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Finite Element Simulation of Two-dimensional Photonic Crystals

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Abstract

Photonic crystals are artificial microscopic structures that are periodically arranged in some directions. In this paper, we study the two-dimensional photonic crystals, which are easy to fabricate and show many interesting characteristics. We employ the finite element method with triangular cell lattice partition for modelling and simulation. By comparing with the existing literature, the effectiveness of our approach is proved.

Keywords: photonic crystal, finite element method, honeycomb lattice, band gap.

1 Introduction

Photonic crystal is an artificial microstructure formed by periodic arrangement of media with different refractive index. It can be classified into one-dimensional [1–2], two-dimensional, and three-dimensional [3] structures in terms of the periodicity in different directions. In particular, the two-dimensional photonic crystals have many interesting characteristics is relatively easy to fabricated, thus having received widespread attention.

The periodic array results in energy band gap, which restricts the electromagnetic wave transmitting in this material [4]. Due to the special periodic structure of photonic crystals, a blocking effect can be shown on the photons with specific wavelengths and forms the photonic band gap. If the photon energy falls into the photonic band gap, it cannot enter the photonic crystal. The research of photonic band gaps is of great interest, by which one could design certain photonic crystals according to needs, and the light propagation in a certain frequency range can be prohibited through the

photonic band gap. Many applications are based on the position and width of the band gap, such as the narrowband filter and demultiplexer [5]. We refer the interested reader to [6–8] for more mathematical discussion on photonic crystal band gap studies.

This paper studies the two-dimensional photonic crystals. The common methods to simulate the band gap of two-dimensional photonic crystals include the plane wave expansion (PWE) method and the finite difference time domain (FDTD) method, while we use the finite element method (FEM) to carry on the modelling and simulation. We adopts the triangular cell lattice partition in the simulation process for modelling honeycomb lattice photonic crystals. Then, we calculate the band structure of columnar photonic crystals, and show the correctness and effectiveness of our approach by comparing our results with [9].

2 Methods

In classical electromagnetic theory, electromagnetic phenomena can be described by the Maxwell equations

$$\begin{cases} \nabla \times H = J + \frac{\partial D}{\partial t} \\ \nabla \times E = -\frac{\partial B}{\partial t} \\ \nabla \cdot B = 0 \\ \nabla \cdot D = \rho \end{cases}. \tag{2.1}$$

For more details on computational photonics, see [10-12]. For photonic crystals, we study the electromagnetic waves. Their propagation in photonic crystals satisfies (2.1). Here, E is the strength of the electric field, H is the strength of the magnetic field. For two-dimensional photonic crystals, it is difficult to solve Maxwell equation directly, because electromagnetic waves can be decomposed into simple harmonics, namely, $\mathrm{E}(\mathrm{r},\mathrm{t}) = \mathrm{E}(\mathrm{r})\mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}$, $\mathrm{H}(\mathrm{r},\mathrm{t}) = \mathrm{H}(\mathrm{r})\mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}$, where ω denotes the frequency. The Helmholtz equation independent of time can be obtained by substituting (2.1) by the electric field equation (2.2) and magnetic field equation (2.3)

$$\nabla \times \nabla \times \mathrm{E}(\mathrm{r}) - \left(\frac{\omega}{c}\right)^2 \varepsilon(\mathrm{r})\mathrm{E}(\mathrm{r}) = 0, \tag{2.2}$$

$$\nabla \times \left(\frac{1}{\varepsilon(\mathrm{r})} \cdot \nabla \times H(r)\right) - \left(\frac{\omega}{c}\right)^2 H(r) = 0, \tag{2.3}$$

When considering the band gap problem of 2-D photonic crystal, it can be simplified into two polarization cases called TM and TE [13]. In TM mode, $E(r) = (0,0,E_R(\tilde{r}))$. In TE mode, $H(r) = (0,0,H_R(\tilde{r}))$, where $\tilde{r}$ denotes the two-dimensional coordinate (x,y). Substituting into (2.2) and (2.3), the electric field wave equation of TM mode and magnetic field wave equation of TE mode can be shown respectively by (2.4) and (2.5)

$$\nabla \cdot \left(\nabla E_R(\tilde{r})\right) + \left(\frac{\omega}{c}\right)^2 \varepsilon(\tilde{r})E_R(\tilde{r}) = 0, \tag{2.4}$$

$$\nabla \cdot \left(\frac{1}{\varepsilon(\tilde{r})}\nabla H_R(\tilde{r})\right) + \left(\frac{\omega}{c}\right)^2 H_R(\tilde{r}) = 0. \tag{2.5}$$

Due to the periodicity of two-dimensional photonic crystals, which satisfies the Bloch theorem, the above wave equation can be converted to the unit cell. In two dimensions, $\lambda = (\omega/c)^2$, and $\mathbf{k} = (a, b)$ represents the wave vector in two-dimensional plane space, whose value range is irreducible Brillouin region B.

Now, consider the TM module,

$$(\nabla + i\mathbf{k}) \cdot [(\nabla + i\mathbf{k})E_R(\tilde{\boldsymbol{r}})] + \lambda\varepsilon(\tilde{\boldsymbol{r}})E_R(\tilde{\boldsymbol{r}}) = 0. \tag{2.6}$$

The problem is solved in two-dimensional set region with the unit protocell boundary $\Pi = \partial\Omega$, which satisfies

$$E_R|_{\Gamma_1} = E_R|_{\Gamma_3}, E_R|_{\Gamma_2} = E_R|_{\Gamma_4}, \tag{2.7}$$

where Γ_1 and Γ_3 are a set of opposite edges of unit primordial cells, Γ_2 and Γ_4 are another set of opposite edges of unit primordial cells.

The process of solving the TE mode is similar. To facilitate our discussion, the two modes will be uniformly written as the eigenvalue form. Given the the vector $\mathbf{k}$, find λ and u_R satisfying the following equation:

$$\begin{cases} (\nabla + i\mathbf{k}) \cdot [\varrho(\nabla + i\mathbf{k})u_R] + \lambda\vartheta u_R = 0, & \text{in } \Omega \\ u_R|_{\Gamma_1} = u_R|_{\Gamma_3}, u_R|_{\Gamma_2} = u_R|_{\Gamma_4}, & on \ \ \partial\Omega \end{cases}, \tag{2.8}$$

where

$$\varrho = \begin{cases} 1, & TM \\ \frac{1}{\varepsilon(\tilde{r})}, & TE \end{cases}, \vartheta = \begin{cases} \varepsilon(\tilde{r}), & TM \\ 1, & TE \end{cases}, \tag{2.9}$$

and λ is the eigenvalue to be solved.

For Equation (2.8), the variational process and discrete process of a finite element are given below. First, define Sobolev space as follows:

$$\mathrm{H}_{\mathrm{k}}^1(\Omega) = \left\{v \in H^1(\Omega) \mid v|_{\Gamma_1} = v|_{\Gamma_3}, v|_{\Gamma_2} = v|_{\Gamma_4}\right\}. \tag{2.10}$$

The eigenvalue problem to be solved is that $\forall k \epsilon B$, $(k, u) \in C \times \mathrm{H}_{\mathrm{k}}^1(\Omega)$ satisfies

$$\int_\Omega (\nabla + ik)u \cdot \overline{(\nabla + \imath k)v}\ d\Omega = \lambda \int_\Omega \vartheta u \cdot \bar{v}\ d\Omega\,,\ \forall\, v \in H_k^1(\Omega). \tag{2.11}$$

For a fixed wave vector k, when the solution region is a linear trig element, the approximate values of u, v in the element are u_e, v_e, and the basic functions denote N_i^e, N_j^e, N_k^e. Hence, we have

$$\begin{aligned} \int_\Omega \varrho(\nabla + ik)u \cdot \overline{(\nabla + \imath k)v}\ d\Omega &= \int_\Omega \varrho(\nabla + ik)\left(u_i N_i^e + u_j N_j^e + u_k N_k^e\right) \cdot (\nabla - ik)N_x\ d\Omega \\ &= u_i \int_\Omega \varrho(\nabla + ik)N_i^e \cdot (\nabla - ik)N_x d\Omega + u_j \int_\Omega \varrho(\nabla + ik)N_j^e \cdot (\nabla - ik)N_x d\Omega + \\ &\quad u_k \int_\Omega \varrho(\nabla + ik)N_k^e \cdot (\nabla - ik)N_x d\,\Omega, \end{aligned} \tag{2.12}$$

$$\int_\Omega \vartheta u \cdot \bar{v}\ d\Omega = u_i \int_\Omega \vartheta\varepsilon N_i^e \cdot N_x d\Omega + u_j^e \int_\Omega \vartheta\varepsilon N_j \cdot N_x d\Omega + u_k^e \int_\Omega \vartheta\varepsilon N_k \cdot N_x d\Omega, \tag{2.13}$$

Then, according to the local and global numbering relations of linear trigonometric elements, the total stiffness matrices A and B are generated, and the discrete eigenvalue problem corresponding to the variational problem is obtained as

$$AU = \lambda BU. \tag{2.14}$$

When solving the generalized eigenvalue problem, periodic boundary conditions (2.7) should also be considered, to obtain the final generalized eigenvalue problem to be solved:

$$A\tilde{U} = \lambda B\tilde{U}. \tag{2.15}$$

3 Results

Figure 1 shows the photonic crystal array composed of periodic dielectric columns, extending indefinitely in the z-direction. The background medium is air. Figure 2 is the triangular lattice element selected after the photonic crystal parallel to the xoy plane. The side length is a lattice constant a_0, and it is composed of two medium columns in the middle and eight 1/2 medium columns on its four edges. After periodic expansion, the triangular lattice can be consistent with the photonic crystal, as shown in Figure 1.

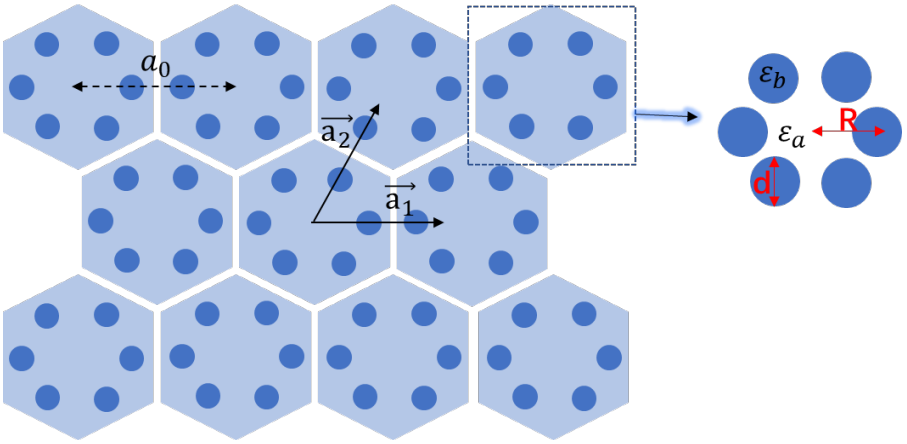

Figure 1: Photonic crystal array.

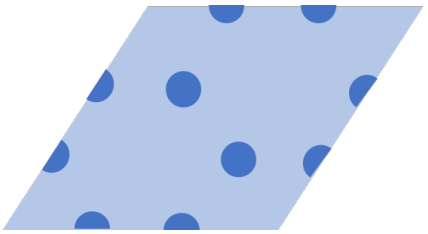

Figure 2: Triangular lattice element.

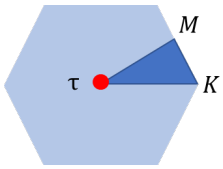

Figure 3: Brillouin region.

Every crystal has two kinds of lattice, the lattice in real space and the lattice in reciprocal space. The basis vectors of the triangular lattice in positive space are $\vec{a_1} = a_0(1,0), \vec{a_2} = a_0(1/2, \sqrt{3}/2)$. The inverted lattice basis vectors are $\vec{b_1} = k_0(\sqrt{3}/2, -1/2), \vec{b_2} = k_0(0,1), k_0 = 4\pi/\sqrt{3}a_0$. An inverted lattice point is selected as the origin, and there are 6 inverted lattice basis vectors closest to the origin. Thus, the first

Brillouin region of the triangular lattice photonic crystal is shown in Figure 3, and the coordinates of the three vertices are $K = k_0(\sqrt{3}/3,0), \tau = (0,0), M = k_0(\sqrt{3}/4,1/4)$.

Figures 4-6 show the 2D photonic crystal dispersion diagram obtained by simulation under the selection of triangular lattice. When a_0/R changes, the width of band gap will change accordingly. When $a_0/R = 3.125$, the normalized frequency is between 0.48 and 0.51. When $a_0/R = 3$, we find that the band gap disappears. Finally, when $a_0/R = 2.8$, the normalized frequency has a band gap between 0.44 and 0.48. These results are all consistent with that in [9], and thus the effectiveness of our approach has been proved.

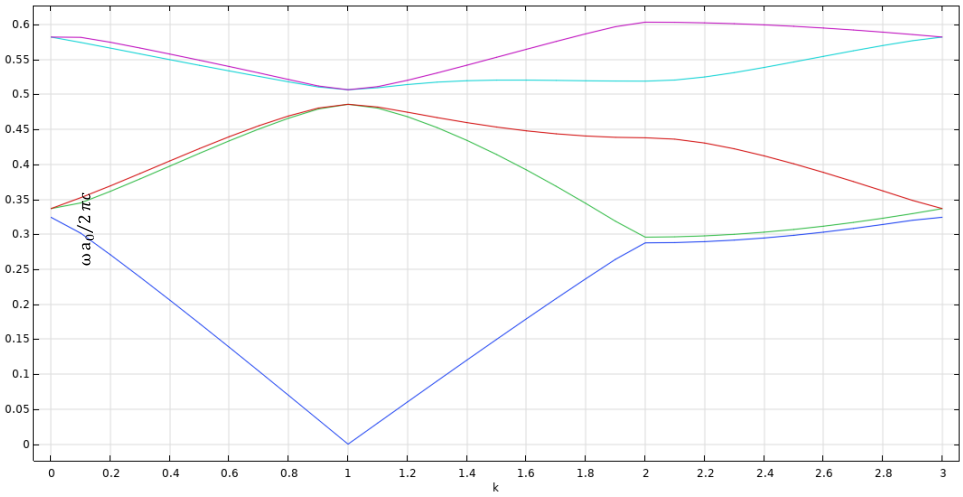

Figure 4: Photonic crystal dispersion diagram, $a_0/R = 3.125$.

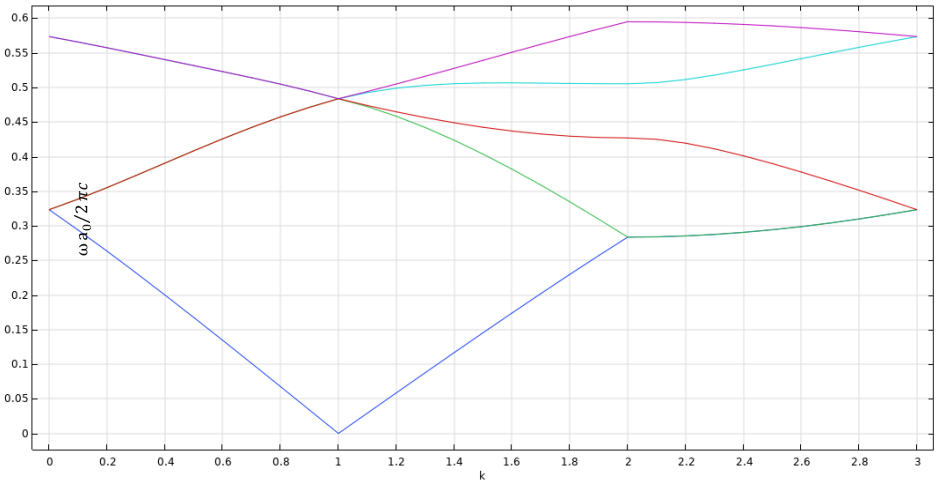

Figure 5: Photonic crystal dispersion diagram, $a_0/R = 3$.

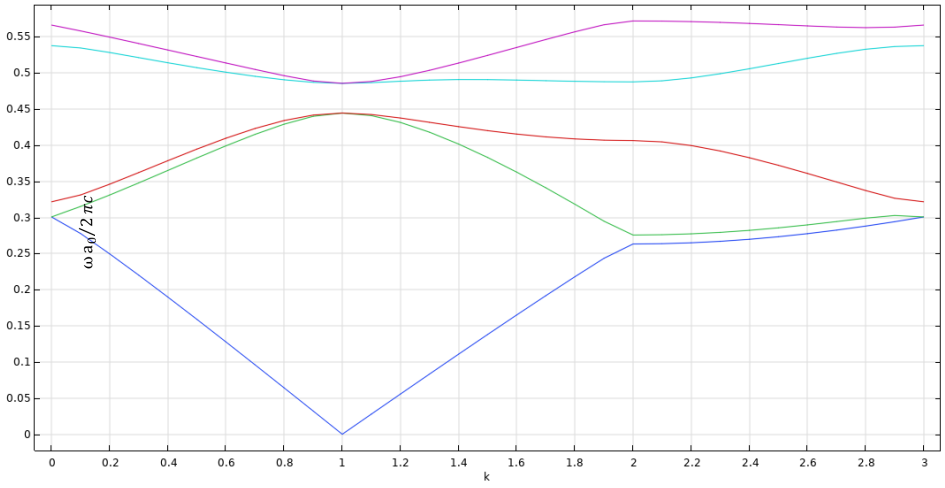

Figure 6: Photonic crystal dispersion diagram, $a_0/R = 2.8$.

4 Conclusions and Contributions

There exist other methods for the simulation of two-dimensional photonic crystals, such as PWE [14] method and FDTD method [15]. In this paper, we exploit the finite element method and choose the triangular lattice as the unit of honeycomb photonic crystal lattice. The dispersion diagram of the photonic crystal is successfully simulated, and the numerical results are consistent with that in the literature, which proves the feasibility and effectiveness of our modelling and simulation processes.

Acknowledgements

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