The Fourteenth International Conference on
Computational Structures Technology
23–25 August, 2022 | Montpellier, France

Proceedings of the Fourteenth International Conference on
Computational Structures Technology
Edited by B.H.V. Topping and J. Kruis
Civil-Comp Conferences, Volume 3, Paper 19.1
Civil-Comp Press, Edinburgh, United Kingdom, 2022, doi: 10.4203/ccc.3.19.1

Nonlinear quasi-static modelling of cable structures including geometric constraints

M.B. Wesdou[1,2], N. Azouz[1], P. Joli[1] and J. Neji[2]

[1]Université Paris-Saclay, Univ Evry, LMEE, Evry, France.
[2]University of Tunis El Manar, LR-MOED, Tunis, Tunisia.

Abstract

The purpose of this paper is the modelling in large displacement of systems composed of a rigid platform suspended by flexible cables, as can be observed in lifting systems of a construction crane or in cable-driven parallel robots (CDPRs). A recent approach has been proposed in the literature to model the nonlinear behavior of a cable element based on a three dimensional catenary elastic modelling and the general displacement control method (GDCM) as solver. In this paper, two options of this method are proposed to take into account the geometric constraints coupling the large displacements of the cable extremities. These two methods are tested and compared by numerical examples.

Keywords: nonlinear modelling, general displacement control method, geometric constraints, elastic catenary model, penalty method.

1 Introduction

The emergence of cable-driven parallel robots (CDPRs) in various fields of industry has generated renewed interest in the study of cables. Indeed, these manipulators have a great advantage of lightness compared to conventional rigid robots. This has allowed the design of long-range robots, in particular for the precise guidance of mobile cameras in stadiums, but also opened up other perspectives such as the use of these manipulators in Large Capacity Airships (LCA) [1, 2]. The first studies of CDPRs adopted the strong hypothesis of the undeformability of cables neglecting their masses at the same time. It turned out that this reducing hypothesis, although very useful for minimizing computations, comes up against an undeniable reality regarding the extension, bending and sagging of cables. This is particularly highlighted if these robots are used to handle heavy loads. These simplifying assumptions in these cases generate more or less significant errors in the location of the end-effector, which affects the accuracy of these robots and limits their field of application. It is therefore essential to carry out a larger study of the cables forming the robot in order to take into account the weight and elastic behavior of the latter and thus improve the precision of the robot. Recent studies have looked at this aspect, we can cite the work [3] where the emphasis has been placed on taking into account the effect of the weight and the deformability of cables. It is in this context that our present study is situated, where the objective is to develop a cable modelling methodology that is of a high level of generality while optimizing the precision/computation time ratio.

The approximated finite element method to formulate the cables is one possible approach but we consider in this paper only the exact analytical method. In this approach each cable is represented by a single cable element composed of only two nodes located at the two extremities of the cable (end-points). This second method is better than the first one because it is not necessary to consider internal nodes to control the geometric non linearity.

It exists several exact analytical models available in the book by Irvine [4] as the parabolic model, the catenary model and finally the elastic catenary model in which the cable is assumed to be perfectly flexible and linearly elastic with the self-weight uniformly distributed along the length of the curve.

Base on this last analytical approach, many authors since 1981 introduced nonlinear cable analysis [5], [6].

We consider in this paper, the elastic catenary model introduced by Yang and Tsay [7,11], a three-dimensional two-node element in which the geometric nonlinear effects are taken into account by an incremental-iterative analysis using a Generalized Displacement Control Method (GDCM). This method has been chosen because it is more robust than the Newton Raphson Method (NRM). On the contrary of the NRM, the GDCM can control the numerical solution around limit points when the stiffness matrix is singular [8, 9].

Moreover to simulate a CDPR, it is necessary to control the relative positions of the cable end-points connected to the moving platform such as the distances between these end-points keep constant. Many approaches to solve geometric constraint problems have been reported in the literature. The most popular approach to handle geometric constraints is to use penalty functions. The penalty function method transforms a constrained extremum problem into a single unconstrained optimization problem by inserting into the objective function, quadratic terms, which control the violation of the constraints thanks to adapted penalty parameters [10].

In this paper, the objective is to analyse the penalty-based method combined with GDCM through two options. The first one consists to combine the geometric constraints by an explicit formulation, which means by considering the constraint forces as external forces and the second one by an implicit formulation, which means by considering the constraint forces as internal forces.

2 Methods

A CDPR is a specific type of robot where several cables are connecting a moving platform to fixed points A_i, A_j,…A_m,. The attachment points of the cables on the platform are denoted B_i, B_j,…B_m.

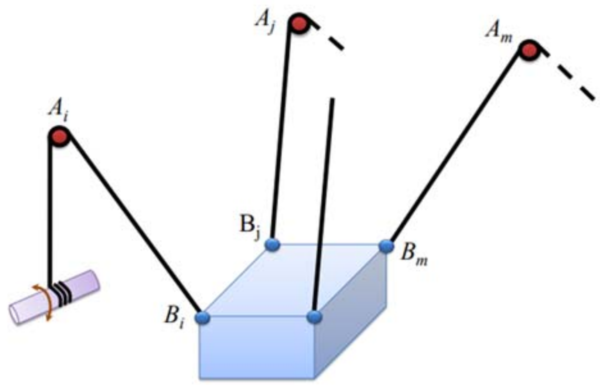

Figure 1: A general m-cable CDPR

It is necessary, to conserve a constant distance between two end-points among B_i, B_j,…Bm, to take into account geometric constraints in the CDPR's modeling, as follows:

$$\left\{\tilde{R}(\{U\}\right\} = \{F_{ext}\} - \{F_{int}\} + \{F_{\Phi}\} \quad (1)$$

$$\{\Phi\} = \{0\} \quad (2)$$

where $\{F_{\Phi}\}$ is the generalized nodal constraint forces associated to the geometric constraints represented by the algebraic equations (2) . The constraints vector $\{\Phi\}$ is defined as follows:

$$\{\Phi\}^T = \left\{(d_{ij}^2 - l_{ij}^2) \quad \dots \quad (d_{jm}^2 - l_{jm}^2)\right\}^T \quad (2)$$

$$d_{ij}^2 = \left\{\overrightarrow{B_iB_j}\right\}^T . \left\{\overrightarrow{B_iB_j}\right\} \quad (3)$$

Where $l_{ij}, \dots, l_{jm}$ are all the necessary lengths that must be kept constant because of the rigid motion of the platform. Based on the formulation of the virtual work δW, we have the following relation:

$$\delta W = \{\delta U\}^T . \{F_{\Phi}\} = \{\delta\Phi\}^T . \{\mu\} \Rightarrow \{F_{\Phi}\} = [C]^T\{\mu\} \quad (4)$$

With $[C] = \left[\frac{\partial\Phi}{\partial U}\right]$ is the Jacobean matrix of the geometric constraints and $\{\mu\}$ is the Lagrangian multipliers vector in which each component represents the closure force associated to one geometric constraint.

From these previous considerations, the equations of the CDPR's modelling becomes:

$$\{\tilde{R}(\{U\})\} = \{F_{ext}\} - \{F_{int}\} + [C]^T\{\mu\} \quad (5)$$

To eliminate the Lagrangian multipliers, one way is to consider each closure force as proportional to the violation of the corresponding geometric constraint, as follows:

$$\{\mu\} = -k\{\Phi\} \quad (6)$$

Physically, it is like adding virtual springs between the cables and the moving platform at the attachment points. Fundamentally these forces are not explicit because they depend on the unknown nodal displacements. So they have to be considered as internal forces added to the others due to the cable stiffness.

$$\{\tilde{F}_{int}\}= \{F_{int}\} + [C]^T k\{\Phi\};\ \{\tilde{R}(\{U\})\} = \{F_{ext}\} - \{\tilde{F}_{int}\} \quad (7)$$

$$\left[\tilde{K}_t(\{U\})\right] = [K_t(\{U\})] + k[C]^T[C] \quad (8)$$

This approach is the well-known penalty method in which the components of $\{\Phi\}$ are called the penalty functions and k the penalty factor. From now on, It is easy to apply straightforward the generalized displacement control method (GDCM). We have respected the formulation introduced in [8] where i is referring to the current increment step j is referring to the current iteration of the GDCM. Readers can find all the details of the GDCM in [8].

First we solve at each iteration the following algebraic system:

$$\begin{cases} \{\Delta F\} = \left[\tilde{K}_t(\{U\})\right]_{j-1}^{i}\left\{\Delta\widehat{U}\right\}_{i}^{j} \\ \{\tilde{R}(\{U\})\}_{j-1}^{i} = \left[\tilde{K}_t(\{U\})\right]_{j-1}^{i}\{\Delta\overline{U}\}_{j}^{i} \end{cases} \quad (9)$$

Then, after having defined the right load incremental parameter λ_i^j, we calculate:

$$\{\Delta U\}_j^i = \lambda_j^i \left\{\Delta\widehat{U}\right\}_j^i + \{\Delta\overline{U}\}_j^i \quad (10)$$

And finally we update the quantities:

$$\begin{cases} \{\tilde{F}_{int}\}_j^i = \{\tilde{F}_{int}\}_{j-1}^i + \left[\tilde{K}_t(\{U\})\right]_{j-1}^i . \{\Delta U\}_j^i \\ \{U\}_j^i = \{U\}_{j-1}^i + \{\Delta U\}_j^i \end{cases} \quad (11)$$

Because the augmented tangent stiffness $\left[\tilde{K}_t(\{U\})\right]$ has to be updated at each iteration, we propose another approach that we call stiffness method. We consider now the closure forces as external forces which means that must be calculated explicitly, so we propose the following increment form:

$$\{\tilde{F}_{ext}\}_i= \{\tilde{F}_{ext}\}_{i-1} + \Delta\{F\} \quad (12)$$

with $\{\tilde{F}_{ext}\} = \{F_{ext}\} + [C]^T k\{\Phi\}$ and consequently $\{\tilde{R}(\{U\})\} = \{\tilde{F}_{ext}\} - \{F_{int}\}$

From now on, it is easy to apply GDCM as follows:

First we solve at each iteration the following algebraic system:

$$\begin{cases} \{\Delta F\} = K_t(\{U\})]_{j-1}^{i}\left\{\Delta\widehat{U}\right\}_{i}^{j} \\ \{\tilde{R}(\{U\})\}_{j-1}^{i} = [K_t(\{U\})]_{j-1}^{i}\{\Delta\overline{U}\}_{j}^{i} \end{cases} \quad (13)$$

Then, after having defined the right λ_i^j, we calculate:

$$\{\Delta U\}_j^i = \lambda_j^i \left\{\Delta\widehat{U}\right\}_j^i + \{\Delta\overline{U}\}_j^i \quad (14)$$

And finally we update the quantities in the same way than (11).

$$\begin{cases} \{F_{int}\}_i^j = \{F_{int}\}_i^{j-1} + [K_t(\{U\})]_i^{j-1} . \{\Delta U\}_i^j \\ \{U\}_j^i = \{U\}_{j-1}^i + \{\Delta U\}_j^i \end{cases} \quad (15)$$

We call this approach, the stiffness method. As we can see the closure forces are considered in this approach like feedback external forces to control the violation of the geometric constraints which can induce numerical instabilities if these forces become too strong relative to the internal forces of the cables. Another problem is encountered at the first increment force. Indeed, as there is not yet violation of the constraint, so there are no feedback forces. It is necessary to reduce $\{\Delta F\}$ at the first increment in order to limit the violation geometric constraints at the next increment. Another possibility, safer, it is to use the penalty method at the first increment.

3 Results

Example 1: cable net

Figure 2 shows a twelve-node cable net using a non-dimensional unit. At first, the cable network is in the horizontal plane (x, y). The cable net's attributes are expressed in consistent units as the following: the cross-sectional area A is 1 unit, the elastic modulus E is 29.10^5 units, the initial length of each cable is $L_0 = 40$ units and the self-weight w is 1 unit. At node 8, a load of 1000 units is applied in the opposite direction of z.

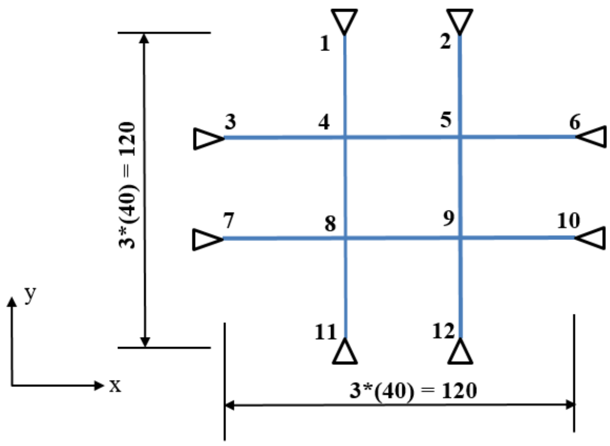

Figure 2: Twelve cables in 3D

Table 1 shows the comparison of our results with them found to references [12] and [13] on the displacements of internal nodes 4.

	u_x	u_y	u_z
Error percentage compared to Damir Seldar et al [13] for node 4	0,2817 %	0,27%	0,107%
Error percentage compared to Y.B.Yang et al [8] for node 4	1,71%	1,1%	0,0754 %

Table 1: Error percentage compared to references for node 4.

This first example is considered as the reference test to validate our numerical algorithm implementation. The main focus for the next two examples is to verify the results found in example 1 by considering now geometric constraints.

Example 2: Cable net with linear geometric constraints

In the example 1, all the degrees of freedom of the nodes 1, 2, 3, 6, 7, 10, 11, 12 are set to zero and so are eliminated in the modelling of the cable net. In this example the node 1 is now considered to be attached to the fixed support. Its three degrees of freedom in translation are not eliminated but constrained by three forces in translation. The three algebraic constraints that we have to take into account in this modelling, are u_{1x} =0, u_{1y} =0 and u_{1z} =0. We can note that they are linear relative to the DOF and not nonlinear as previously defined in the case of maintaining distance between two nodes. Just like in Example 1, a load was applied to the node 8 with the value of 1000 units in the opposite direction of the z axis.
The results found in this example were compared and verified by the results found with the original GDCM.

	u_x	u_y	u_z
Error percentage for node 4 with the stiffness method	0,0 %	-7,43%	-0,12%
Error percentage for node 4 with the penalty method	0,0 %	-0,34%	-0,01%

Table 2: Results found after fixing node 1.

Example 3: Cable net with nonlinear geometric constraints

This last example (see Figure 3) is carried out merely to verify that both methods function for nonlinear geometric constraints when there is a distance separating the nodes. A load was applied to node 3 with the value of 1000 units in the opposite direction of the z axis. 6 distance constraints were applied between nodes 1, 2, 3 and 4 as demonstrated in the following figure.

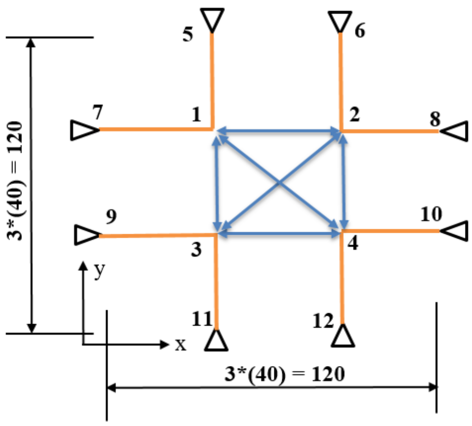

Figure 3: Six geometric constraints on distance

As it shown in table 3, the results found between the two methods are similar and coherent with the result from the original example. Similar to other tests, the stiffness method has a limit value for k (no more than 10^6N/m). In addition, the value of k for the penalty method is limited to no more than 10^{12}.

Nodes	**Stiffness method**			**Penalty method**		
	u_x	u_y	u_z	u_x	u_y	u_z
1	-0,02683	-0,02762	-1,32615	-0.0218	-0.0218	-1.525
2	-0,02980	-0,02980	-0,77438	-0.0369	-0.0369	-0,427
3	-0,00814	-0,00814	-2,67144	-0,0067	-0,0067	-2,628
4	-0,02762	-0,02683	-1,32615	-0,0218	-0,0218	-1,526

Table 3: Displacements of nodes 1,2,3,4.

Table 8 Example 4: the initial and the final computed distance between nodes 1, 2, 3 and 4

	Initial results	**Final results**	
		Stiffness method	Penalty method
1 et 2	40	40,0008	40
1 et 3	40	40,0031	40
1 et 4	56,5685	56,5674	56,5686
2 et 3	56,5685	56,5697	56,5686
2 et 4	40	40,0008	40
3 et 4	40	40,0031	40

Table 4: The initial and the final computed distance between nodes 1, 2, 3 and 4.

The table 4 presents the initial and the final computed distance between nodes 1, 2, 3 and 4. Evidently, the penalty method proves to be the more accurate method. These results indicate the suggested programs' good computational efficiency in different cases.

4 Conclusions and Contributions

Due to the flexible nature of cable-supported structures, the geometric nonlinear impact must be considered while analyzing them. The Generalized Displacement Control method (GDCM), was used to perform incremental-iterative analysis where the loads are not kept constant in the iterative steps and general numerical stability is maintained when passing limit points and snap-back points. Three different tests on cable structures were presented, where the results were compared to previous results found by other researches. Through this comparison, it was found that the GDCM using a 3D elastic catenary model is verified. In addition, two modifications of this method were applied to take into account geometric constraints equations coupling the large displacements of the cable ends. The first method presented to eliminate the geometric constraints consisted in adding external explicit elastic forces while in the second method, the penalty function method was considered. These two methods were tested and verified by different numerical examples. For future work, the aim is to implement the augmented Lagrangian technique to apply the nonlinear geometric constraints. In addition, the main future objective is to model a CDPR using the model and methods presented earlier in this paper.

Acknowledgements

This work is supported by a public grant overseen by the French National research Agency (ANR) as part of the “Investissements d’Avenir” program, through the “ADI 2019” project funded by the IDEX Paris-Saclay, ANR-11-IDEX-0003-02” and the University of Tunis El Manar.

References

[1] F. BenAbdallah, N. Azouz, L. Beji and A. Abichou, "Modeling of a heavy lift airship carrying a payload by a cable driven parallel manipulator," International Journal of Advanced Robotic Systems, 2019.

[2] N. Azouz, M. Khamlia, F. B. Abdallah and F. Guesmi, "Modelling and design of an airship crane," in ASME-IMECE Conference, Pittsburgh, USA, 2018.

[3] H. Yuan, E. Courteille and D. Deblaise, "Static and Dynamic Stiffness Analyses of Cable-Driven Parallel Robots with non-negligible cable mass and elasticity," Mechanism and Machine Theory, pp. 64-81, 2015.

[4] M. Irvine, Cable Structures, New York: dover publications, Inc, 1981.

[5] H. Jayaraman and W. Knudson, "A Curved Element for the Analysis of Cable Structures," Computers & Structures, vol. 14, no. 3-4, pp. 325-333, 1981.

[6] H.-T. Thai and Seung-EockKim, "Nonlinear Static and Dynamic Analysis of Cable Structures," Finite Element in Analysis and Design, vol. 47, pp. 237-246, 2011.

[7] Y. B. Yang and J.-Y. Tsay, "Geometric Nonlinear Analysis of Cable Structures with a Two-node Cable Element by Generalized Displacement Control Method," International Journal of Structural Stability and Dynamics, vol. 7, no. 4, pp. 571-588, 2007.

[8] M. A. Torkamani and M. Sonmez, "Solution Techniques for Nonlinear Equilibrium Equations," in 18th Analysis and Computation Specialty Conference, 2008.

[9] Y.-B. Yang. and M.-S. Shieh, "Solution Method for Nonlinear Problems with Multiple Critical Points," AIAA Journal, vol. 28, no. 12, pp. 2110-2116, 1990.

[10] C. M. Hoffmann and R. Joan-Arinyo, Handbook of Computer Aided Geometric Design, 2002

[11] Y. B. Yang and J.-Y. Tsay, "Geometric Nonlinear Analysis of Cable Structures with a Two-node Cable Element by Generalized Displacement Control Method," International Journal of Structural Stability and Dynamics, vol. 7, no. 4, pp. 571-588, 2007.

[12] S. Damir, L. Zeljan and B. Andjela, "Nonlinear static isogeometric analysis of cable structures," Archive of Applied Mechanics, vol. 89, p. pages 713–729, 2019.