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Optimizing Transportation Plans of Designated Radioactive Waste Using Quantum Annealing

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Abstract

To optimize the transportation of designated waste from the Great East Japan Earthquake, which contains radioactive material and poses risks to the environment and public safety, a cost function was formulated to create a transportation plan using quantum annealing, a computational method that specializes in solving combinatorial optimization problems. This method can be performed using actual quantum annealing machines with more than 5,000 qubits that are currently available. The study evaluates the feasibility of using quantum annealing to optimize the transportation planning of designated waste and increase efficiency while minimizing risks.

Keywords: transportation planning, quantum annealing, cost function, optimization

1 Introduction

Radioactive designated waste from the Great East Japan Earthquake is transported with Global Navigation Satellite System (GNSS)-based central management to avoid disruption, reducing the risk of waste pillage and air dose increase. Approximate solutions are used to optimize the pre-established plan due to the problem's complexity.

Interest in quantum computers has been rapidly increasing in recent years, with the development of quantum gate-based computers and applications such as Variational Quantum Eigensolver (VQE) and Quantum Approximate Optimization Algorithm

(QAOA), but their practical significance for conventional computing is yet to be confirmed due to the requirement for a quantum computer with over one million qubits that does not currently exist [1], [2], [3]; meanwhile, quantum annealing, a computational method for solving combinatorial optimization problems, has been proposed and commercialized, with machines with over 5,000 qubits now available, which works by gradually weakening a transverse magnetic field applied to an Ising model to obtain a combination of qubits with the lowest energy, with the objective function and constraints of the problem expressed as Quadratic Unconstrained Binary Optimization (QUBO) using this model, and is being studied for solving real-world social problems [4], [5], [6], [7].

In this study, we used quantum annealing to optimize the transportation planning of designated waste by formulating a cost function that considers the surrounding environment. The study aims to evaluate the feasibility of this approach.

2 Methods

Quantum annealing Quantum annealing was proposed in 1989 [4] as an optimization method inspired by simulated annealing [8] uses quantum fluctuations to search for the ground state with the lowest energy and has potential in solving complex optimization problems in various fields, as demonstrated by studies such as optimizing vehicle routes to mitigate traffic jams [9], optimizing travel routes of Automated Guided Vehicles (AGVs) in a factory [10], and portfolio optimization [11]. Therefore, quantum annealing has potential in solving complex optimization problems, while simulated annealing is a global optimization method that uses thermal fluctuations.

In this study, a cost function was formulated to optimize the transportation of designated waste while considering constraints such as an equal number of transports from each loading facility, completing planned transports within working hours, and avoiding simultaneous arrivals/departures at unloading sites. Quantum annealing was used to solve the QUBO cost function for both normal and delayed transportation plans. This study demonstrates the potential of quantum annealing in optimizing transportation plans that satisfy specific constraints related to designated waste transport.

Figure 1 depict the problem created through quantum annealing optimization. The transportation time from the loading facility S_n to unloading facility E_m is denoted as $R_{n,m}$ and is calculated based on the number of squares in the transportation plan, with a 30-minute assumption for loading, transportation, and unloading time.

The transportation planning problem for designated waste from multiple loading facilities to multiple unloading facilities is transformed into a cost function equation (1) that can be solved using quantum annealing for combinatorial optimization.

$$H = \lambda_{obj}H_{obj} + \lambda_1 H_1 + \lambda_2 H_2 + \lambda_3 H_3 + \lambda_4 H_4 \tag{1}$$

here, H is the cost function in QUBO form that is optimized using quantum annealing, H_{obj} is the objective function, and H_1, H_2, H_3, H_4 are the constraints in the problem. $\lambda_{obj}, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ are the weights on each objective function term and constraint term.

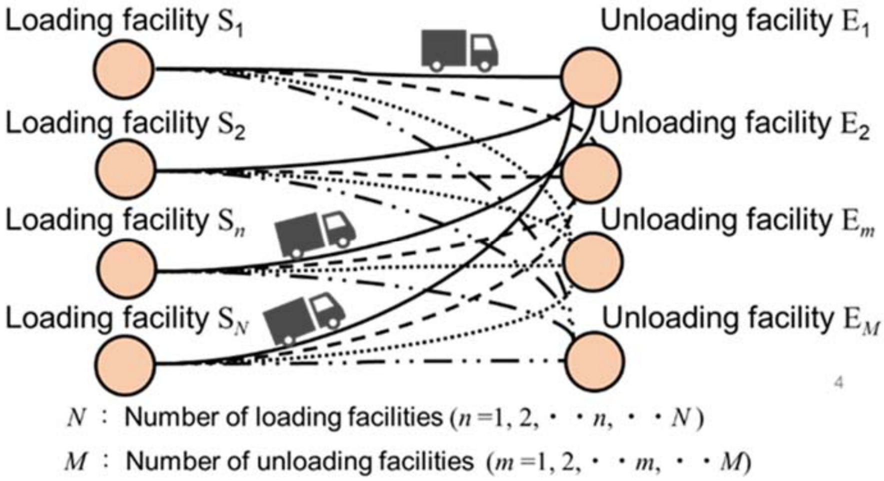

Figure 1: Schematic diagram of the study problem.

The objective function H_{obj} for the optimization problem of interest is expressed in Equation (2).

$$H_{obj} = -\sum_{k=1}^{NMT} q_k \tag{2}$$

Here, q_k is a qubit that outputs 0 or 1, N is the number of loading facilities, and T is the number of times at which transportation is to start. When the qubit q_k outputs 1, it means that the transport will be performed at the specified time, so when the objective function H_{obj} is the smallest, the plan is to perform the most transports. The constraint H_1, which adjusts the number of transports from the loading facility S_1 to the same level as the number of transports from other loading facilities, is expressed in Equation (3).

$$H_1 = \sum_{n=2}^{N}\left(\sum_{m=1}^{M}\sum_{k=(m-1)NT+1}^{(m-1)NT+T} q_k - \sum_{m=1}^{M}\sum_{k=(m-1)NT+(n-1)T+1}^{(m-1)NT+nT} q_k\right)^2 \tag{3}$$

When H_1 is the smallest, the number of transports from each loading facility is the same, and a transportation plan without bias by the facility is created. The constraint H_2 to avoid transportation that cannot arrive within working hours is expressed by Equation (4).

$$H_2 = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=(m-1)NT+nT-R_{n,m}}^{(m-1)NT+nT} q_k \tag{4}$$

When H_2 is the smallest, all transportation is planned to be completed within the working hours. The constraint H_3, which prevents the overlapping of the unloading times of multiple transportations, is expressed in Equation (5).

$$H_3 = \sum_{m=1}^{M} \sum_{n=2}^{N} \sum_{l=1}^{n-1} \sum_{k=1}^{T+R_{l,m}-R_{n,m}} \left(q_{k-R_{l,m}+R_{n,m}+(m-1)NT+(l-1)T} + q_{k+(m-1)NT+(n-1)T} - \frac{1}{2} \right)^2 \tag{5}$$

In Equation (5), $q_{k-R_{l,m}+R_{n,m}+(m-1)NT+(l-1)T}$ and $q_{k+(m-1)NT+(n-1)T}$ represent transports that start from different loading facilities and arrive at the unloading facility at the same time. Therefore, when $q_{k-R_{l,m}+R_{n,m}+(m-1)NT+(l-1)T}$ and $q_{k+(m-1)NT+(n-1)T}$ are both equal to 1, overlapping unloading times occur. In Equation (5), the value of $\left(q_{k-R_{l,m}+R_{n,m}+(m-1)NT+(l-1)T} + q_{k+(m-1)NT+(n-1)T} - \frac{1}{2} \right)^2$ is equal to the value of $q_{k-R_{l,m}+R_{n,m}+(m-1)NT+(l-1)T}$ and $q_{k+(m-1)NT+(n-1)T}$ are both 1, the value is 9/4; when either of them is 1, the value is 1/4; and when both of them are 0, the value is 1/4. As a result, when H_3 is minimized, the number of vehicles arriving at the unloading facility is either one or zero at each time, thus preventing duplication of unloading times. The constraint H_4 to prevent the initiation of multiple types of transport from one loading facility to different unloading facilities at the same time is expressed in Equation (6).

$$H_4 = \sum_{n=1}^{N} \sum_{k=1}^{T} \left(\sum_{m=1}^{M} q_{(m-1)NT+k+(n-1)T} - \frac{1}{2} \right)^2 \tag{6}$$

We optimize the transportation plan of the designated waste by using quantum annealing to obtain the combination of outputs of qubits q_k that makes Equation (1), which is expressed in detail in Equations (2)-(6), the smallest. In this paper, we assign qubits q_k to each arrival point of the unloading facility at each transportation start time for one loading facility, so that NMT variables are required in the optimization of the transportation plan for the designated waste.

The optimization using the formulated cost function is performed on a superconducting qubit quantum annealing machine provided via the cloud by D-Wave [6], [11]. The machine has 5760 qubits, but it is not fully coupled and uses auxiliary qubits to represent the interaction between each qubit [12], [13]. As a result, the size of the problem that can be solved is smaller than the total number of qubits available.

3 Results

In an experiment to verify the validity of the cost function formulated in Chapter 3, optimization using quantum annealing and simulated annealing was conducted under the conditions in Table 1. The cost function was set to have 16 time cells for each 30-minute interval between 9:00 and 17:00 working hours.

In this section, we evaluate the accuracy of optimization by the number of qubits of the actual quantum annealing machine, by specifying the number of loading and unloading facilities from two to four points and creating a cost function for each of them. Previous studies have provided guidelines for determining the weights of each term of the cost function used in the optimization by quantum annealing [14], [15]. Empirically, the weights should be set so that the amount of decrease in the objective function term due to the failure to satisfy the constraints does not exceed the amount of increase in the constraint term. In Equation (1), λ_2, λ_3, and λ_4 should be set larger than λ_{obj} to avoid the overlap of unloading times and complete the transportation within working hours. The weights λ of each term are determined in the order of 1 to 4 below, as completing the transportation within working hours is crucial for protecting the working environment of workers and ensuring the timely completion of transportation. Avoiding the overlap of unloading times is also essential to prevent the increase of air dose and the risk of waste leakage due to vehicles staying near the unloading facilities.

1. Transportation must be completed during working hours.
2. Avoid duplication of loading and unloading times.
3. Maximize the number of transports per day.
4. Maintain the same number of transports from each loading facility.

	Parameters	Value
Weight of the objective function term	λ_{obj}	10
Weight of the constraint term	λ_1	5
Weight of the constraint term	λ_2	100
Weight of the constraint term	λ_3, λ_4	50
Transportation time	$R_{1,1}$	1
Transportation time	$R_{1,2}$	2
Transportation time	$R_{1,3}$	3
Transportation time	$R_{1,4}$	4
Transportation time	$R_{2,1}$	2
Transportation time	$R_{2,2}$	3
Transportation time	$R_{2,3}$	4
Transportation time	$R_{2,4}$	5
Transportation time	$R_{3,1}$	3
Transportation time	$R_{3,2}$	4
Transportation time	$R_{3,3}$	5
Transportation time	$R_{3,4}$	6
Transportation time	$R_{4,1}$	4
Transportation time	$R_{4,2}$	5
Transportation time	$R_{4,3}$	6

Table 1: Execution conditions.

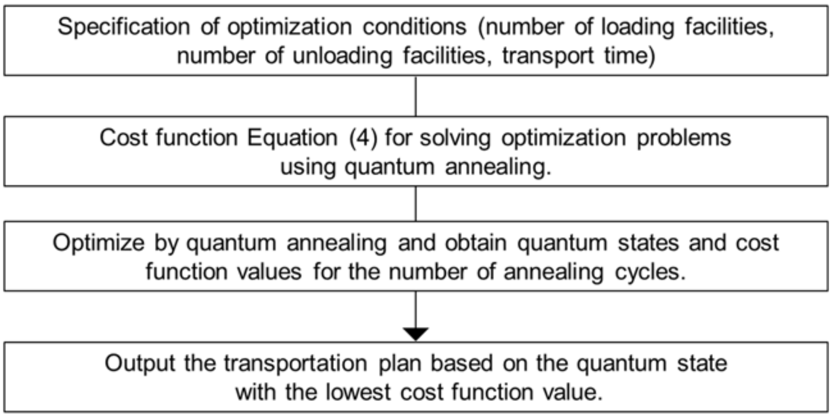

Figure 2: Optimization flow.

The weight of the second term in constraint condition 4 was set to be smaller than that of the objective function term to account for the variation in the number of units transported from each loading facility. Figure 6 illustrates the optimization process using the D-Wave superconducting qubit machine.

The transportation plan was created based on the quantum state with the smallest value of the cost function obtained from repeated annealing. For experiments using simulated annealing, dwave-neal [16], one of D-Wave's Ocean software development kits, was used. The execution environment for simulated annealing is based on Windows 10 Home, with the CPU of Intel® Core™ i5-9400, and RAM of 8.0 GB.

Table 2 summarizes the experimental results of optimization using the actual quantum annealing machine. The results show the probability of satisfying the constraint conditions and the optimal solution for each number of loading facilities. The number of variables used in the cost function differs from the number of qubits used in the actual machine, and the probability of completing transportation within working hours and avoiding overlap between loading and unloading times are important constraint conditions. The optimal solution is the output with the smallest value of the cost function while satisfying the constraints.

Number of loading facilities	Number of unloading facilities	Variables	The actual number of qubits	Probability of finding the optimal solution [%]	The difference in cost function value from the optimal solution	Probability of satisfying the constraint [%].
2	2	56	545	0	35	64.4
3	2	96	1033	0	100	6.7
4	2	128	1780	0	105	1.9
3	3	144	2471	0	235	0.6
4	3	192	3891	0	250	0

Table 2: Optimization results by quantum annealing.

Parameter	Value
Initial state	Random
Initial temperature	10
Endpoint temperature	0.1
Number of inner loops	1000
Number of outer loops	1000

Table 3: Parameter settings for simulated annealing.

Simulated annealing was compared to quantum annealing for optimization using 1000 runs per cost function for various numbers of loading and unloading facilities, with parameter settings for simulated annealing listed in Table 3 and results shown in Table 4.

Table 2 shows that the optimization using quantum annealing satisfied the constraints for samples with variables up to 144 but failed for the samples with 4 loading and 3 unloading facilities due to overlapping unloading times. Simulated annealing, on the other hand, detected the optimal solution in all samples. Figure 2 shows the time required for both methods, with QA referring to quantum annealing and SA to simulated annealing.

Number of loading facilities	Number of unloading facilities	Variables	Probability of finding the optimal solution [%]
2	2	56	99.7
3	2	96	99.9
4	2	128	99.9
3	3	144	91.1
4	3	192	98.7

Table 4: Optimization results by simulated annealing.

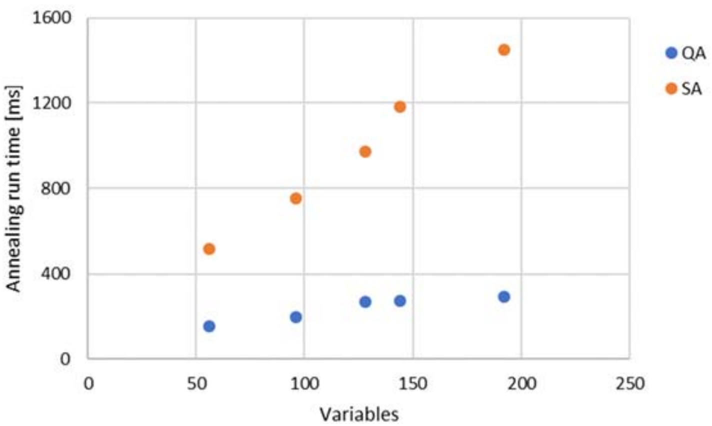

Figure 2: Annealing execution time for optimization.

4 Conclusions and Contributions

This study compared quantum annealing and simulated annealing for optimizing transportation plans for designated waste. Quantum annealing struggled with convergence as the number of qubits increased, while simulated annealing produced the best results. The study highlights the importance of designing an appropriate cost function and demonstrates the current limitations of quantum annealing in solving complex optimization problems. Future work includes evaluating optimization results with various parameter settings and defining additional constraints in the cost function. Overall, this study emphasizes the importance of carefully designing the cost function for desired outcomes.

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