The Fifth International Conference on Railway Technology
RAILWAYS 2022
22–25 August 2022
Montpellier, France

Proceedings of the Fifth International Conference on
Railway Technology:
Research, Development and Maintenance
Edited by J. Pombo
Civil-Comp Conferences, Volume 1, Paper 10.22
Civil-Comp Press, Edinburgh, United Kingdom, 2022, doi: 10.4203/ccc.1.10.22

A statistical inverse approach for load identification on in-service tunnel structures

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Abstract

Under complex underground environment, earth pressures on many in-service tunnel structures have far exceeded values expected in design stage, leading to severe structural diseases. Identification of the pressures on these structures is the basis for digital monitoring, residual capacity estimation, and health assessment of them. Here, a statistical inversion approach is proposed to identity the current earth pressures on tunnel structures based on the easily observed deformation data. To deal with the well-known non-uniqueness and ill-conditioning issues in a load inversion problem, this approach is based on Bayes' theorem to obtain the complete posterior probability densities (PPD) of inversion results. Accordingly, non-uniqueness is recognized and quantified by the PPDs, based on which ill-conditioning can also be flattened by a statistical integration. Numerical cases are carried out to test this approach in detail and future extensions are discussed in the last.

Keywords: Inverse problem; Statistical inversion; Load identification; Tunnel structure.

1 Introduction

Identification of external loads on structures is vital for in-service structures [1], especially for those large-deformed tunnel structures, on which the earth pressures have far exceeded values expected in design stage due to the complex service environment [2].

As tunnels are buried in the ground, direct measurement of the external loads by measuring devices is rather difficult [3]. That is, sensors are unavailable to be installed if they weren't pre-embedded during the pouring period, let alone the huge cost of large-scale sensor installation. Alternatively, inversion of the load pressures based on easily observed structural response, say deformation, is a more desirable.

Shown in Equation (1), forward problems compute the structural response given a specific load condition:

$$\mathbf{d}=C(\mathbf{q}) \tag{1}$$

where **d** is a vector of deformation data, **q** is a vector of load pressures, and C is a forward mapping function. On the contrary, inversion problems estimate the load condition on structures given a set of observed structural response. Previous researches tried to inverse Equation (1) directly (Equation (2)) or find a best fitted solution by minimizing a loss function in a deterministic way (Equation (3)) [3−5]:

$$\mathbf{q}=C^{-1}(\mathbf{d}) \tag{2}$$

$$\mathbf{q}=\min_{\mathbf{q}}||\mathbf{d}-C(\mathbf{q})||^{r} \tag{3}$$

where $||.||^r$ indicates the rth-order norm of a vector. However, for load inversion problems, two major issues are ill-conditioning and non-uniqueness of solutions [6]. That is, a slight error in the observation can lead to a large bias in the solution due to the large condition number of C (ill-conditioning); and vastly different solutions usually give rise to similar structural responses (non-uniqueness). Regularization techniques were further introduced [4−5]. To penalize undesired components by imposing regularized constraints on the solution space, a best-fitted, unique, and smooth solution can be obtained. However, in practical cases, it is difficult to tune a suitable regularized factor (constraint), which deeply relies on researchers' experiences [7]. In comparison, non-uniqueness is recognized in statistical inversion and quantified by probability densities (PDs). Additionally, the ensemble of feasible solutions, with corresponding PDs, are jointly used for further engineering decision, which can be more robust than a potential ill-conditioned solution in deterministic inversions [8]. However, few researches have been conducted on statistical inversion of load pressures for underground structures.

Here, a statistical approach for load inversion on in-service tunnel structures is proposed which aims to deal with ill-conditioning and non-uniqueness issues in the inversion process.

2 Methods

This approach is based on Bayes' theorem to obtain the posterior probability densities (PPDs) of the load pressures by incorporating the likelihood function and prior distribution. As shown in Figure 1, parameterization and numerical sampling are pre-processing and post-processing steps for the Bayes' theorem, respectively.

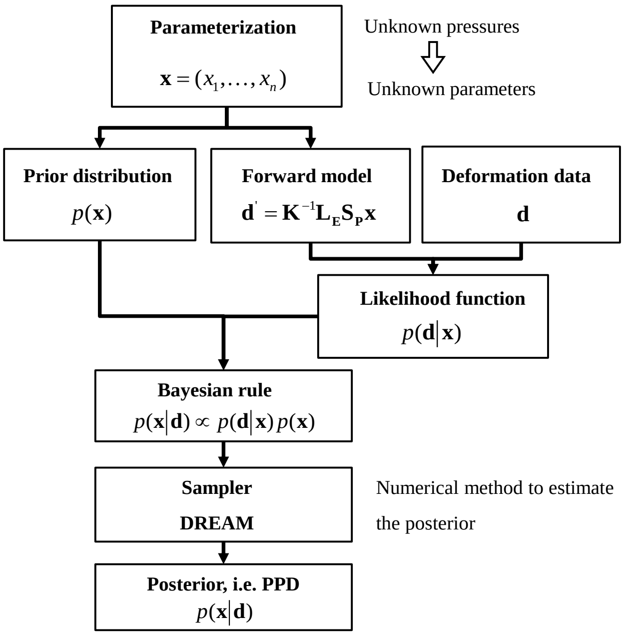

Figure 1: Flowchart of this approach.

Parameterization: The unknown pressures must be represented mathematically before inversion. Here, linear interpolation (Equation (4)), interpolated by n evenly-distributed unknowns $\mathbf{x}=(x_1,\ldots,x_n)$ in a polar coordinate, is adopted to represent the unknown pressures $\mathbf{q}$ (Figure 2). That is, any shape of pressures can be fitted with appropriate values of $\mathbf{x}$. Accordingly, inversion of unknown pressures has been transferred into inversion of unknowns $\mathbf{x}$.

$$\mathbf{q}=\mathbf{I_p x} \tag{4}$$

see the derivation of matrix $\mathbf{I_p}$ in [9].

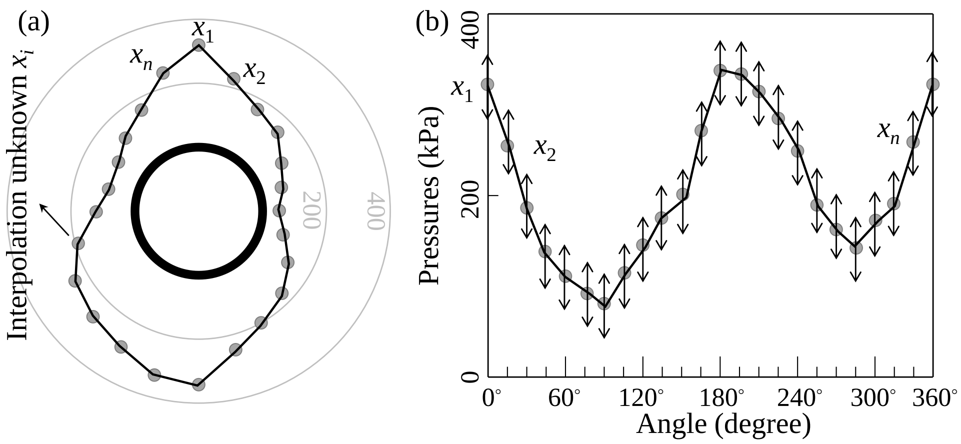

Figure 2: Parameterization of unknown pressures, (a) polar coordinate; (b) unfold view.

Bayes' theorem: Inference of $\mathbf{x}$ is based on Bayes' rule given observed data $\mathbf{d}$:

$$p(\mathbf{x}|\mathbf{d})=p(\mathbf{d}|\mathbf{x})\cdot p(\mathbf{x})\cdot k^{-1} \tag{5}$$

where $p(\mathbf{x})$ is the prior distribution of $\mathbf{x}$, $p(\mathbf{d}|\mathbf{x})$ is the likelihood function, and k is the normalizing factor.

Prior distribution: The prior represents ones' prejudgments on the unknowns before measuring the data. It is often the case that one knows little about $\mathbf{x}$. Thus, a weakly-informative prior, i.e., uniform distribution is adopted, indicating that every values has equal probability within a pre-defined bound before inversion.

Likelihood function: The likelihood measures how well a set of given parameter values can give rise to the observed data, determined by $\mathbf{e}$:

$$\mathbf{e}=C(\mathbf{x})-\mathbf{d} \tag{6}$$

according to the Central Limit Theorem, a zero-mean Gaussian distribution is typically assumed for $\mathbf{e}$, and we get:

$$p(\mathbf{d}|\mathbf{x})=N_\sigma \exp[-\|\mathbf{e}\|^2/(2\sigma_d^2)] \tag{7}$$

where N_σ is a constant, σ_d is the estimated variance of the data noise.

Forward model: For a tunnel structure, the well-known finite element method can be adopted:

$$C(\mathbf{x})=\mathbf{K}^{-1}\mathbf{f}=\mathbf{K}^{-1}\mathbf{L_E}\mathbf{q}=\mathbf{K}^{-1}\mathbf{L_E}\mathbf{I_p}\mathbf{x} \tag{8}$$

where $\mathbf{K}$ is the global stiffness matrix, and $\mathbf{f}$ is the equivalent nodal forces, equivalent to the pressures $\mathbf{q}$ with the matrix $\mathbf{L_E}$ based on virtual work. Derivation of $\mathbf{K}$ and $\mathbf{L_E}$ for a Euler beam can be seen in [10].

The sampler: In the absence of analytical solution (Equation (5)), a practical way to obtain the posterior is Bayesian sampling. Markov Chain Monte Carlo (MCMC) is an iterative method to generates samples to reach the PPDs. As the forward model for tunnel structures can be computationally intensive, a more efficient version of MCMC, called DREAM, is adopted. See the details in [11].

3 Results

A numerical example is presented. Referring to [3], the earth pressures (Figure 3) were assumed to act on a tunnel lining. Under the pressures, deformation data were computed based on a 2-D elastic beam model (Equation (8)). Accordingly, the goal is to inverse the actual pressures (assumed unknown now) based on the observation in Figure 3(b).

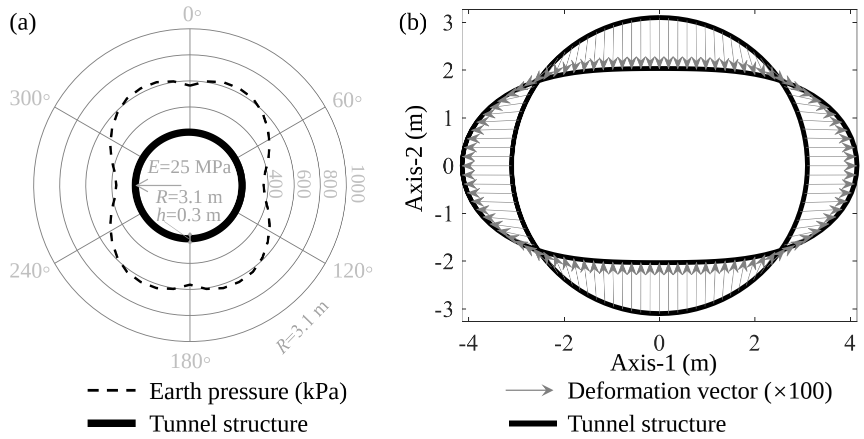

Figure 3: Numerical example, (a)assumed pressures; (b)deformation data.

22 evenly-distributed parameters $\mathbf{x}=(x_1,\ldots,x_{22})$ parameterized the unknown pressures. A uniform prior is adopted for x_i~Uniform(0,1000) (i=1,…,22). Different cases (Table 1) were run simultaneously, in which some observations were contaminated by noises. For necessary comparison, deterministic inversions were also run for every cases based on the rule of Equation (3).

Cases	**Inversion approach**	**Noise**
Case1	Statistical	0
	Deterministic	
Case2	Statistical	1%Gaussian noises
	Deterministic	
Case3	Statistical	5%Gaussian noises
	Deterministic	
Case4	Statistical	10%Gaussian noises
	Deterministic	

Table 1: Cases for testing this approach (i=1,…,22).

Markov chains were run to estimate the PPDs. Results of our approach for Case1 are shown in Figure 4(a). The inferences (PPDs) of pressures at any points on the structure are presented by different colours: hotter colours indicate higher probability. Intuitively, for example, at angle 0˚, 60˚, and 120˚, inference of values of pressures are presented by the PPDs, respectively in Figure 4(b).

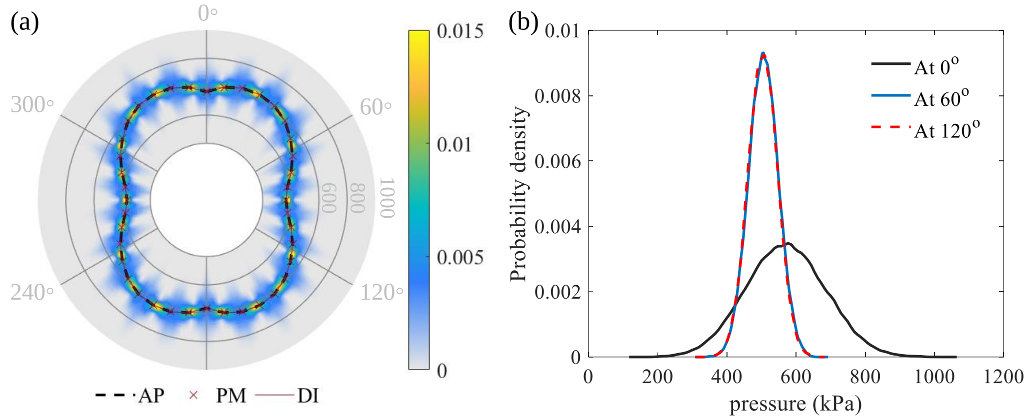

Figure 4: Results for case1, (a)PPDs; (b)examples for PPDs. (AP=actual pressure; PM=posterior mean; DI=deterministic inversion, same for figure 5).

All the potential values of the pressures are recognized while our confidence on the values are quantified by PPDs. Non-uniqueness is recognized in statistical inversion and quantified by the PPDs, suggesting a different view of inference from deterministic inversions. Noting that the actual pressures are bounded by hot areas. In addition, the posterior means (PM), which are the integration of all potential solutions based on PPDs, fit perfectly with the actual pressures indicating effectiveness of our approach. Certainly, deterministic inversion also did a great job in the no-noise case.

When slight noises were added in the observation (Figure 5), ill-conditioning occurs in the results of deterministic inversion. In comparison, PM fit relatively well with the actual pressures. Although the fit is getting worse with the increase of noise level, it performs far better than solutions in deterministic inversion, suggesting its power to deal with ill-conditioning.

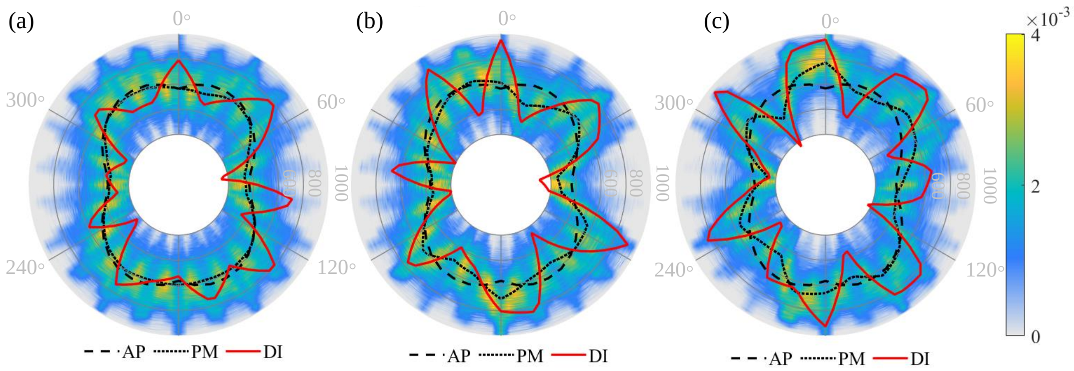

Figure 5: Results (a)case2; (b) case3; (c) case4. (AP=actual pressure; PM=posterior mean; DI=deterministic inversion).

4 Conclusions and Contributions

A statistical inverse approach for load identification on in-service tunnel structures is proposed in this paper. With this approach, load identification on tunnel structures can be conducted based on the easily observed deformation data rather than unavailable measuring devices.

This approach is based on statistical inversion, which suggests a new view of inference that is different from deterministic inversions. That is, every potential solutions is recognized in the inversion process and the confidence on different solutions is quantified by probability densities. It suggests that this approach recognizes and quantifies non-uniqueness and encourages one to make further engineering decision based on probability.

The posterior means integrate the ensemble potential solutions based on the corresponding probability densities. This statistical averaging process flattens unstable features of any individual solutions, and is proved to be powerful to deal with ill-conditioning in an inversion process. At least the numerical examples show that under 5% Gaussian noises (maximum value of 4 mm in absolute terms, which is quite large in practice), a relatively good inversion result can still be obtained by this approach.

It is noted that the forward model for the numerical cases are chosen as 2-D elastic beam, which can underestimate the structural response in reality and lead to unacceptable errors in the inversion results in practical cases. As a result, more complex forward model, considering the geometric and material nonlinearity, should be introduced in the near future.

Acknowledgements

This study was supported by Natural Science Foundation of China (Grant No. 51978523). Z.T. acknowledges support from China Scholarship Council.

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