The Fourteenth International Conference on Computational Structures Technology
23–25 August, 2022 | Montpellier, France

Proceedings of the Fourteenth International Conference on
Computational Structures Technology
Edited by B.H.V. Topping and J. Kruis
Civil-Comp Conferences, Volume 3, Paper 19.3
Civil-Comp Press, Edinburgh, United Kingdom, 2022, doi: 10.4203/ccc.3.19.3

Modal superposition of non-classically damped transient problems with position and velocity dependent force

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Abstract

The approach to the solution of transient problems of the semidefinite systems with non-classical damping and force dependent on time, position and velocity is proposed and implemented. It provides reliable, efficient and accurate solution algorithm without blurring the physical meaning.
Problem matrix is constant even the problem is nonlinear, its inversion is trivial, and order is controllable. The price is solution of the eignproblem and transformations between natural and principal coordinates. However, in spite of the iterative procedure, modal approach is proved to be highly efficient with maximum two tightening iteration per step for given integration parameters, practically independently on the integration scheme involved.
The contribution is illustrated on the theoretical and practical examples. The proposed procedure eliminates nonphysical behaviour of the semidefinite problems under reactive force e.g. as in the ship power propulsion system under ice conditions when simulating long lasting ice impact conditions.

Keywords: non-classical damping, modal superposition, nonlinear problem, transient vibration, numerical integration, ship power propulsion.

1 Introduction

Modal superposition technique has number of advantages regarding numerical efficiency and accuracy control. However, it can be directly applied to the linear problems, only [1], [2]. The implementation of the modal superposition to the nonlinear problems is linked to the tightening iterative procedures, [3]. In this paper, it is presented an application of the modal superposition technique to the specific problems when non-classical damping can be treated as nonlinearity source. Additionally, external load is position and velocity dependent, [4], what also boils out in nonlinearity. Putting all nonlinear forces in governing equation to the right-hand side, modal superposition technique can be applied providing that nonlinear force is calculated and transformed in modal space in each time step.

The major advantage of the modal superposition technique is in the reduced number of (decoupled) system equations, as e.g. for the complex space is given in [5]-[7]. When the dependencies of the force on displacements and velocities are of such type that some (typically higher) modes have not significant impact to the force and therefore can be missed along with corresponding modal equations from calculation, additional numerical effort reduction is achieved.

The theoretical and practical examples are presented in this paper. The Practical example of the transient problem of the ship power propulsion system is presented. Transient force is induced due to propeller-ice interaction. This force is dependent on position as well as velocity of the propeller. The ship power propulsion system is modelled as semidefinite problem and therefore includes rigid body modes that are filtered out in order to have elastic mode extracted out. Numerical efficacy and accuracy are discussed, too.

2 Methods

The governing Equation (1) of the mechanical dynamic system typically reads:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q}(t, \mathbf{q}, \dot{\mathbf{q}}) \tag{1}$$

where:

$\mathbf{M}$ $n \times n$ *global mass matrix*

$\mathbf{D}$ $n \times n$ global non-classical damping matrix

$\mathbf{K}$ $n \times n$ global stiffness matrix

$\mathbf{q}$ $n \times 1$ displacement vector that depends on time

$\mathbf{Q}(t, \mathbf{q}, \dot{\mathbf{q}})$ $n \times 1$ force vector as function of displacement and time

Let damping matrix be divided to classical and non-classical part according Equation (2):

$$\mathbf{D} = \alpha\mathbf{M} + \beta\mathbf{K} + \mathbf{D}_{NL} \tag{2}$$

where $\boldsymbol{D}_{NL}$ is non-classical damping part matrix. Let, additionally, the related undamped eigenproblem (3):

$$(\mathbf{K} - \lambda\mathbf{M})\Phi = 0 \tag{3}$$

has eigen-pair matrices: $\mathbf{\Phi}$ and $\mathbf{\Lambda}$. Now, the governing Equation (1) can be reshaped in modal space as it follows:

$$\ddot{\mathbf{r}} + \mathbf{D}_L\dot{\mathbf{r}} + \mathbf{\Lambda}\mathbf{r} = \mathbf{f}(t, \mathbf{q}, \dot{\mathbf{q}}) - \mathbf{\Phi}^{\mathrm{T}}\mathbf{D}_{NL}\dot{\mathbf{r}} \tag{3}$$

where is diagonal matrix of classical damping and $\boldsymbol{f}(\mathbf{q},\mathbf{v},t) = \mathbf{\Phi}^{\mathrm{T}}\ \mathbf{F}(\mathbf{q},\mathbf{v},t)$ force in modal space. The whole integration procedure can be summarized in incremental-iterative manner as it follows:

***Algorithm*:** Modal superposition: Non-classical damping and
Time, displacement and velocity dependent force

1: On input: $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K}$, $\boldsymbol{q_0}$, $\dot{\boldsymbol{q}}_\mathbf{0}$, *the rule for calculation of the displacement dependent force*
2: Solution of the eigenproblem: $(\mathbf{K} - \lambda\mathbf{M})\,\boldsymbol{\varphi} = 0$, $\boldsymbol{\Lambda}$, $\boldsymbol{\Phi}$
3: Calculate α and β from known relative coefficients ξ_i
4: Calculate $\mathbf{D}_{NL} = \mathbf{D} - (\alpha\mathbf{M} + \beta\mathbf{K})$
5: Set and select modal equations
6: Perform modal superposition of transient problem with $\boldsymbol{f}(t)=\boldsymbol{\Phi}^T(\boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},t) - \boldsymbol{D}_{NL}\dot{\boldsymbol{q}}\ )$
    6.1 Calculation of the displacement and velocity dependent force
    6.2 Check convergence criteria
    6.3 Do nonlinear tightening iterations until convergence
7: On output: $\mathbf{F}(\boldsymbol{q},\dot{\boldsymbol{q}},t)$,$\boldsymbol{q}(t)$, $\dot{\boldsymbol{q}}(t)$, $\ddot{\boldsymbol{q}}(t)$

The proposed algorithm is implemented and a number of theoretical and practical problems are solved.

3 Results

Example 1: Two degrees of freedom system according Figure 1 is analysed for classical (Figure 2) and non-classical damping case (Figure 3) in order to quantize non-classical damping influence and illustrate procedure on example with significant influence of non-classical damping. Figure 3 with responses of the non-classically damped system solved with direct and modal superposition technique support proposed algorithm approach and for the time step close to the convergent solution it shows overlapping solutions for direct and modal approach for a series of integration schemes.

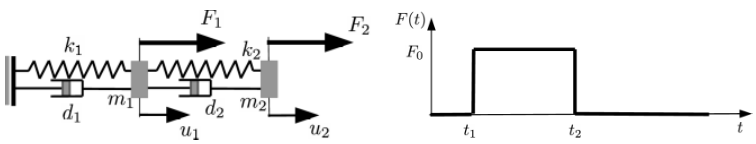

Figure 1: Example 1 - 2 dof lumped mass system $F_2(t)$ in time.

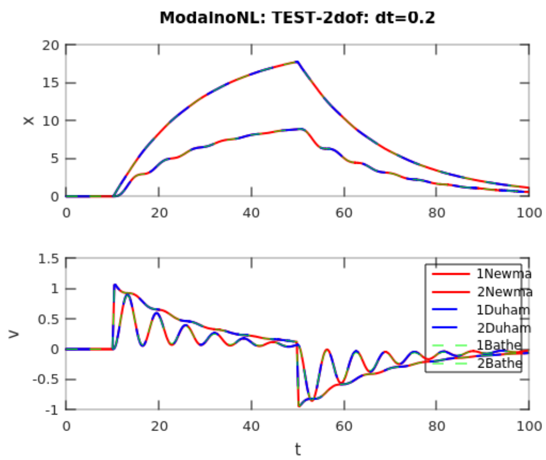

Figure 2: Example 1 - Classically damped model solution.

The available space does not allow detailed breakdown of the results (Problem matrix is constant even the problem is nonlinear, its inversion is trivial, and order is controllable. The price is solution of eignproblem and transformations between natural and principal coordinates.). However, in spite of the iterative procedure, modal approach is proved to be highly efficient with maximum two tightening iteration per step for given integration parameters, practically independently on the integration scheme involved.

Example 2: Practical example of the electrically powered ship propulsion system according scheme in Figure 4 has been analysed in run up case continuously exposed to the ice conditions (although case is not foreseen in rules, [4], basic ice load inputs are correlated to them).

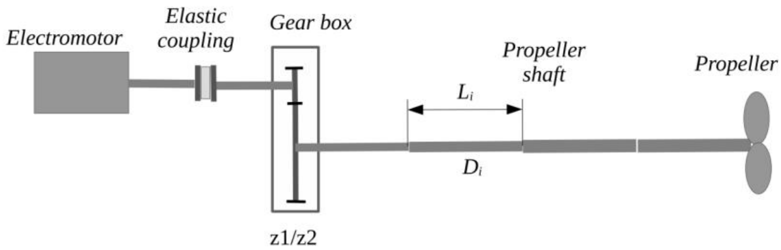

Figure 4: Ship power propulsion system scheme.

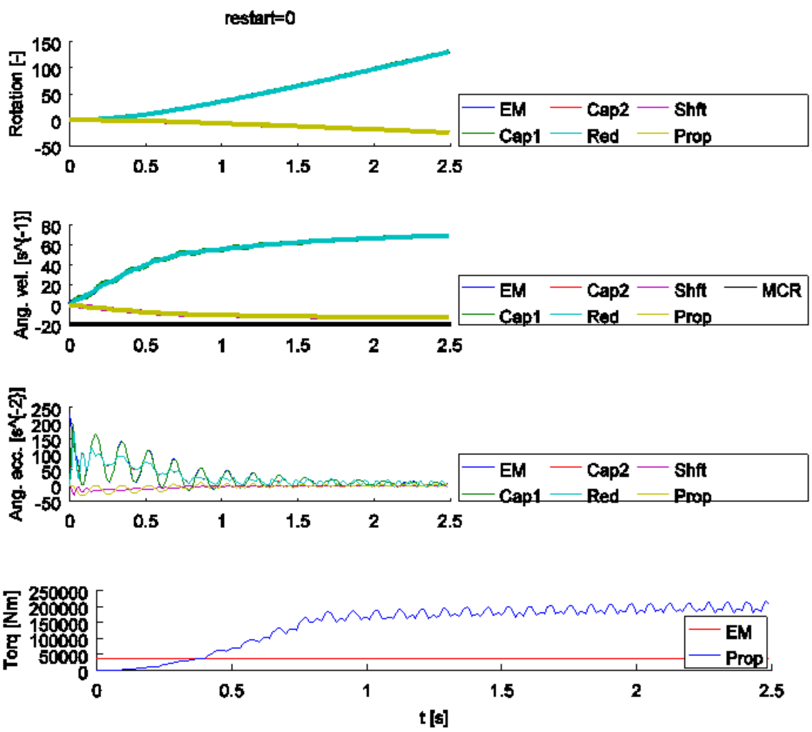

Figure 5: Example 2: Non-classically damped, semidefinite problem, loaded with position and velocity dependent force solution: Run up in the ice conditions.

Figure 5 presents solution of the practical implementation of the proposed algorithm to the torsional vibration of the ship power propulsion system (Figure 4) solved for position and velocity dependent force. Modal superposition technique facilitates filtering rigid body modes from the response of this semidefinite problem. However, it is important to note that filtering out of the rigid body mode can in this case be performed only partially (afterwards) because rigid boy mode takes part in the calculation of the force hammering to the propeller due to ice.

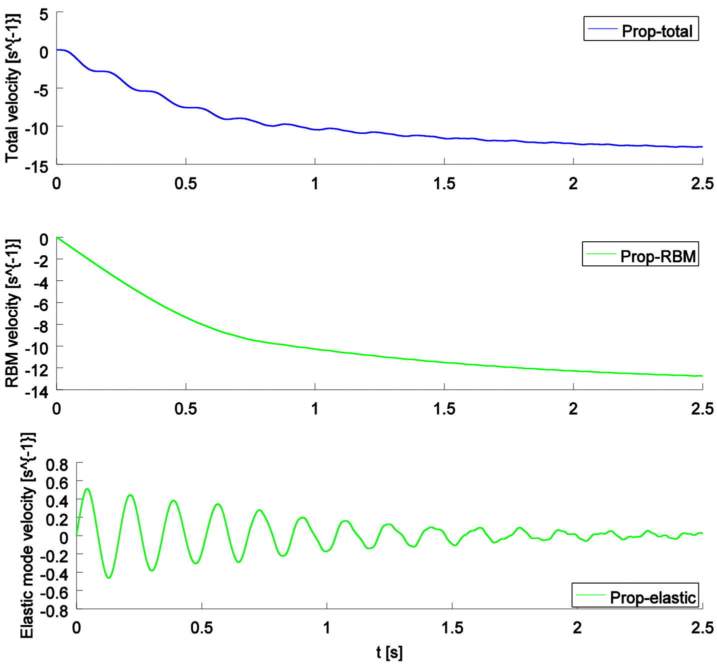

Figure 6: Example 2 - Decoupled rigid body mode and elastic modes contribution to the propeller motion.

With this technique, without lost physical meaning like in [5]-[7], we can have clear insight to elastic modes and corresponding stress harmonics important e.g. in fatigue analysis, Figure 6.

4 Conclusions and Contributions

The proposed approach to the solution of transient problems of the semidefinite systems with non-classical damping and force dependent on time, position and velocity provides reliable, efficient and accurate solution algorithm without blurring the physical meaning.

The position dependent (single sense) reactive force in combination with semidefinite system can even boils out in negative rotation what is nonphysical. Introducing the gradual scaling of the reactive force intensity controlled by velocity can keep physical behaviour even on long runs.

The contribution is illustrated on the theoretical and practical examples. The proposed procedure eliminates nonphysical behaviour of the semidefinite problems under reactive force e.g. as in the ship power propulsion system under ice conditions when simulating long lasting ice impact conditions.

Acknowledgements

This research has been supported by the Croatian Science Foundation grant IP-2018-01-6774, Evolutionary Shape Synthesis with Integral and Partitioned 3D Phenotypes, Dynamic Parameterizations and Meshless Modelling.

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