

Proceedings of the Fifth International Conference on  
Railway Technology:  
Research, Development and Maintenance  
Edited by J. Pombo  
Civil-Comp Conferences, Volume 1, Paper 23.2  
Civil-Comp Press, Edinburgh, United Kingdom, 2022, doi: 10.4203/ccc.1.23.2  
©Civil-Comp Ltd, Edinburgh, UK, 2022

## **Introducing a TOPSIS based quantitative resilience measure for railway systems**

**Zhonglin Wang, Marian Sorin Nistor, Stefan Pickl**

**Universität der Bundeswehr München**

**Werner-Heisenberg-Weg 39**

**85577 Neubiberg, Germany**

### **Abstract**

We tackle the importance of railway systems as part of an interdependent chain of critical infrastructures. One approach is modeling the railway systems as graphs and then applying graph measures to them. The solution in this paper takes the TOPSIS from Multiple-Criteria Decision-Making (MCDM) field and adapts it to produce a new aggregation of different graph measures.

**Keywords:** graph theory, graph measures, TOPSIS, MCDM.

### **1. Introduction**

The "Technique for Order Preference by Similarity to Ideal Solution" (TOPSIS) [1, 2] approach can aggregate different measures (like graph centrality measures [3–6], graph nodal efficiency measures [7, 8], and graph nodal vulnerability measures [9, 10]) into a new one, comprehensively analyzing the graph for identifying the critical nodes from various perspectives. In the process of applying the TOPSIS, the most important step is how to estimate and allocate the weights for different measures. Traditionally, in the MCDM field, there are some existing, well-known, and widely used estimating methods like swing weighting [11], Analytic Hierarchy Process (AHP) [12], Simple Multi-Attribute Rating Technique (SMART) [13], and so on.

However, the problem is that all of them need the experts' knowledge and experiences, but in graph theory, different researchers have different analysis criteria, which thus will lead to different results and draw distinct conclusions. Therefore, in order to eliminate these diversities and make experiments or computations repeatable, here we introduce a new weight estimating method by conducting graph global vulnerability analysis to quantify the process of estimating weights. The analysis is conducted on the German high-speed train system (ICE), and the results show that the proposed aggregation measure is a promising one to identify the critical nodes in a graph.

## **2. Method**

Hwang and Yoon (1981) initially developed TOPSIS to help find the best alternative within a finite number of criteria. As a well-known MCDM approach, the global interest of the TOPSIS method has exponentially grown since the 1980s. The TOPSIS method aims to select the best alternative that simultaneously has the shortest distance from the positive ideal solution and the furthest distance from the negative ideal solution. Here the positive ideal solution means maximum alternative value based on benefit criteria and minimum alternative value based on cost criteria. However, the negative ideal solution represents the minimum alternative value according to benefit criteria and the maximum alternative value according to cost criteria. TOPSIS offers a cardinal ranking of finite distinctive alternatives by making full use of attribute information but without considering the attribute preferences to be independent [12, 14]. Successful applications of the TOPSIS method can be found in fields such as supply chain management and logistics, engineering and manufacturing systems, business and marketing management, human resources management, energy management, and so on [15]. In order to apply the TOPSIS method in a specific area, the attribute values of different criteria must be numeric, monotonically increasing or decreasing, and have commensurable units [15].

TOPSIS is a ranking method based on the closeness between a limited number of evaluation objects and the ideal solutions. It is to evaluate the relative merits of the existing objects. There are two ideal solutions, one is the positive ideal solution or the optimal target; the other one is the negative ideal solution or the worst target. The best object should have the closest distance to the positive ideal solution and the farthest distance from the negative ideal solution. Both optimal and worst targets among multiple targets can be found based on the normalization matrix. Then the closeness between each target and the ideal solution can be obtained by calculating the distance between each evaluated target and the positive (negative) ideal solution. Afterward, according to the value of the closeness, we can obtain a ranking order serving as the basis for evaluating the pros and cons of the target. Here, the closeness value is between 0 and 1. The closer the value is to 1, the closer the corresponding evaluated

target is to the optimal level; otherwise, if the value is closer to 0, the evaluated target is closer to the worst level.

### 3. Results

The TOPSIS-based aggregation measure (AggregTOPSIS) is introduced and compared with nine graph measures, including betweenness centrality measure (BetwCentr) [4], closeness centrality measure (CloCentr) [4], degree centrality measure (DegCentr) [6], eigenvector centrality measure (EigenCentr) [4], nodal efficiency measure (Effi) [7], nodal flow-weighted efficiency measure (FWEffi) [8], nodal betweenness-efficiency vulnerability measure (BetwEffiVul) [10], nodal residual closeness vulnerability measure (ResiduCloVul) [9], to identify the critical nodes in a graph. In order to compare them and tell the differences in which one is more suitable and efficient to identify the critical nodes, we propose a new graph performance to define the resilience. However, before conducting the resilience analysis, we first need to design different attack scenarios based on these distinguished measures. The so-called attack means that when we attack one node, we will remove the given node and its corresponding edges from the original graph. Therefore, the procedure to conduct resilience analysis is similar to the process of how to determine and allocate the weights for each measure during the calculation of TOPSIS.

There are four steps to follow:

The first step is to rank the nodes of the graph based on each measure to prepare each attack scenario.

Secondly, deleting the same top number of critical nodes from the graph based on the order derived from the first step.

Thirdly, in this step, we should also stepwise calculate the resilience of a graph [16]. For example, when deleting the top one node from the graph, we compute the first group of graph resilience values based on different measures. If removing the top two nodes, then we need to calculate the second group of graph resilience values also based on these measures, and so on. That means, when we are deleting top nodes, we should calculate and get the group of graph resilience values.

Fourthly, based on the graph resilience values within a group, i.e., after we have removed top nodes from the graph, we can compare and conclude which measure is more suitable and efficient to identify the critical nodes.

The results are shown in Figure 1. According to this figure, we could find that with the largest frequencies, the attacks based on aggregation measure can almost always lead to lower resilience; in such cases, the TOPSIS-based aggregation measure can be seen as a promising and suitable measure.

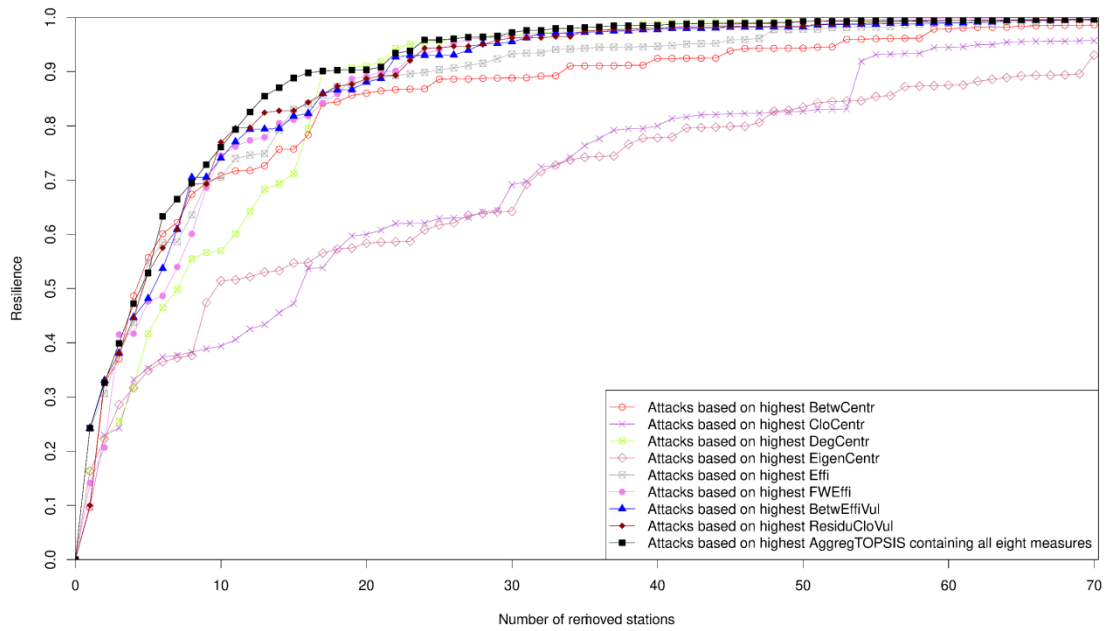


Figure 1: Results of resilience analysis under different targeted attacks.

#### 4. Conclusions

In order to quantify graph resilience properly, here we proposed a new graph performance considering three factors, including traveling time, the number of people, and the train flow. The new TOPSIS-based aggregation measure showed promising results in comparison with other individual measures like graph centrality measures, graph nodal efficiency measures, and graph nodal vulnerability measures.

Different attack scenarios were designed based on the selected measures to compare the results. However, in practice, it is less likely that many stations of a railway system are simultaneously disrupted. Therefore, further analysis should consider the essential stations to be disrupted. Thus, if one measure can lead to lower resilience when deleting a small number of nodes from the graph, we can say this measure is more suitable and effective in identifying the critical stations in a transportation system.

Further study should also consider the situation of aggregating fewer measures and whether the new simplified TOPSIS-based aggregation measure can lead to lower resilience with higher frequencies when deleting even just a small number of nodes.

## References

1. Hwang, C.-L., Yoon, K.: Methods for multiple attribute decision making. In: Multiple attribute decision making, pp. 58–191. Springer (1981)
2. International Society on MCDM. <http://www.mcdmsociety.org/> (2020). Accessed 20 February 2020
3. Tsiotas, D., Polyzos, S.: Introducing a new centrality measure from the transportation network analysis in Greece. *Annals of Operations Research* **227**, 93–117 (2015)
4. Boudin, F.: A comparison of centrality measures for graph-based keyphrase extraction. In: Proceedings of the sixth international joint conference on natural language processing, pp. 834–838 (2013)
5. Newman, M.E.J.: The mathematics of networks. *The new palgrave encyclopedia of economics* **2**, 1–12 (2008)
6. Freeman, L.C.: Centrality in social networks conceptual clarification. *Social networks* **1**, 215–239 (1978)
7. Latora, V., Marchiori, M.: Economic small-world behavior in weighted networks. *The European Physical Journal B-Condensed Matter and Complex Systems* **32**, 249–263 (2003)
8. Nistor, M.S., Pickl, S., Raap, M., Zsifkovits, M.: Network efficiency and vulnerability analysis using the flow-weighted efficiency measure. *International Transactions in Operational Research* **26**, 577–588 (2019)
9. Dangalchev, C.: Residual closeness in networks. *Physica A: Statistical Mechanics and its Applications* **365**, 556–564 (2006)
10. Wang, Z., Zsifkovits, M., Pickl, S.W.: Analyzing vulnerabilities of the German high-speed train network using quantitative graph theory. *International Journal of Safety and Security Engineering* **8**, 59–64 (2018)
11. Winterfeldt, D. von, Edwards, W.: Decision analysis and behavioral research (1993)
12. Yoon, K.P., Hwang, C.-L.: Multiple attribute decision making: an introduction, vol. 104. Sage publications (1995)
13. Barron, F.H., Barrett, B.E.: The efficacy of SMARTER—Simple multi-attribute rating technique extended to ranking. *Acta Psychologica* **93**, 23–36 (1996)
14. Chen, S.-J., Hwang, C.-L.: Fuzzy multiple attribute decision making methods. In: Fuzzy multiple attribute decision making, pp. 289–486. Springer (1992)
15. Behzadian, M., Otaghsara, S.K., Yazdani, M., Ignatius, J.: A state-of the-art survey of TOPSIS applications. *Expert Systems with applications* **39**, 13051–13069 (2012)
16. Wang, Z., Nistor, M.S., Pickl, S.W.: Analysis of the definitions of resilience. *IFAC-PapersOnLine* **50**, 10649–10657 (2017)