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# A gradient-based approach for wheel-rail contact point determination

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## Abstract

The mainstream of solving wheel-rail contact problem is employing geometry constraints to locate contact point with discrete steps, which is difficult to optimize wheel-rail contact behaviour in their lifecycle. This work shows the approach developed to determine wheel-rail contact point with gradient-based optimization model, which converts the discrete problem into a continuous one by introducing optimization pipeline including Helmholtz partial differential and step differential function for design variables filtering and projection, respective. The effectiveness of the proposed method is demonstrated with wheel profile under various service stages, which can be used for continuous optimization of the wheel-rail contact behaviour.

**Keywords:** contact points, gradient-based approach, conformal contact, wheel profile.

## 1 Introduction

The wheel-rail contact geometry parameters determination is a basis for the analysis of contact mechanics and the wheel-rail contact tribology, which significantly affects the running safety and stability of a train. Therefore, it is of vital significance to model wheel-rail contact behaviour for exploring the mechanism of wheel-rail tribology [1,2].

The typical wheel-rail contact situations including single contact point, two-point contact, and conformal contact are commonly appeared during service time. The current progress regarding wheel-rail contact point determination can be boarding divided into two aspects [3-5]: rigid and elastic contact hypothesis. The former one locates the wheel-rail contact points by solving geometry-constrained equation and the latter one uses maximum penetration as the contact point of the wheel-rail [5].

However, many advancements to single point and two-point contact theories were made, the analysis of wheel-rail conformal contact situation is less exploited. Meanwhile, the current progress on wheel-rail contact analysis mainly relay on the dynamic discrete contact model, which cannot be used for explore the continuous optimization of the wheel-rail contact behaviour [6,7]. Therefore, this paper presents a continuous optimization pipeline for determining wheel-rail contact point, aiming at solving wheel-rail conformal contact problem with gradient-based optimization.

## 2 Methods

The main idea of the proposed method is based on minimizing the energy of the full domain that is consisted of potential wheel-rail contact pairs, namely, the normal contact node pairs and virtual contact node pairs, the solid green one is used for describing the normal contact node pair and the virtual contact node pair is denoted by the red dash one, which is shown in Fig. 1. For each node pair in the potential wheel-rail contact pairs, a binary value $\omega \in \{0,1\}$ is introduced for indicating the solid ($\omega=1$) and pseudo ($\omega=0$) contact pair, which leads to a binary field to construct the wheel-rail contact points distribution in the potential link relationship

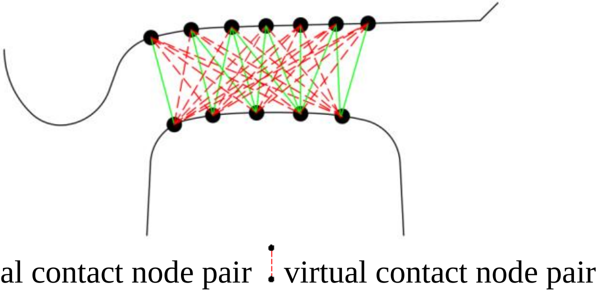


Fig. 1 Illustration of the normal contact and virtual contact node pair.

Since the wheel-rail contact situation is binary status, namely, connected or not. Thus, the location for determining the wheel-rail contact points is an integer programming problem with large-scale parameters which is hard to solve by a conventional algorithm. Consequently, the relaxations for the optimization problem, filtering and projection operators, are performed by adopting a continuous variable $\phi$ which varies from 0 to 1.

**Filtering:** The purpose of filtering is to remove checkboard patterns which should be avoided in the process of optimization. To improve the numerical stability, we introduce Helmholtz partial differential equation shown as Equation 1 which is convenient for smoothing the design variables. where r is the filter radius and the filtered design variable is denoted by $\bar{\phi}$.

$$-r^2\Delta\bar{\phi}+\bar{\phi}=\phi \tag{1}$$

**Projection:** The connection relationship of wheel-rail contact should be exactly determined while the filtered link parameters consist of intermediate values. To ensure 0-1 solution, in other words, to obtain physical link parameters, the projection operation is conducted to achieve the clear relationship between the wheel and rail. In order to allow for the use of a gradient-based optimization scheme, the differentiable step function is employed here to obtain the clear wheel-rail connection relationships for the threshold projection which is expressed as Equation 2:

$$\omega = \frac{\tanh(\frac{\beta}{2}) + \tanh(\beta(\bar{\phi} - \frac{1}{2}))}{2\tanh(\frac{\beta}{2})} \tag{2}$$

**Objective:** As the wheel-rail contact node pair $\omega$ has been defined, the optimization formulation for determining the wheel-rail contact points can be constructed as:

$$\begin{aligned} & \textit{find } \Phi = \{\phi_1, \phi_2, \ldots, \phi_n\} \\ & \text{minimize } \left| h - v(\omega) \right|_2 \\ & s.t. \begin{cases} v(\omega(\bar{\phi}(\phi))) = k\dfrac{p.s}{H}.\omega \\ -r^2 \Delta\bar{\phi} + \bar{\phi} = \phi \\ \omega = \dfrac{\tanh(\frac{\beta}{2}) + \tanh(\beta(\bar{\phi} - \frac{1}{2}))}{2\tanh(\frac{\beta}{2})} \\ 0 \le \phi \le 1 \end{cases} \end{aligned} \tag{3}$$

The aim of the above formulation is to minimize the discrepancy between the measured profile wear *h*, and the physical driven profile wear *v(x)*, where the friction coefficient, normal pressure, slide velocity and the hardness of the material are denoted by k, p, s and H [1]. Since the wheel-rail contact parameter optimization is a continuous problem with relaxation constraints, Equation 3 can be iteratively solved by a numerical optimization solver, such as the method of moving asymptotes.

## 3 Results

The measured service wheel profiles in a one-round lifecycle, shown in Fig. 2, are adopted for evaluating the performance of the proposed method to locate the wheel-rail contact geometry parameters. Figure 3 gives the optimization results of the wheel-rail contact model under various service stages of the measured wheel profile. Here 500 points of the measured wheel profiles are extracted and then assembled into 500x500 potential corresponding contact points.

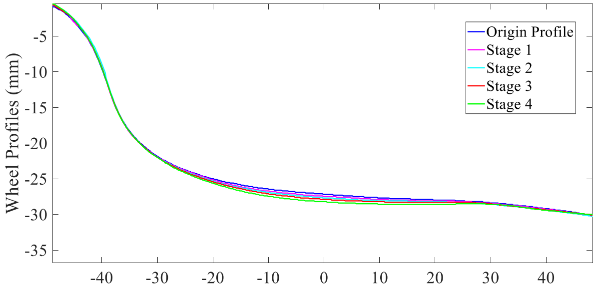
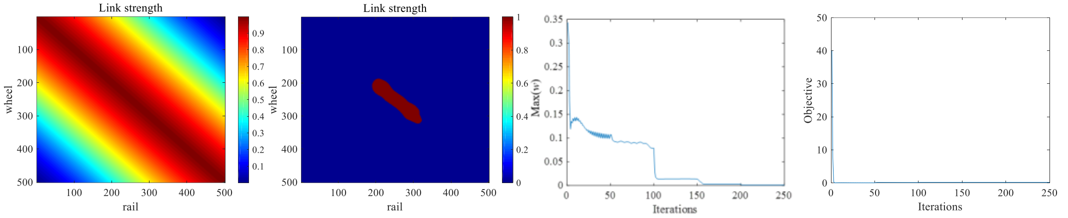
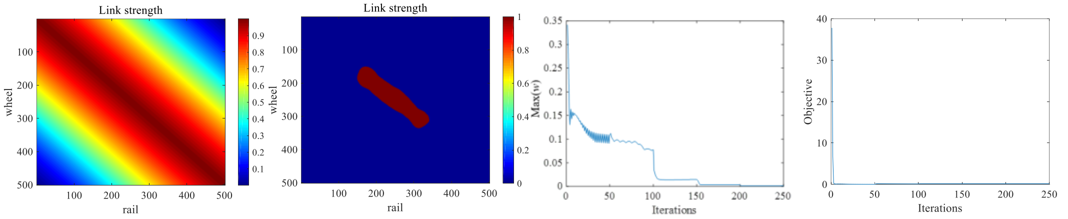


Fig. 2 The measured wheel profile at various service stages.

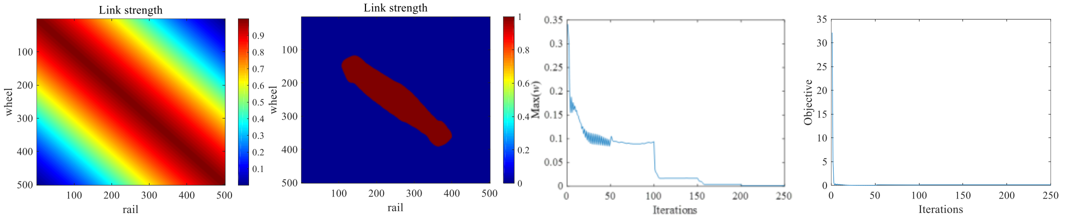
As depicted in the second to the fourth column of Fig. 3, it can be found that the proposed method can efficiently and effectively solve the optimization problem of the wheel-rail contact parameters, the objective function converges quickly at the first few steps even from fairly large error, so as the design variables largely change and gradually decreased with iterations. Although it does not guarantee that the change of the design variables reduces in each iteration, it guarantees the reduction in the average error, the reason causes such situation is that we start to use a relatively small parametrical value for the projection function, which relaxes the solution space for searching optimal wheel-rail contact parameters, the maximum changes of the design variables gradually converge which acts in accordance with the property of the projection function as increment of the value beta, which is of function for speeding up the convergence of the algorithm.



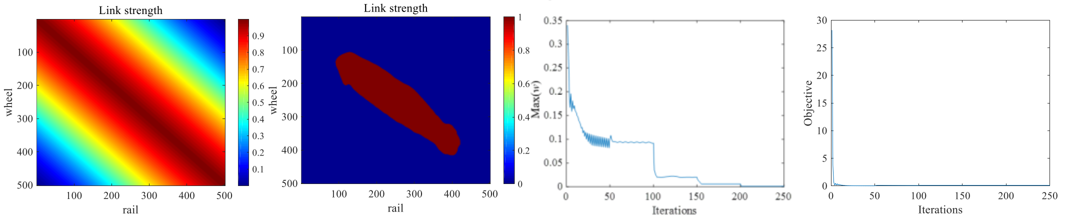
(a) stage 1



(b) stage 2



(c) stage 3



(d) stage 4

(i) (ii) (iii) (iv)

Fig. 3 wheel-rail contact parameters under various service profiles. (i) initialization, (ii) final results, (iii) converging history of the design variable and (iv) converging history of the objective function.

As for the initial service stage of the high-speed train, the wheel-rail mainly contacts between the rolling circle and the top surface of the rail, then the wheel profile forms the typical concave wear pattern and it expands as the service time increases, finally the wheel profile will be reprofiled because the large concave pattern causes the shaking of the train which severally affect the running stability of the high-speed train. Overall, the proposed method is of significant performance for determining the wheel-rail contact parameters with physical-driven profile estimation.

## 4 Conclusions and Contributions

In the present work a continuous optimization pipeline is proposed for determining wheel-rail contact geometry parameters, which transform the discrete problem into a continuous one by introducing filtering and projection technology. The effectiveness of the proposed optimization model is evaluated with service wheel profile data, the experimental results shows that the proposed method can determine the wheel-rail contact geometry parameters with measured wheel profile under various service stages and well performed.

## Acknowledgements

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