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## **Railroad track model parameter estimation by frequency response function analysis**

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### **Abstract**

This paper shows a procedure to determine the mechanical properties of a simple track flexible model to be used in the multibody simulation of railway vehicles. The procedure is based on the frequency response function of the track in the range of frequencies in which the simple model is desired to be used. This work applies, as an example, the proposed procedure to a scaled railroad track available at the Authors' facilities with a 10-DOF simple track model showing its applicability, range of operation, and ease of implementation, which can be easily extended at full-scale tracks with similar simplified track models.

**Keywords:** railway multibody dynamics, vehicle-track interaction, frequency response function, optimization.

### **1 Introduction**

The dynamic analysis of railway vehicles is a challenging task; it is a mechanical system formed by many connecting bodies that interact each other with the special treatment of the wheel-rail contact. In addition, the nonlinearities of the wheel-rail contact interface -i.e., conformal and non-conformal contact, multiple contact points or friction- result in simulation models with a high computational cost that lead many researchers in the field of computational dynamics to develop simulation models that efficiently deal with the dynamics of railway vehicles [1-3]. In this context, simulation models based on computational efficiency usually adopt the rigid body motion assumption because including flexibility in any connecting bodies or track clearly

increases the degrees of freedom of the system and its computational cost, which may lead to unfeasible simulations for dynamic purposes.

However, when the interest is focused on the dynamics of the railway vehicle due to the railway-track interaction (i.e., train-bridge interaction, wear patterns, response to track corrugation, RCF, etc), it is necessary to account for, at least, the track flexibility. In this sense, many research works can be found in the literature about vehicle-track interaction [4-8], which are especially focused on specific ranges of frequencies according to the scope of the study.

In this work, a procedure to determine the mechanical properties of a simple track flexible model to be used in the multibody simulation of railway vehicles is proposed. The procedure is based on the frequency response function of the track in the range of frequencies in which the simple model is desired to be used.

This work applies, as an example, the proposed procedure to a scaled railroad track available at the Authors' facilities with a 10-DOF simple track model showing its applicability, range of operation, and ease of implementation, which can be easily extended at full-scale tracks with similar simplified track models.

## 2 Methods

The main idea of this work is to determine the mechanical properties of a simplified track model to accurately reproduce the dynamics of the system within a specific range of frequencies. Therefore, it is first necessary to determine the system behaviour.

The system to be reproduced in this work is shown in Fig. 1 and it is a 5-inch gauge scaled railroad track located at the roof of the Engineering School of the University of Seville. It is a 90-meter length structure formed by straight, transition and curved sections in similarity to a full-scale track. It has supported mechanisms every 10-cm, to act as sleepers, but with the possibility to mechanically introduce track irregularities as vertical and lateral deviations of the track cross-section. These mechanisms are installed on metallic tables that allow to set up the desired vertical profile of the track regardless of the inclination of the building roof surface.



Figure 1. Scaled railroad track located at the University of Seville.

The vertical and lateral receptance of this system can be obtained by experimental tests or by FEM methods. In this work, a FEM model of the track that consists of 2-meter length rails supported on 21 mechanisms installed on a metallic plate is used to account for the Frequency Response Function (FRF) when a load is acting vertical and laterally on top of the rail surface. Figure 2 shows the CAD model on the left and the mesh of the FEM model at the right.

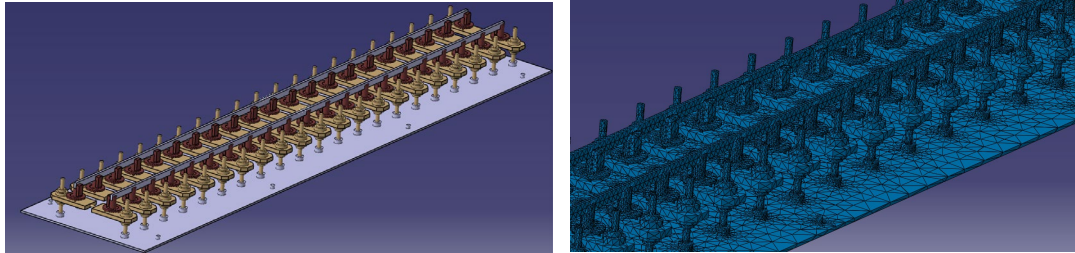


Figure 2. CAD model (left) and FEM mesh (right) of the 2-meter length scaled track model.

As a preliminary assumption, it is considered that the plate is rigidly attached at the 8 bolted connections. Such hypothesis leads to the following FRF (receptance) when a vertical load (Fig. 3 left) and lateral load (Fig. 3 right) is acting on the center of the rail.

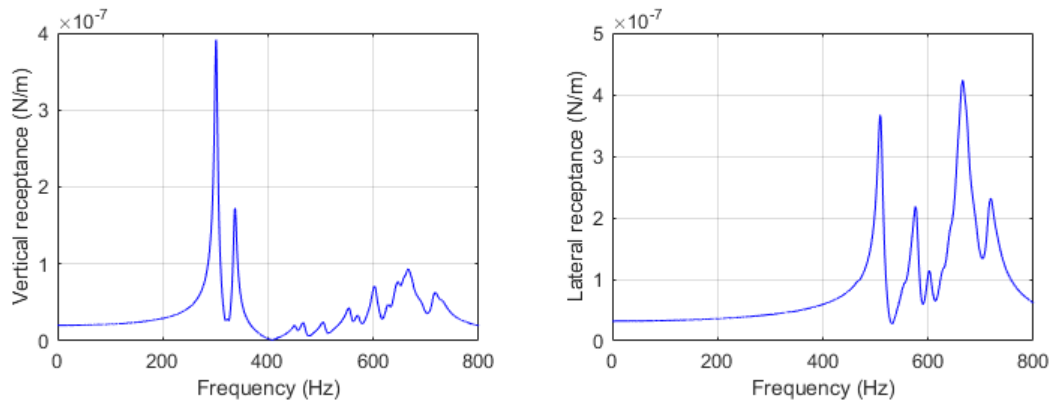


Figure 3. Vertical (right) and lateral (left) receptance of the track under vertical and lateral loads respectively

Once the system response is obtained, the next step is to derive the flexible track model whose mechanical parameters should be found to provide a similar response that the one obtained in Fig. 3. This will be derived in the following section.

### 3 Results

The simple flexible track model that will be used in this work is the one shown in Fig. 4. It is formed by two rails supported along three different suspension planes, with a total of 10 DOFs shown in blue font. In addition, there is a total of 19 parameters that define the model as follows:

- Mass and inertia properties of rails and suspension planes:  $m_1, m_2, m_3, m_4, I_3, I_4$ .
- Vertical and lateral stiffness:  $k_{1v}, k_{2v}, k_{3v}, k_{1l}, k_{2l}, k_{3l}$ .
- Vertical and lateral damping coefficients:  $c_{1v}, c_{2v}, c_{3v}, c_{1l}, c_{2l}, c_{3l}$ .
- Distance between rails:  $L_r$ .

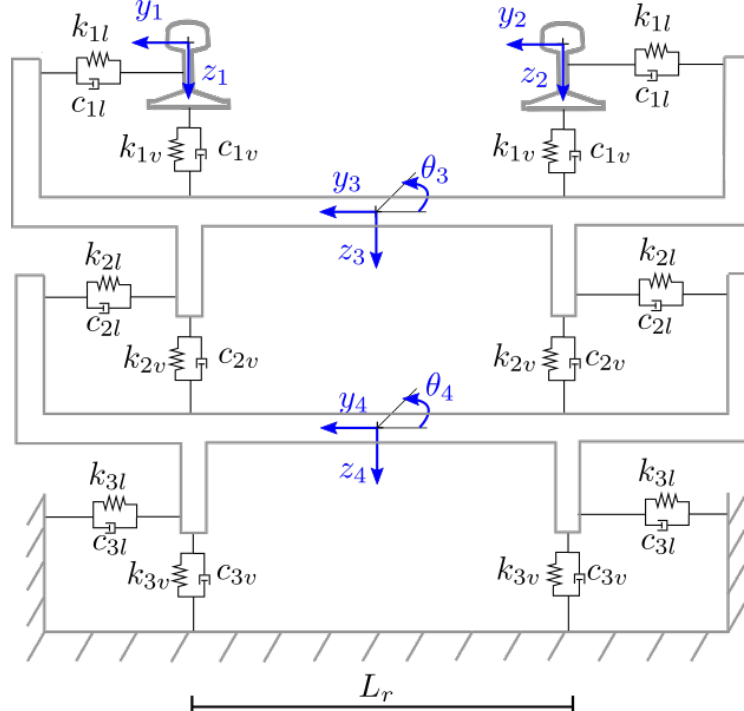


Figure 4. Flexible track model

The vertical and lateral receptance of the proposed track model for different harmonic loads  $\mathbf{P}$  applied on the vertical or lateral rail DOF can be easily obtained using the generalized mass, damping and stiffness matrices as follows:

$$FRF = \mathbf{H}(\omega) \cdot \mathbf{P} = [-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K}]^{-1} \cdot \mathbf{P} \quad (1)$$

where  $\mathbf{P}$  is the load vector, being 1 in the desired DOF (rail vertical or lateral displacement) and 0 in the others,  $\omega$  is the load frequency, and  $\mathbf{M}$ ,  $\mathbf{C}$ ,  $\mathbf{K}$  the generalized mass, damping and stiffness system matrices. This way, the mechanical parameters of the proposed track model can be adjusted such that the vertical or lateral receptance of the proposed track model coincides to the receptance obtained using the FEM model.

In this work, a least square fit subject to boundary constraints of the FRF between the FEM model and the proposed track model provides the values of the mechanical parameters. The boundary conditions imposed in the least square fit are that all parameters remain positive, rail masses are equal and  $L_r$  is fixed to 135 mm.

The parameter optimization is derived in two steps:

1. Optimize the system parameters (mass, inertia and vertical stiffness and damping coefficients) up to the first natural vertical frequency of 302 Hz given in Fig. 3 left.
2. Optimize the system lateral stiffness and damping coefficients up to the same frequency.
3. Combine both parameters and compare the whole vertical and lateral receptance with respect to Fig. 3.

Figure 5 shows the vertical (left) and lateral (right) receptance of the proposed model after the optimization. The vertical receptance is clearly reproduced up to 320 Hz while in case of the lateral one, realistic results can only be expected to be obtained up to 100 Hz.

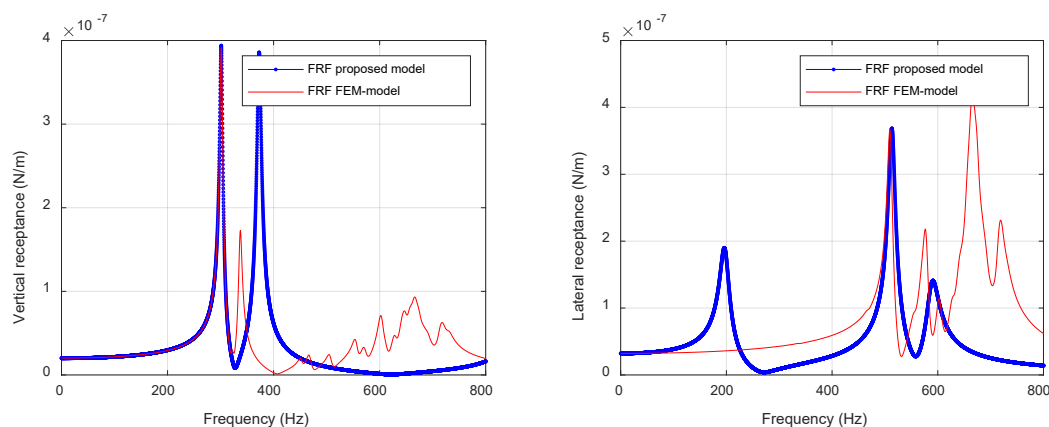


Figure 5: Vertical (left) and lateral (right) track receptance

## 4 Conclusions and Contributions

A procedure to determine the mechanical parameters of simple flexible track models is proposed to achieve accurate vehicle track dynamic analysis at specific frequency ranges. In this work, this procedure is applied to a 5-inch track gauge scaled track available at the University of Seville with a simple 10 DOF track model, but it could be easily applied to any other track topologies and more detailed simplified track models.

The track parameter optimization procedure is referred to the results of a FEM-model of the scaled track, although other sources of validation, such as experimental results, can be used as a reference.

Numerical results show that a simple least square fit of the track model mechanical parameters can lead to accurate estimation of vehicle track dynamics within specific frequency ranges. In this sense, an optimization of the inertial and vertical stiffness and damping ratio is applied based on the vertical FRF, while the lateral parameters are estimated separately with the lateral FRF. This approach shows that the vertical receptance of the simplified system is clearly reproduced up to 320 Hz while the

lateral one is only achieved up to 100 Hz. As future work, this procedure, and specially the FEM-model, will be validated with experimental results for a more accurate determination of the track parameters.

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## References

- [1] Escalona, J. L., & Aceituno, J. F. (2019). Multibody simulation of railway vehicles with contact lookup tables. *International Journal of Mechanical Sciences*, 155, 571-582.
- [2] Aceituno, J. F., Urda, P., Briaies, E., & Escalona, J. L. (2020). Analysis of the two-point wheel-rail contact scenario using the knife-edge-equivalent contact constraint method. *Mechanism and Machine Theory*, 148, 103803.
- [3] Escalona, J. L., Aceituno, J. F., Urda, P., & Balling, O. (2020). Railway multibody simulation with the knife-edge-equivalent wheel-rail constraint equations. *Multibody System Dynamics*, 48(4), 373-402.
- [4] Knothe, K. L., & Grassie, S. L. (1993). Modelling of railway track and vehicle/track interaction at high frequencies. *Vehicle System Dynamics*, 22(3-4), 209-262.
- [5] Nielsen, J. C., Lundén, R., Johansson, A., & Vernersson, T. (2003). Train-track interaction and mechanisms of irregular wear on wheel and rail surfaces. *Vehicle System Dynamics*, 40(1-3), 3-54.
- [6] Wu, T. X., & Thompson, D. J. (2004). On the parametric excitation of the wheel/track system. *Journal of Sound and Vibration*, 278(4-5), 725-747.
- [7] Martínez-Casas, J., Giner-Navarro, J., Baeza, L., & Denia, F. D. (2017). Improved railway wheelset-track interaction model in the high-frequency domain. *Journal of Computational and Applied Mathematics*, 309, 642-653.
- [8] Zhai, W., Han, Z., Chen, Z., Ling, L., & Zhu, S. (2019). Train-track-bridge dynamic interaction: a state-of-the-art review. *Vehicle System Dynamics*, 57(7), 984-1027.