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Enhancing thermal topology optimization with an elasto-plastic algorithm

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Abstract

This paper presents an approach to optimize the design of structures under high temperature conditions by employing bi-directional evolutionary structural optimization (BESO) in elasto-plastic limit analysis. The elasto-plastic design approach considers plastic ultimate load multiplier. By adopting the BESO method, the optimal material distribution within the design domain is identified to achieve the desired structural performance while minimizing material usage. Different thermal loads were considered for elastic and elasto-plastic designs, and the results show how material layouts, mean stress, and complementary work resulted differently according to those thermal loads. The effectiveness of the proposed approach was demonstrated through numerical results, highlighting the potential to improve the optimization of elasto-plastic design of structures under high temperature conditions. The presented approach offers an efficient and robust design method for structures subjected to high temperature conditions, which can improve their safety and durability.

Keywords: elasto-plastic, geometrically nonlinear, thermal analysis, topology optimization, BESO

1 Introduction

Structural components are often exposed to high temperatures during their lifetime, which can lead to irreversible changes in their material properties, including reduced strength, stiffness, and ductility. High temperature exposure can be caused by various environmental factors, such as fire or extreme heat exposure, and can pose a significant risk to the structural integrity of the component. Nan et al. [1] modelled and analyzed a steel-composite floor structure considering various fire scenarios while considering the impact of a concrete slab, using a 3D finite element model. Also, Qian et al. [2] conducted an experimental investigation of standard steel extended end-plate beam-to-column joints under elevated temperatures.

To ensure that structural elements can withstand high temperature conditions, their design must account for elasto-plastic behavior. The elasto-plastic design approach considers the nonlinear behavior of materials under load, particularly in the plastic range. This approach is essential to accurately predict the behavior of the structure under extreme loading conditions [3,4].

To further enhance the design of structures for high temperature conditions, topology optimization (TO) methods have been developed to identify the optimal material distribution within the design domain to achieve the desired structural performance. Bi-directional evolutionary structural optimization (BESO) is one such TO method that has been proven effective in designing optimal structural systems under complex loading and boundary conditions[5–7]. Movahedi et al.[8] presented a methodology for conducting reliability-based geometrically nonlinear topology optimization using elasto-plastic limit analysis. Furthermore, BESO method was considered for geometrically nonlinear elasto-plastic design in the work of Habashneh and Movahedi [9].

Recently, thermoelastic designs were considered for structural optimization by adopting BESO method in the work of Habashneh and Movahedi [10]. However, the present paper contributes to the field of structural engineering by addressing the need for more efficient and robust design methods for structures which are subjected to high temperature conditions. Thus, improve the safety and durability of structures in various applications. In particular, the proposed approach developed BESO method to optimize the design elasto-plastic temperature problems, which is a novel application of this method. The results of this paper demonstrate the effectiveness of the proposed approach in achieving optimal designs that maximize structural performance while minimizing material layout. By presenting these original contributions, the proposed algorithm makes a significant contribution to the field of structural design for high temperature conditions.

The following sections will discuss the methodology, and numerical results, highlighting the potential of the proposed approach to enhance the optimization of elasto-plastic design of structures under high temperature conditions.

2 Methods

The proposed methodology contains an optimization problem with the objective function of mean compliance minimization and the defined constraints which are related to volume fraction, applied loads and plastic-limit load multiplier. Furthermore, it should be noted that the optimization problem is done in the form of iterative scheme where at each iteration step the topology of the structure is updated.

The mentioned optimization problem can be mathematically written as follows:

$$Minimize: C = u^T K u \tag{1.a}$$

$$Subject\ to:\ V^* - \sum_{i=1}^{N} V_i x_i = 0 \tag{1.b}$$

$$\frac{V^*}{V_0} - V_f \leq 0 \tag{1.c}$$

$$x_i \in \{0,1\} \tag{1.d}$$

$$Ku = f \tag{1.e}$$

$$m_s - m_p \leq 0 \tag{1.f}$$

The mean compliance is denoted by C, displacement vectors are represented by u, and the global stiffness matrix is symbolized by K. Additionally, V_i refers to the volume of each individual element, V^* represents the total volume of the structure, and N denotes the total number of elements present.

Furthermore, the volume of the design domain is indicated by V_0, while the proportion of volume fraction is denoted by V_f. In addition, the binary design variable x_i takes a value of (0) to indicate the absence of an element, or a value of (1) to indicate its presence. It is worth noting that the loading vector includes both mechanical loading f_m and thermomechanical loading f_{heat}, which are applied in combination as $f = f_m + f_h$.

Equation $(1.f)$ imposes a constraint on the plastic limit load multiplier. Based on the principle of statics, any load multiplier that is statically admissible, denoted as m_s, must be less than or equal to the plastic limit load multiplier, m_p, which corresponds to the entire design domain.

It is important to note that the displacement parameter u in Equation $(1.a)$ is the outcome of a nonlinear thermal elastoplastic analysis. The plastic limit load multiplier constraint $(m_s - m_p \leq 0)$ determines when the structure will experience complete plastic collapse and the optimization process will terminate.

3 Results

To demonstrate the effectiveness of the optimization procedure outlined, two numerical examples will be examined. The first example involves a pinned plate, which is considered for geometrically nonlinear thermoelastic topology optimization as shown in Figure 1. Young's modulus of the material is $210\ GPa$, and Poisson's ratio is 0.30. In addition, the selected material has a thermal expansion coefficient of 12×10^{-6} and the thickness of the plate is $10\ mm$. Considered applied loads include $F = 15\ kN$ (mechanical load) and three distinct temperature conditions, $\Delta T = 0\mathrm{C}^0, 20\mathrm{C}^0, and\ 35\mathrm{C}^0$. In this example, the value of volume fraction (V_f) is set to be 0.40.

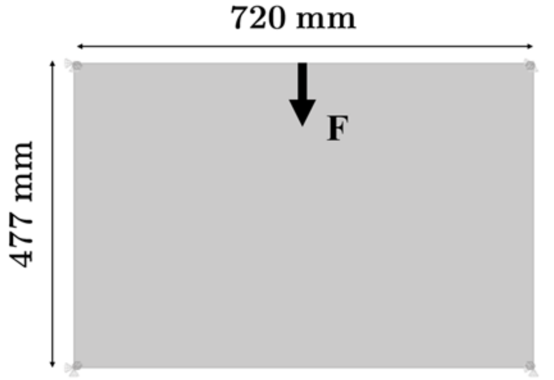


Figure 1. design domain of pinned plate problem

According to the applied loads, the optimal solution and corresponding complementary work (W^c) values are presented in Table 1. By considering different thermal loads (ΔT), various topologies can be achieved, as evident from the produced layouts. It is worth mentioning that the displacements of all models are analyzed using W^c as a basis, thus a comparison between the obtained W^c is also considered. The results indicate that an increase in the applied thermal load leads to a corresponding increase in the complementary work. For example, it was found that complementary work increased by 30.96% from $3.59kJ$ in case of $\Delta T = 0C^0$ to $5.20kJ$ in case of $\Delta T = 35C^0$.

Applied loads		Optimized shape	Complementary work (W^c) - kJ
Mechanical load (kN)	Thermal load (ΔT)- C^0		
15	0		3.59
	20		4.04
	35		5.20

Table 1: The results of optimized shapes and complementary work of pinned plate example

The second example, depicted in Figure 2, involves a U-shaped structure and is subjected to topology optimization for elasto-plastic thermal design. In this example, the initial predetermined load for the model is $F_0 = 5\ kN$, and the plastic limit load multiplier is $m_p = 3.15$. Therefore, the ultimate load for the entire design is $F_{\text{ult}} = m_p \times F_0 = 15.75\ kN$. To illustrate the impact of the load multiplier, three scenarios are presented: $F_1 = 0.10 \times F_0$, $F_2 = 2.20 \times F_0$, and $F_3 = 3 \times F_0$. Also, three distinct temperature conditions are considered which are $\Delta T = 0\text{C}^0\ , 20\text{C}^0, and\ 35\text{C}^0$. The selected Young's modulus of the material is $70\ GPa$, yield stress of $195\ MPa$, and Poisson's ratio is 0.30. The thermal expansion coefficient is 23×10^{-6} and the thickness of the plate is $10\ mm$. The volume fraction (V_f) is set to be 0.50. It should be noted that the considered BESO parameters for both examples are $ER = 2\%$, $AR_{max} = 1\%$, $r_{min} = 60\ mm$ and $\tau = 0.1\%$.

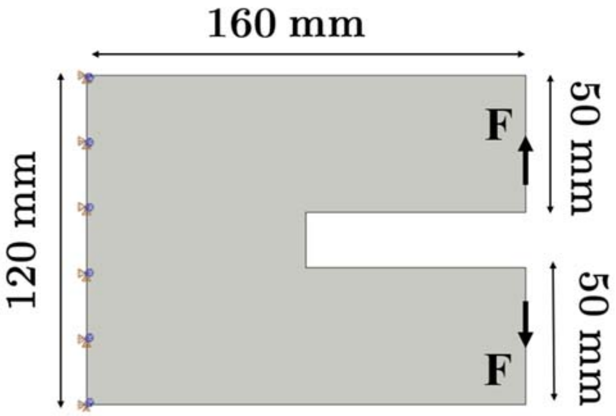


Figure 2. design domain of U-shape problem

Table 2 presents the results of analyzing the impact of the plastic-limit load multiplier in thermal designs. As anticipated, the lightest load condition exhibited the least occurrence of plastic zones. In contrast, plastic areas were observed in the second scenario, while significant plastic zones were generated in the third scenario. The obtained layouts revealed that varying the load multiplier resulted in distinct material distributions for different values of ΔT. Additionally, each load multiplier generated different topologies. As expected, the absence of plastic zones is most apparent at the lightest load condition.

In the second scenario, however, we see plastic areas. Large plastic areas are produced in the third scenario. Also, it can be noted that the mean stress within the resulted shape increases as ΔT increase for each loading scenario. For example, when ΔT changed from 0C^0 to 35C^0, the mean stress in the case of F_1 increased by 22.16% from $21.95\ MPa$ to $28.20\ MPa$.

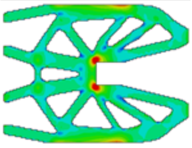
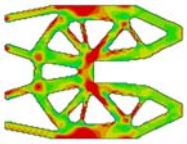
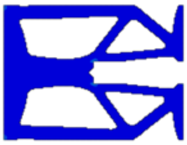
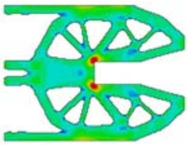
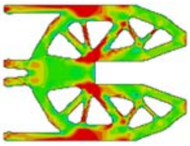
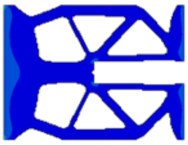
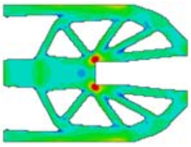
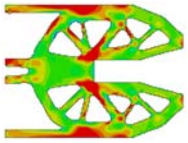
Applied loads		Optimized shape	Mean stress (MPa)	Applied loads		Optimized shape	Mean stress (MPa)	Applied loads		Optimized shape	Mean stress (MPa)
F1(kN)	$\Delta T = 0C^0$		21.95	F2 (kN)	$\Delta T = 0C^0$		77.32	F3 (kN)	$\Delta T = 0C^0$		134.17
	$\Delta T = 20C^0$		24.15		$\Delta T = 20C^0$		80.12		$\Delta T = 20C^0$		136.90
	$\Delta T = 35C^0$		28.20		$\Delta T = 35C^0$		84.61		$\Delta T = 35C^0$		139.24
											

Table 2: The results of optimized shapes and mean stress of U-shape example

4 Conclusions and Contributions

In this paper, the optimal solutions in the case of elastic and elasto-plastic thermal designs were proposed by utilizing the developed BESO algorithm. The analysis of different thermal loads revealed that different topologies can be achieved, and an increase in thermal load results in an increase in complementary work. The impact of plastic-limit load multiplier on thermal designs was also evaluated, and the results showed that plastic zones were most apparent under high loads. It was also observed that varying the load multiplier generated different topologies and distinct material distributions. Finally, the mean stress within the resulted shape increased with an increase in thermal load. These findings provide valuable insights into the behavior of thermal designs under different load conditions, which can aid in the development of more efficient and robust topological designs.

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