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Modelling the Critical Speed Amplification Effect on Railway Track-Ground Systems

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Abstract

The railway track vibration generated by the train passage depends upon the individual and combined effect of its various components: the train, the track and the supporting soil. In the case of the train, higher speeds result in larger vibrations on the railway structure. Moreover, amplification occurs when approaching the so-called ‘critical speed’, and the response is further increased when considering the multiple axles effect. Regarding the track, the type and properties also influence the overall structure response, increasing the wheel-rail interaction forces and magnifying the vibrations. Similarly, the supporting ground features directly affect track behaviour, propagating the total railway system response and causing disturbances in nearby structures and increasing the localisation of energy at transition zones. Although different numerical approaches meant to study these amplifications effects have been developed, deep-wave propagating problems induced by high-speed trains require more elaborated simulations. Thus, this paper investigates the railway track amplifications due to speed, focusing on the different track types and soil layering effects, for which semi-analytical approaches simulating the track-ground dynamic complex behaviour are employed.

Keywords: critical speed, track-ground response, beam on elastic foundations, track types, semi-analytical models, equivalent stiffness.

1 Introduction

The railway track structure vibration resulting from the train passage, depends on the different train, track types and supporting soil behaviour. In the first case, the tendency to use high-speed trains in order to supply the growing traffic demand results in deflections in the railway system. Nonetheless, when approaching the so-called 'critical speed' [1-5], dynamic amplification occurs, and the response is further increased, leading to faster degradation mechanisms, poor ride quality, and ultimately more frequent and higher maintenance costs [4,6-7].

Similarly, the various track properties play a crucial role in the overall railway response. Important variations in the track properties, such as rail irregularities, transitions zones, etc., result in wheel-track interaction forces, which amplify the structure response [8-9]. Regarding the supporting soil, magnification in its response depends upon its properties and the dynamic excitation [10-12]. Overall, when the railway system is subject to a high-speed excitation, the response propagates within the soil, affecting the track system and causing disturbances in the vicinity structures.

Therefore, this paper investigates the amplification of the railway system due to speed. For this, different track types and soil layering effects are studied, and their dynamic track-ground response is analysed via a combination of analytical and semi-analytical approaches.

2 Methods

The track-ground coupling behaviour is simulated by combining analytical and semi-analytical methodologies. The ballasted and the slab tracks are represented via analytical beam on elastic foundation, BOEF, formulations. In contrast, the 3D ground is simulated through the thin layer method, TLM [13-15]. Once both systems have been modelled independently, coupling techniques are enforced, and the track-ground dynamic response can be obtained – see Figure 1.

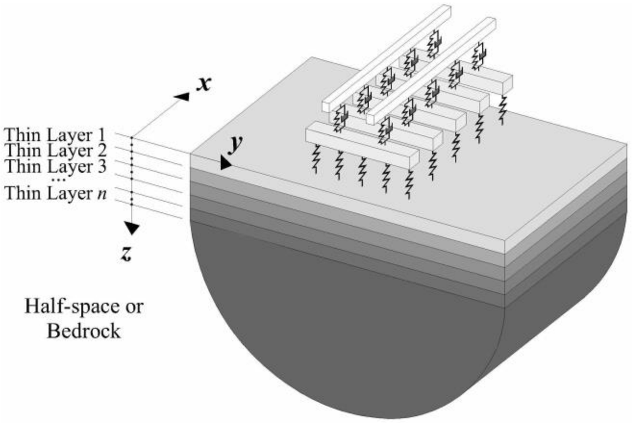

Figure 1: Track coupled with TLM soil model.

Regarding the tracks, analytical two-layer BOEF models are employed. The rail is simulated as a continuous Euler-Bernoulli beam supported by a continuous viscoelastic layer representing the railpads. In the ballasted case, the rail-pads are resting on two continuous masses simulating the sleepers and the ballast –Figure 2a. In contrast, a beam representing the slab supports the railpads in the slab track type – Figure 2b. Both track models rest on a continuous layer of spring-in-series corresponding to the soil.

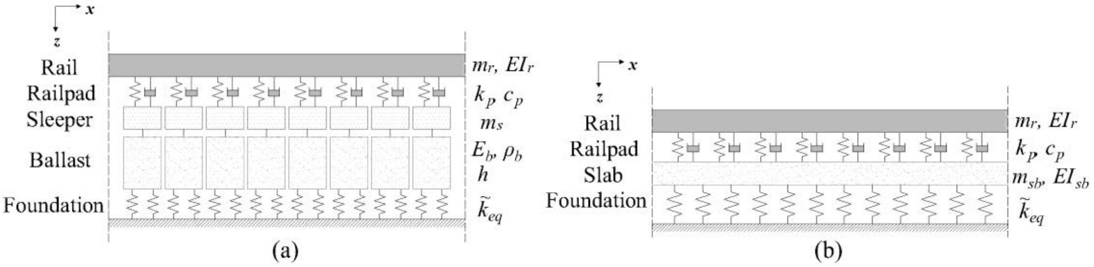

Figure 2: BOEF models, (a) ballasted track, (b) slab track.

The track response is computed using Equation (1) inversion, representing its dynamic behaviour in the wavenumber-frequency domain.

$$[\tilde{D}]_{b,sb}\{\tilde{U}\}_{b,sb} = \{\tilde{F}\}_{b,sb} \tag{1}$$

where *'b'* and *'sb'* corresponds to the ballasted and slab track, respectively; $\tilde{D}$ is the dynamic stiffness matrix of the system – see [2,13] for further information. $\tilde{U}$ and $\tilde{F}$ are the vectors of displacements and forces respectively: $[\tilde{u}_r, \tilde{u}_s, \tilde{u}_{bf}]^T$ and $[\tilde{p}, 0, 0]^T$ in ballast, and $[\tilde{u}_r, \tilde{u}_{sb}]^T$ and $[\tilde{p}, 0]^T$ in the slab track, where the applied force is $\tilde{p}$. Subscripts *'r', 's', 'bf'* and *'sb'* represents the rail, sleeper, ballast and slab components.

Regarding the soil, the TLM allows for the 3D layered soil representation via discretisation of the domain according to the smallest relevant wavelength [13]. In addition, finite element techniques are employed in the solution of the soil system equation of motion involving a stiffness matrix relating the displacements and the stresses at both sides of a single layer.

Once simulated the soil behaviour, the equivalent stiffness foundation $\tilde{k}_{eq}$ is obtained and used to couple the track and the ground – see Equation (2):

$$\tilde{k}_{eq}(\beta_x, \omega) = \frac{2\pi}{\int_{-\infty}^{\infty} \tilde{u}^G(\beta_x, \beta_y, z=0, \omega)\, C_{tg}\, d\beta_x} \tag{2}$$

where $\tilde{k}_{eq}$ is computed via Green's functions related to the soil deflection $\tilde{u}^G$ which can be included in the BOEF models as its foundation parameter [13,16]. In general, $\tilde{u}^G$ is computed at the soil surface $(z = 0)$ in the wavenumber-frequency domain $(\beta_x, \beta_y, \omega)$, and C_{tg} is a scaling factor for the track-soil coupling which depends upon the track type, its width, and the track-soil compatibility conditions [11].

3 Results

In order to study the effect of train speed on track amplifications, both ballast and slab BOEF track models resting on layered soils are considered. The track models are excited by a non-oscillating load (i.e. zero riding frequency, $\bar{f} = 0$ Hz) of 150 kN moving at a constant speed. In addition, the track is supported and coupled through semi-analytical methods to a layered ground that rests on a half-space medium. Table 1 presents the soil properties employed, respectively –for more information regarding the track and soil parameters refer to [11].

Layer	Depth [m]	Young's modulus [MPa]	Poisson's ratio [-]	Density [kg/m^3]	Loss factor [-]
1	2	60	0.35	1500	0.06
2	∞	200	0.35	1800	0.06

Table 1: Soil parameters.

Figure 3 shows the dynamic amplification curves and the critical speed values (c_{cr}) of both track types resting on layered soil and subject by a constant moving force. It can be seen that the ballasted track results in larger rail deflections compared to the slab case. This track behaviour is also evidenced in the critical speed of each case, 135 m/s for ballasted and 171 m/s for slab track. Overall, this is due to the additional bending stiffness of the slab track. Similarly, Figure 4 shows the track type effect and compares the rail deflection at 100% and 50% of the critical speed for both track cases. As expected, the track response is amplified at higher speeds.

Figure 5 and Figure 6 compares the ground contours of the layered soils supporting the ballasted and the slab track, respectively. In both cases, results are shown for 50% and 100% of the critical speed, thus evidencing the amplification effect with speed. It can be seen that at lower speeds, contour shapes are uniform. On the contrary, the contours display conical-shaped waves with trailing oscillations behind the load at higher speeds.

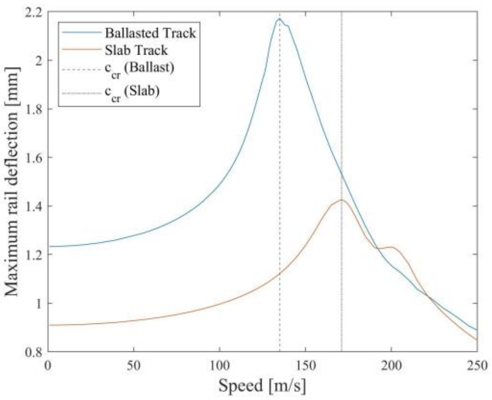

Figure 3: DAF of ballasted and slab tracks resting on layered soil.

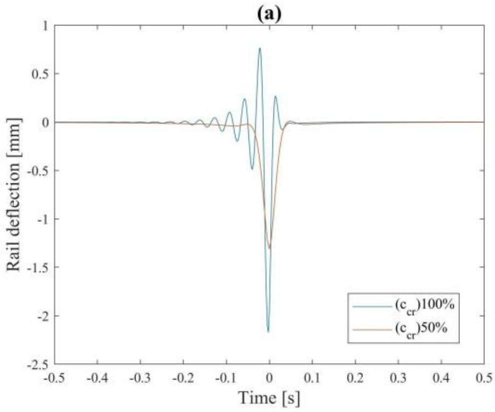

Figure 4: Track response on layered soil, at 100% and 50% of the critical speed, (a) ballasted track, (b) slab track.

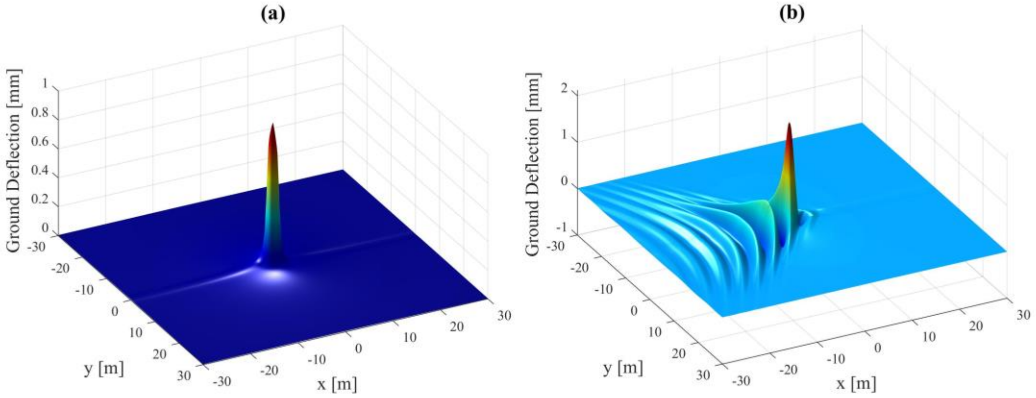

Figure 5: Ground contour due to ballasted track, resting on layered soil at (a) 50% of the critical speed, and (b) 100% of the critical speed.

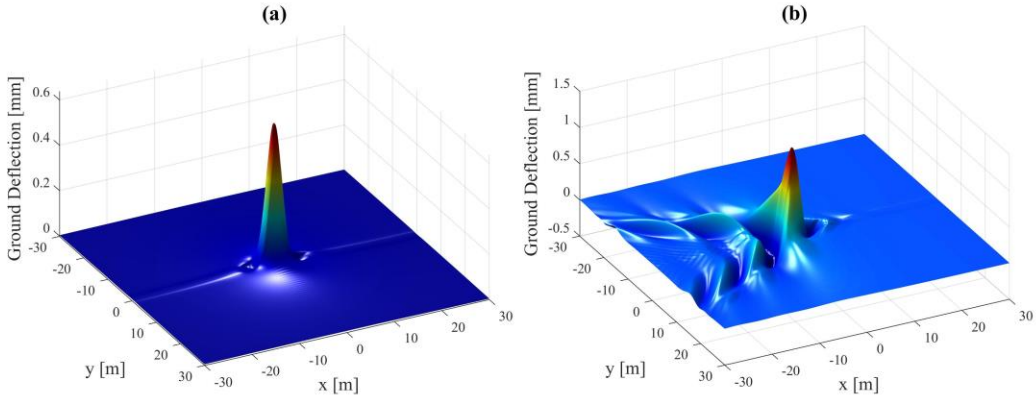

Figure 6: Ground contour due to slab track resting on layered soil, at (a) 50% of the critical speed, (b) 100% of the critical speed.

4 Conclusions

The railway track vibration generated by the train passage depends upon the individual and combined effect of its various components: the train, the track and the supporting soil. When the train approaches higher speeds, larger vibrations on the railway structure arise. Moreover, amplification occurs when approaching the so-called ‘critical speed’, leading to faster track deterioration and maintenance costs increment.

Despite the different numerical approaches developed to study these amplification effects, deep-wave propagating problems induced by high-speed trains require more elaborated simulations. Thus, this work investigates the railway track amplifications due to speed, focusing on the different track types and soil layering effects, for which semi-analytical approaches simulating the track-ground dynamic complex behaviour are employed.

Overall, results show the influence of speed on the track and layered ground response. In the case of the track type, it is observed that the additional bending stiffness of the slab track results in lower rail deflections than those of the ballasted case. This behaviour also leads to lower critical speeds in the ballasted case compared to the slab case. Similarly, higher speeds amplify the ground response. It is observed that at speeds closer or equal to the critical, ground response is larger and exhibit a conical shape with trailing oscillations, a behaviour which is not displayed at lower speeds.

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References

[1] D.P. Connolly, P. Alves Costa, “Geodynamics of very high speed transport systems”, Soil Dynamics and Earthquake Engineering, 130, 1–13, 2020. doi:10.1016/j.soildyn.2019.105982.

[2] P. Alves Costa, A. Colaço, R. Calçada, A.S. Cardoso, “Critical speed of railway tracks. Detailed and simplified approaches”, Transportation Geotechnics, 2, 30-46, 2014. doi:10.1016/j.trgeo.2014.09.003.

[3] K. Dong, D.P. Connolly, O. Laghrouche, P.K. Woodward, P. Alves Costa, “Non-linear soil behaviour on high speed rail lines”, Computers and Geotechnics, 112, 302-318, 2019. doi:10.1016/j.compgeo.2019.03.02.

[4] S.B. Mezher, D.P. Connolly, P.K. Woodward, O. Laghrouche, J. Pombo, P.A. Costa, “Railway critical velocity - Analytical prediction and analysis”, Transportation Geotechnics, 6, 84–96, 2016. doi:10.1016/j.trgeo.2015.09.002.

[5] P. Alves Costa, P. Soares, A. Colaço, P. Lopes, D. Connolly, “Railway critical speed assessment: A simple experimental-analytical approach”, 134, 106156, 2020. doi:10.1016/j.soildyn.2020.106156.

[6] P.K. Woodward, O. Laghrouche, S.B. Mezher, D.P. Connolly, “Application of Coupled Train-Track Modelling of Critical Speeds for High-Speed Trains using Three-Dimensional Non-Linear Finite Elements”, International Journal of Railway Technology, 4, 1-35, 2015. doi:10.4203/ijrt.4.3.1.

[7] D.P. Connolly, K. Dong, P. Alves Costa, P. Soares, P.K. Woodward, “High

speed railway ground dynamics: a multi-model analysis", International Journal of Rail Transportation, 8, 324-346, 2020. doi:10.1080/23248378.2020.1712267.

[8] P. Chumyen, D.P. Connolly, P.K. Woodward, V. Markine, "The effect of soil improvement and auxiliary rails at railway track transition zones", Soil Dynamics and Earthquake Engineering, 155, 107200, 2022. doi:10.1016/j.soildyn.2022.107200.

[9] C. Charoenwong, D.P. Connolly, P.K. Woodward, P. Galvín, P. Alves Costa, "Analytical forecasting of long-term railway track settlement", Computers and Geotechnics, 143, 104601, 2022. doi:10.1016/j.compgeo.2021.104601.

[10] A. Paixão, E. Fortunato, R. Calçada, "Design and construction of backfills for railway track transition zones", Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, 229, 58–70, 2020. doi: 10.1177/0954409713499016.

[11] A.C. Lamprea-Pineda, D.P. Connolly, M.F.M. Hussein, "Beams on elastic foundations – a review of railway applications and solutions", Transportation Geotechnics, 33, 2022. doi:10.1016/j.trgeo.2021.100696.

[12] S. Zhang, G. Cheng, X. Sheng, D.J. Thompson, "Dynamic wheel-rail interaction at high speed based on time-domain moving Green's functions", Journal of Sound and Vibration, 488, 115632, 2020. doi: 10.1016/j.jsv.2020.115632.

[13] K. Dong, D.P. Connolly, O. Laghrouche, P.K. Woodward, P. Alves Costa, "The stiffening of soft soils on railway lines". Transportation Geotechnics, 17A, 178-191, 2018. doi:10.1016/j.trgeo.2018.09.004.

[14] E. Kausel, "Thin-layer method: Formulation in the time domain", International Journal for Numerical Methods in Engineering, 37, 927–41, 1994. doi:/10.1002/nme.1620370604.

[15] E. Kausel, "Wave propagation in anisotropic layered media", International Journal for Numerical Methods in Engineering, 23, 1567–78, 1986. doi:10.1002/nme.1620230811.

[16] H.A. Dieterman, A. Metrikine, "The equivalent stiffness of a half-space interacting with a beam. Critical velocities of a moving load along the beam", European Journal of Mechanics, A/Solids, 15, 67–90, 1996.