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Analytical model to study the influence of sleeper width on the dynamic responses of railway track

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Abstract

The dynamics of infinite beams resting on a homogenous foundation subjected to moving loads has been investigated by analytical methods. However, these methods cannot be applied for non-homogenous foundations. Here, we present an analytical model for infinite beams resting on a periodical viscoelastic foundation, of which the constitution law is periodic along the beam. In the steady-state, the reaction forces of the foundation to the beam are supposed to repeat when the loads recover a length of the foundation period. This periodicity condition existed also for periodically supported beam. By using the Fourier transform, the dynamic equation of the beam and this condition lead to a linear differential equation with a periodic coefficient. Therefore, the Floquet theory has been applied to work out the response. Moreover, the numerical examples show the influence of the foundation periodicity to the dynamic response of the beam.

Keywords: Railway, Dynamic structure, Analytical model, Floquet theory

1 Introduction

The dynamic response of a railway track has been studied by analytical and numerical methods. Since the 1970s, a railway track has been modelled by an infinite beam supported by an elastic foundation or periodical supports. By using a free wave propagation, Mead et al. [1] developed an analytic model for periodically supported infinite beams. Metrikine et al. [2] extended this model which includes the foundation behavior. Recently, Hoang et al. [3-4] have presented a characteristic relation between the reaction force and displacement of supports in a periodically supported beam. By using this relation, Tran et al. [5-6] has developed an analytical model for the dynamic of railway sleepers. Meanwhile, the numerical model has been also developed by several authors [7-9] to calculate the dynamic response of a railway track on transition zones. Combinations of analytic and numeric models (so-called semi-analytical models) have been used in several articles [10].

In this paper, a new analytical model is developed to study the influence of the support width to the dynamic response of a railway track. The rail is considered as an Euler-Bernoulli beam subjected to moving loads. The reaction force of the supports is written with the help of a rectangular function. The Fourier transform of the dynamic equation leads to a 4th order linear differential equation and then the solution can be expressed in a periodic form. The numerical results show that the model agree well with the existing model when the support width is very small. Otherwise, the model shows the influence of the support with on the response of the railway track. the response of the rail.

2 Methods

Let's consider an infinite beam supported by periodical supports of width a shown in Figure 1. The length of one period is l. The beam is subjected to moving loads Q_j of speed v. The position of the moving load j is characterized by the distance D_j to the first load. Each support is modelled as a spring-damper system with stiffness k_s and damping coefficient η_s.

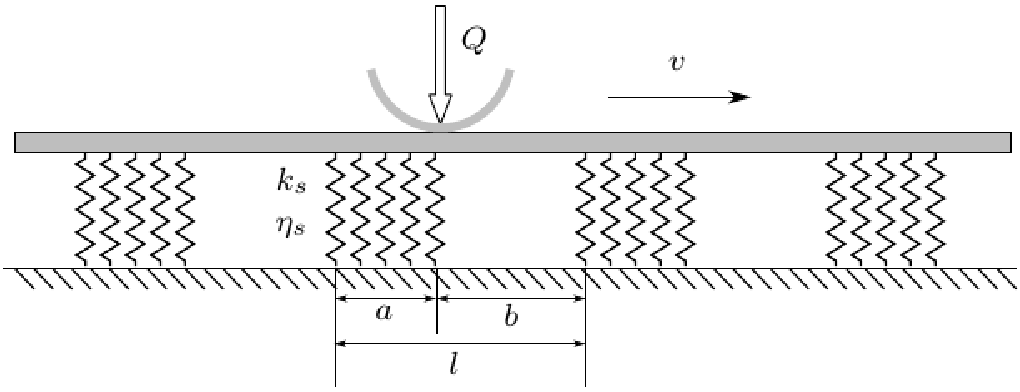

Figure 1. Analytical model presentation

The total force applied on the beam can be rewritten by using the Dirac distribution. The dynamic equation of the Euler-Bernoulli beam is the following:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho S\frac{\partial^2 w(x,t)}{\partial t^2} = \sum_{j=1}^{K} Q_j \delta(x + D_j - vt) - \sum_{n=-\infty}^{\infty} f_n(x,t)\Pi_a(x - nl)$$

where E, I, ρ, S are respectively Young modulus, moment of inertia, density and section area of the beam. $\prod_a(x)$ is the rectangular function which is equal to 1 if $x = [0, a]$ and 0 otherwise. The function $f_n(x,t)$ represents for the reaction force applied on the beam.

$$f_n(x,t) = k_s w(x,t) + \eta_s \frac{\partial w(x,t)}{\partial t} \quad nl \leq x \leq nl + a$$

By using the Fourier transform, the dynamic equation of the beam is written in the frequency domain as follows:

$$EI\frac{\partial^4 \hat{w}(x,\omega)}{\partial x^4} - \rho S \omega^2 \hat{w}(x,\omega) = \frac{1}{v}\sum_{j=1}^{K} Q_j \mathrm{e}^{-\mathrm{i}\frac{\omega}{v}(D_j+x)} - \sum_{n=-\infty}^{\infty} \hat{f}_0(x,\omega)\mathrm{e}^{-\mathrm{i}\frac{\omega}{v}nl}\Pi_a(x-nl)$$

If we put:

$$\hat{w}_r(x,\omega) = \mathrm{e}^{-\mathrm{i}\frac{\omega}{v}x} s(x,\omega)$$

the dynamic equation in the frequency domain of the rail becomes:

$$EI\left(s^{(4)} - 4\mathrm{i}\frac{\omega}{v}s^{(3)} - 6\frac{\omega^2}{v^2}s^{(2)} + 4\mathrm{i}\frac{\omega^3}{v^3}s^{(1)} + \frac{\omega^4}{v^4}s\right) - \rho S\omega^2 s =$$
$$\frac{1}{v}\sum_{j=1}^{K} Q_j \mathrm{e}^{-\mathrm{i}\frac{\omega}{v}D_j} - \sum_{n=-\infty}^{\infty} \hat{f}_0(x-nl,\omega)\mathrm{e}^{\mathrm{i}\frac{\omega}{v}(x-nl)}\Pi_a(x-nl)$$

where $s^{(n)}$ stands for the nth derivation of the function s with regard to x. We see that the right-hand side of the previous equation is periodic, and the left-hand side is linear, therefore, we find the solution in periodic form with the boundary conditions:

$$\begin{cases} s^{(n)}(0,\omega) = s^{(n)}(l,\omega) & \forall n = 0,1,2,3 \\ s^{(n)}(a^+,\omega) = s^{(n)}(a^-,\omega) & \forall n = 0,1,2,3 \end{cases}$$

The boundary conditions are the continuity of the displacement, slope, moment, and shear force of the beam in one period $[0, l]$.

In the interval $[0, a]$, the solution is:

$$s(x,\omega) = \sum_{k=1}^{4} A_k \mathrm{e}^{\alpha_k x} - \frac{\sum_{j=1}^{K} Q_j \mathrm{e}^{-\mathrm{i}\frac{\omega}{v}D_j}}{vEI\left(\frac{\omega^4}{v^4} - \lambda_a^4\right)}$$

where $\lambda_a = \sqrt[4]{\frac{\rho S \omega^2 - K_s}{EI}}$ and $K_s = k_s + \mathrm{i}\omega\eta_s$

In the interval $x = [a, l]$, the solution is:

$$s(x,\omega) = \sum_{k=1}^{4} B_k \mathrm{e}^{\beta_k x} - \frac{\sum_{j=1}^{K} Q_j \mathrm{e}^{-\mathrm{i}\frac{\omega}{v} D_j}}{vEI\left(\frac{\omega^4}{v^4} - \lambda_b^4\right)}$$

where $\lambda_b = \sqrt[4]{\frac{\rho S \omega^2}{EI}}$.

The constants A_k and B_k $(k = [1, 2, 3, 4])$ in the two last equations can be determined numerically or analytically by applying the boundary conditions into the dynamic equation of the beam in the frequency domain.

3 Results

3.1 Validation of model

We compare the results obtained by this model while $a << 1$ (the reaction force can be described by a point force) and the periodically supported beam model [4]. The track parameters are given in the Table. 1. The beam is subjected to 2 moving loads which corresponds to the passing of 1 axle. Each wheel load is $Q = 100\ kN$ and the speed is $v = 90$ kmh^{-1}.

Parameters	Notation	Value	Unity
Section mass of rail	ρS	60	kgm^{-1}
Section stiffness of rail	EI	6.3	MNm^2
Length of one period	l	0.6	m
Sleeper width	a	0.25	m
Stiffness of support	k_s	192	MNm^{-1}
Damping of support	η_s	1.97	MNsm^{-1}

Table 1. Track parameters

Figure. 2 show the displacement and the reaction force applied on the rail. The blue lines present the responses calculated by this model while $a << 1$ and the red ones are obtained by the existing model (see [4]). We see that the two models agree well.

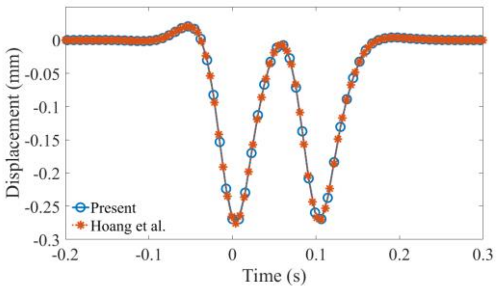

Fig. 2a. Displacement

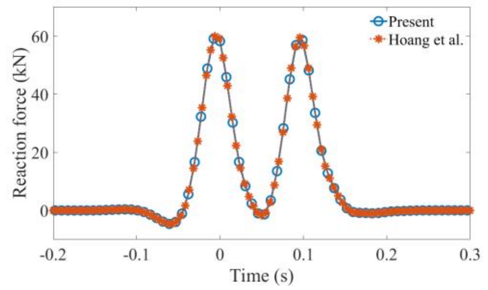

Fig. 2b. Reaction force

Figure 2. Comparison of the new model and the periodically supported beam model

3.2 Numerical example

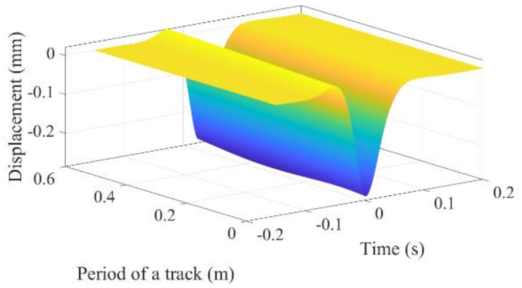

Fig 3a. Displacement

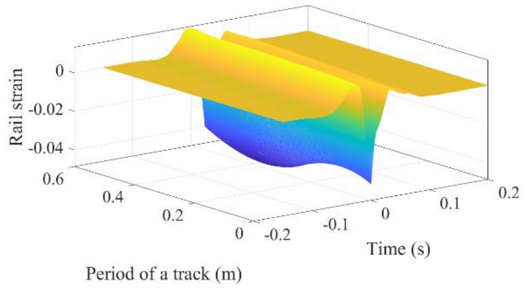

Fig 3b. Rail strain

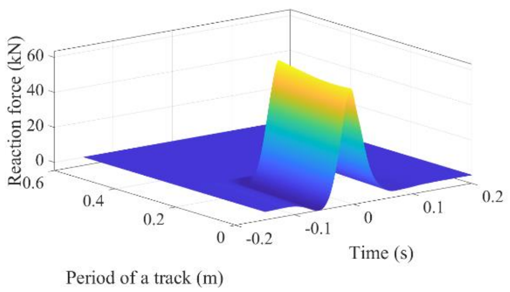

Fig 3c. Reaction force

Figure 3. Track responses

A numerical example is calculated with the sleeper width $a = 0.264$ m, which corresponds to the mono block sleeper used. The period of the track is $L = 0.6$ m.

Figure 3 shows the displacement, the strain and reaction force of the rail (the strain is calculated at under the rail head). We see that the reaction force is only at the supported interval *[0, a]*. In addition, the strain and the displacement of the rail in this interval is less that the unsupported interval.

3.3 Influence of the sleeper width on the beam responses

Now we study the influence of the support width on the beam responses. The track parameters are those given in Table 1. Figures 4 shows the track responses with different sleeper widths. We see that when the beam is continuously supported (a = 0.6), the rail response is homogenous. When the support is a point force (a = 0), the variation of response is larger between the supported point and non-supported position on the rail. This result is interesting: when we need a “smooth” track, the rail with a large support has more advantage.

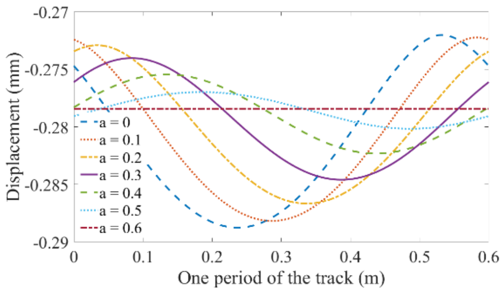

Fig 4a. Rail displacement

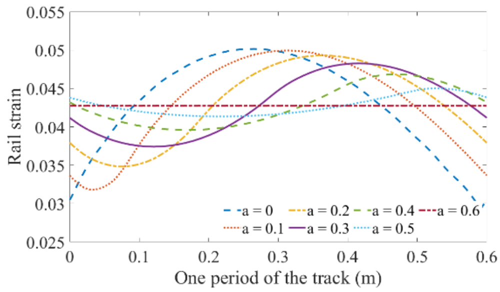

Fig 4b. Rail strain

4 Conclusions and Contributions

This paper presents an analytic model for the dynamic responses of the railway track to study the effect of the support width. This model agrees well with existing models with the continuous support or concentrated point supports. Otherwise, it permits to calculate fast the track responses when the support width is not small. The application shows that the support width does not influence the force distribution, but the rail responses have a variation important between the supported and non-supported interval.

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