Proceedings of the Sixth International Conference on
Soft Computing, Machine Learning and Optimisation in
Civil, Structural and Environmental Engineering
Edited by: P. Iványi, J. Lógó and B.H.V. Topping
Civil-Comp Conferences, Volume 5, Paper 1.3
Civil-Comp Press, Edinburgh, United Kingdom, 2023
doi: 10.4203/ccc.5.1.3

Computational framework for a family of methods for stress-constrained topology optimization

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Abstract

Topology optimization is a complex engineering problem that requires significant computational effort. In this study, we propose a unified computational framework that combines stress-limited topology optimization with various types of constraints. These constraints may include plastic material, reliability analysis or low-cycle fatigue, all of which taking into account inherent uncertainties. Our framework utilizes efficient code implemented in MATLAB environment, which is based on the stress intensity in each finite element. We demonstrate the advantages of using the object-oriented programming paradigm, which is often used in numerical computations.

The proposed framework incorporates safety assessment in the topology optimization process, while also considering the number of cycles for plasticity involving fatigue. We apply the First Order Reliability Method (FORM) for safety control with a performance function based on the number of failure cycles under a complex, multi-level load program. We also use the Reliability Index Approach (RIA) and Performance Measure Approach (PMA) algorithms to account for uncertainties involved in the design problem.

The presented numerical examples show the dependence of the volume fraction on the probability of failure. Our framework is validated on a real experiment and utilizes cubic shape functions, which makes the experimental and numerical results

almost identical in the case of fatigue-resistant design of structural joint under bi-axial tension.

Overall, the proposed software architecture provides a robust and efficient solution for topology optimization and low-cycle fatigue analysis in engineering design. Its object-oriented class hierarchy provides several advantages such as ease of code maintenance and scalability.

Keywords: topology optimization, stress constraints, fatigue, reliability analysis, probabilistic design, robust design, object-oriented programming,plasticity.

1 Introduction

In the recent years, structural optimization has become the very popular engineering research topic. The starting date of structural optimization and in particular topology optimization, cannot be precisely determined. However, as it was stated in review paper by Lógó and Ismail [1] Maxwell's idea presented in the paper published in 1870 is considered as the first publication in this area. For many researchers the beginning of the topology optimization begins with Michell's optimization paper, published in 1904.

Nevertheless these historical milestones nowadays bring us to the research on topology optimization of 3D multi-scale structures and its application in additive manufacturing [2].

Currently, several approaches for topology optimization has been implemented among which there are Solid Isotropic Material with Penalisation (SIMP), level sets or Bi-Directional Evolutionary Structural Optimization (BESO). The Authors of this study, however, worked out their own method based on fully stress design heuristic. This approach is described in paper by Błachowski et al [4] and detailed software architecture for the approach is presented by Tauzowski et al in [3].

After successful application of the method for elastic and elastoplastic structures, the Authors extended their stress constrained approach to reliability-based topology optimization [5].

In this study, a unified framework for stress constrained topology optimization will be presented allowing for solving such engineering problems as design of structures made of elastoplastic material, design of structures with prescribed safety level or design of structures with assumed number of cycles in plasticity involving fatigue analysis.

2 Methods

This study formulates topology optimization problem as a size optimization under stress constraints. The derivation of the method starts with general formulation of structural optimization:

Find $\boldsymbol{x} = [x_1, x_2, \dots, x_M]^T$ which minimizes $f(\boldsymbol{x})$
subject to the constraints $g_i(\boldsymbol{x}) = 0,\ i = 1,2, \dots, N$

$$h_j(\boldsymbol{x}) \leq 0,\ j = 1,2, \dots, P \tag{1}$$

where
$\boldsymbol{x}$ is an M- dimensional vector called the ***design vector***
$f(\boldsymbol{x})$ is termed the ***objective (cost) function***
$g_i(\boldsymbol{x})$ and $h_j(\boldsymbol{x})$ are known as ***equality*** and ***inequality constraints***, respectively.

In the proposed approach the objective function $f(\boldsymbol{x})$ represents overall volume of the structure, equality constraints $g_i(\boldsymbol{x})$ are equilibrium equations and the inequality constraints $h_j(\boldsymbol{x})$ depend on the specific problem. Graphical representation of the initial and optimal topology is shown in Figure 1.

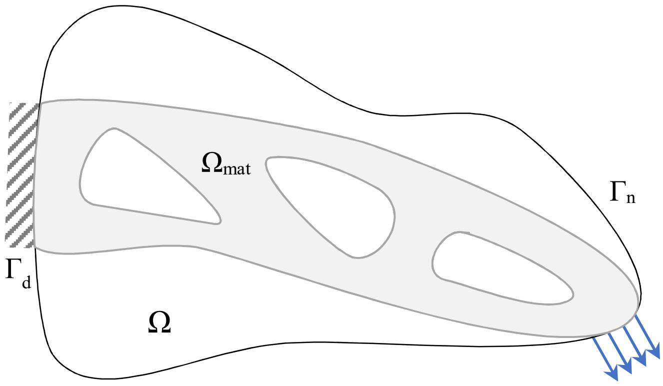

Figure 1: Design domain Ω vs optimal topology Ω_{mat}

Three particularly useful and frequently used type of inequality equations are:

1) $|\boldsymbol{\sigma}(\mathbf{x})| - \sigma_0 \leq 0$, in the case of stress constrained optimization
   where $\boldsymbol{\sigma}(\mathbf{x})$ denotes stress concentration at the optimal solution and σ_0 is allowable stress level

2) $\Pr(\boldsymbol{\sigma}(\mathbf{x})) - R_0 \leq 0$, in the case of reliability based optimization
   where $\Pr(\sigma(\mathrm{x}))$ represents probability of occurrence of given stress level at the optimal topology and R_0 is a threshold value

3) $N_{\mathrm{R}}(\mathbf{x}) - N_{\mathrm{t}} \leq 0$, in the case of low-cycle fatigue
   where $N_{\mathrm{R}}(\mathbf{x})$ is number of cycles at optimal design and N_{t} represents permissible value

Next, based on the various constraints mention above one can build general framework for topology optimization driven by stress intensity.

3 Results

Based on the methodology proposed in the previous section the following computer implementation can be presented:

```
Template Method (design pattern) for topology optimization

abstract class TopologyOptimization {
  public void AlgorithmTemplate {
    while( error > threshold ) {
      feModel = createFiniteElementModel();
      objectiveFunction = determineObjectiveFunction( ... );
      constraintFunctions = determineCostraints();
    }
  }
  double determineObjectiveFn ( elementVolumes, elementDensities) {
     return sum( elementVolumes, elementDensities )
  }
  abstract void determineConstraints() { }
}
```

The above code utilizes design pattern known as Template Method. It allows to isolate the general topology optimization problem in a form of an abstract class and then the particular concrete optimization problem under specific constraints can be defined separately. This implementation allows for well-organized way of extending the proposed framework.

```
Topology optimization for specific engineering problems

// Topology Optimization for elastic or elastoplastic constraints
class StressConstrainedOptimization extends TopologyOptimization {
  determineConstraints() {
     vonMisesCriteron ...
     plasticDeformation ...
  }
}

// Topology Optimization for reliability constraints
class ReliabilityBasedOptimization extends TopologyOptimization {
  determineConstraints() {
    FirstOrderReliabilityMethod() ...
  }
}

// Topology Optimization for fatigue constraints
class FaitgueConstrainedOptimization extends TopologyOptimization {
  determineConstraints() {
    numberOfCycles ....
  }
}
```

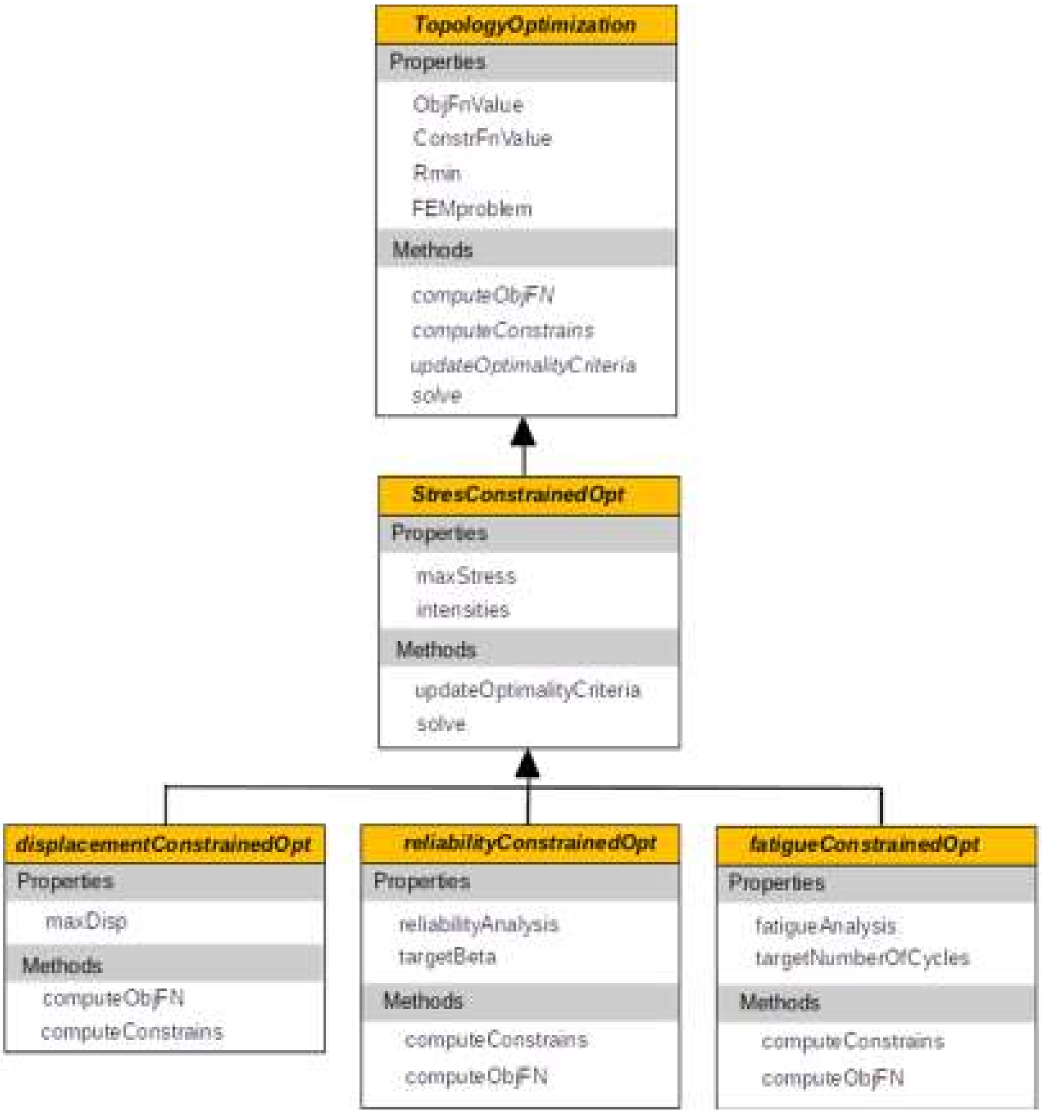

Figure 2: Class hierarchy of the proposed implementation

4 Conclusions and Contributions

In conclusion, the Authors proposed a unified computational framework for topology optimization that incorporates stress-constrained topology design with various constraints: including deterministic and probabilistic displacements or low-cycle fatigue. The proposed framework addressed also uncertainties inherent in design problems by employing the reliability assessment. The Authors demonstrated that their framework provides an efficient solution for topology optimization in various engineering design problems. The presented implementation shows that the framework's object-oriented class hierarchy provides ease of code maintenance and scalability, making it a robust solution. Finally, the proposed framework was validated as a useful tool for solving frequently occurring problems in engineering practise.

Acknowledgements

The present study was supported by the NKFI (grant K 138615) - Joint grant of the Hungarian and the Polish Academy of Sciences.

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