated on the B-spline solution, whilst the NURBS one is able to provide an optimised topology with smoother boundary. In agreement with results presented in [12], the NURBS solution is characterised by better performances in terms of generalised compliance but at the price of a higher number of design variables (twice the number of design variables of the B-spline solution).

6.2 Influence of the linear filtering technique on the optimised topology

In agreement to the statements in [17], filtering the density gives bad results as shown in Fig 2. This is mainly due to the coupling introduced among the densities evaluated at the CPs due to the nature of applied filters, which makes convergence very difficult. In this context, the least-squares filters like Savitzky-Golay are the worst choice when filtering densities since they require a minimal window of 9 CPs.

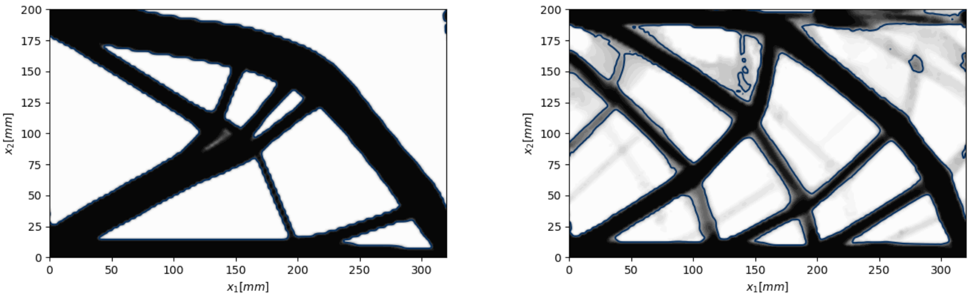
6.3 Influence of the gradient smoothing on the optimised topology

As expected, the smoothing of the sensitivity data, is a case-dependent approach. One must choose the radius of the filter, B-spline degrees and number of CPs carefully in order to converge. For our test case we used a uniform distribution of CPs, and we set $\delta = \max(\frac{a_1}{n_1+1}, \frac{a_2}{n_2+1})$ and $r_{\min} = c\delta$. As shown in Fig. 3, the optimised topology has been determined by using different values of the coefficient c. From the analysis of these results, one can infer that the higher is the smoothing radius the smoother the boundary. However, as the radius increases, convergence problems arise and the



$C = 63.33\,\mathrm{N}$ $C = 522.04\,\mathrm{N}$

Figure 2: Computed final filtered densities using shepard's filter [19] (left), and Savitzky-Golay filter [25] (right).



$C = 57.56\,\mathrm{N}$ $C = 78.04\,\mathrm{N}$

Figure 3: Computed final filtered densities using a smooth gradient with radius $r_{\min} = 1.5\delta$ (left), and $r_{\min} = 2\delta$ (right)

optimised solution is characterised by meaningless intermediate values of the pseudo-density.

7 Conclusions

The use of filters during the optimisation process does not seem to be a good approach for topology optimization using B-spline density-based method. Indeed, both B-spline and NURBS entities are characterised by a local support property, which constitutes a kind of filter that strongly reduces mesh-dependency of the optmised topology and checker-board effect. Therefore adding another layer of filtering makes convergence very difficult. It is the authors opinion that the post-processing of the computed densities is a better strategy, which is not case-dependent unlike the use of filters. We shall present comparison results for post-processing strategies along with a comparison of their performances during the presentation.

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