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Adaptive coupled discrete/continuous approach for the forming of materials

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Abstract

During the manufacturing process of train wheels, industries encounter limitations to study the fatigue cracking problem with the finite element method (FEM). Another method, the discrete element method (DEM), is more adapted to study these types of problems, but it is very expensive.
This paper shows a strategy to switch from an initially purely FEM computation to a combined FEM-DEM computation. This switching made it possible to take advantage of both methods. To achieve this strategy, a coupling method and a field transfer method between FEM and DEM have been developed. To validate these methods, numerical test cases are carried out.

Keywords: discrete element method, finite element method, coupling, fields transfer, crack, fatigue, train wheels.

1 Introduction

The fatigue cracking in forming tools during the manufacturing process of train wheels is a concern in the industry. The numerical simulations should be used to avoid cracks too severe to manufacture good quality wheels. The finite element method (FEM) is extensively used in industrial computational services. However, FEM suffers from limitations for crack propagation problems. Indeed, different mesh sizes are involved in such problems, and the mesh size is often particularly small in the vicinity of the crack tip. Moreover, the fields are singular in this area which is in contradiction with the continuum mechanics framework. In fact, FEM discretizes the topology of the discontinuity with the mesh, i.e., the mesh follows the geometrical discontinuity. Other methods such as the eXtended Finite Element Method, or the Cohesive Zone Model suffer from multiple difficulties, such as considering multi-cracks, crack deviations and closure. Unlike FEM, the discrete element method (DEM) is widely used to model cracked media such as raw materials or granular materials such as concrete, but this method is very expensive. In order to take advantage of both the FEM and the DEM, some authors developed coupling methods, generally overlapping, to decrease computation times while benefiting from DEM accuracy in crack regions.

The aim – to enhance cost-effectiveness – is to propose a strategy to switch from an initially purely FEM computation to a combined FEM-DEM computation, with DEM employed only when and where required. It involves the non-overlapping coupling of FEM and DEM via Lagrange multipliers, the transfer of FEM fields to DEM, and dynamic modifications of the computational domain.

The short paper is organized as follows. In the first part, the FEM and DEM discretizations, the coupling by Lagrange multipliers and the field transfer method are briefly presented. In the second part, some test cases are proposed to validate the steps of the implemented strategy.

2 Methods

2.1 FEM and DEM discretizations

The finite element method is based on a discrete mathematical algorithm allowing to find an approximate solution of a set of partial differential equation on a continuous compact domain. Within the element, the value of a function is determined using a polynomial interpolation of the values at the nodes [1]:

$$f(x,y,z) \simeq \sum_{i=1}^{n} \mathrm{N}_i(x,y,z) \cdot f_i \tag{1}$$

f: unknown function; N_i:shape function on nodes i; f_i value of f on nodes i.

The discrete element method (DEM) make it possible to simulate a set of solids in interaction in a discontinuous domain [2]. There are several forms to define the interaction between the discrete elements (contact laws, spring or beam). In this study, the interaction is controlled by beams of Euler-Bernoulli type (figure 1).

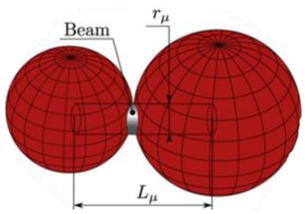

Figure 1: Cohesive beam bond configuration [3]

The beam shape is chosen cylindrical. So, the geometry is described with only two parameters: the length L_μ and radius r_μ. Two mechanical parameters are also associated: Young's modulus E_μ and Poisson's ratio v_μ [3].

2.2 Non-overlapping coupling by Lagrange multipliers

A large number of coupling methodologies have been developed in recent years. These methodologies are classified into two main families "overlapping coupling" [4]–[6] and "non-overlapping coupling"[7]–[10].
The comparison criteria are the computing time [11] and the accuracy of the solution. Here, the idea is to use a non-overlapping FEM-DEM coupling by Lagrange multipliers for cost-efficiency.

In the chosen method, the interaction forces between coupled sub-domains are determined based on the action/reaction principle and the constrained dynamic equilibrium reads:

$$Ma = f^{int} - f^{ext} + C^T\lambda \tag{2}$$

M: Weight tensor; a: acceleration; f^{int}:internal forces; f^{ext}:external forces; C:Coupling matrix; λ: Lagrange multiplier

2.3 Field Transfer

When switching from FE to DE it is necessary to transfer the fields FE to fields DE. Some evaluations performed on the type of quantity transferred (strain, stress and displacements), has shown that the transfer of displacement was sufficient.

The displacement field transfer is made using the FE polynomial interpolation. Indeed, the DE unknown displacements can be obtained based on the interpolation functions of the FE and the known nodal displacements. For example, for a Q4 FE containing a DE located in x:

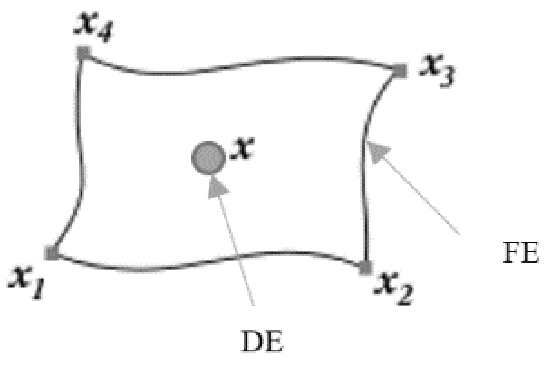

Figure 2: Transfer principle.

$$u(x) = N_1(x)u(x_1) + N_2(x)u(x_2) + N_3(x)u(x_3) + N_4(x)u(x_4) \tag{3}$$

$u(x)$: unkown displacement; N_i:shape function on nodes i;$u(x_i)$ value of $u(x)$ on nodes i.

3 Results

3.1 Initially coupled FEM-DEM computation

A first evaluation concerned the necessity to couple only translational degrees of freedom or both translational and rotational degrees of freedom at the FEM/DEM interfaces. The studied structure is a linear elastic bending beam, built using two finite element sub-domains initially coupled to a discrete sub-domain (figure 3), clamped at the left end and loaded at the right end.

To study the rotation, it is interesting to compare the section rotation in the coupling zone, with the reference results (full beam discretized with FEM only). To simplify the problem, in the case of small perturbations hypothesis, the relation between the deflection y and the section rotation θ is:

$$\theta = \frac{dy}{dx} \tag{4}$$

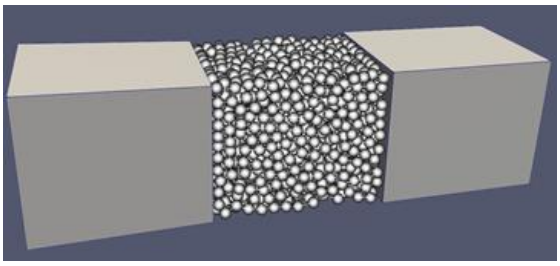

Figure 3: Coupled FEM/DEM model.

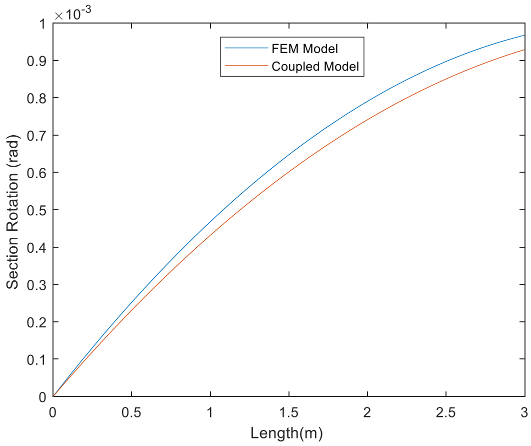

Figure 4: Section rotation comparison for pure FEM and for FEM/DEM coupled model.

The figure 4 shows that in case of small perturbation, the coupling of rotations is not absolutely necessary in a first step. The results are close between the reference and the coupled model. Indeed, the coupled model is 7% away from the reference at most. The evaluation of field transfer during the computation can thus be performed. More accurate coupling strategies will be developed later.

3.2 Initially not coupled, then field transfer switching from FEM to DEM during computation

After checking the influence of the rotation, the next step is to validate the transfer of the fields. The same bending problem as in the previous paragraph (figure 3) is studied, but this time with a field transfer model from a FEM sub-domain to a DEM one.

The beam is subjected to a linear ramp loading. The transfer is performed when the simulation reaches half of the loading as presented in figure 5.

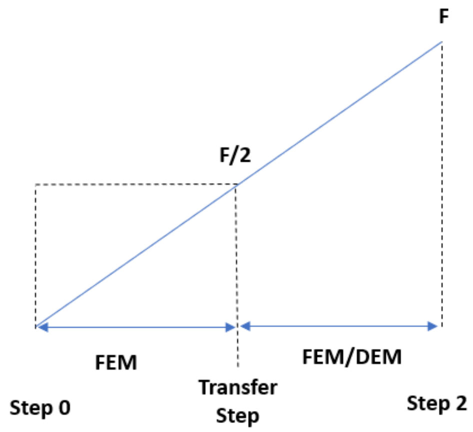

Figure 5: Loading ramp.

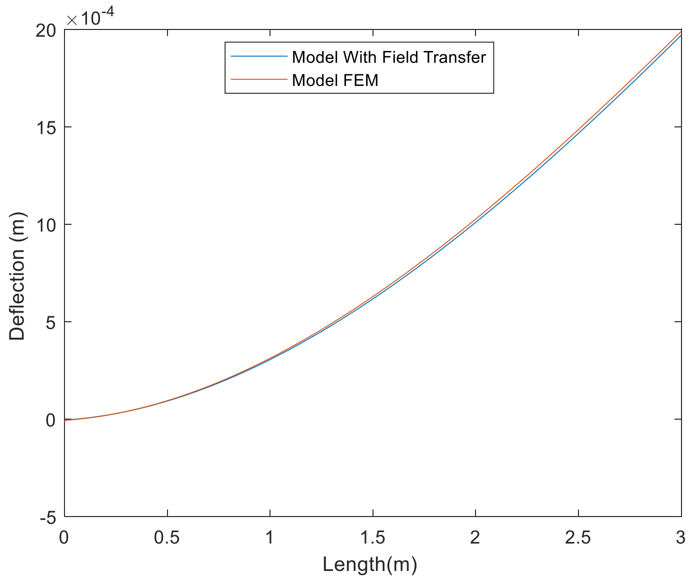

Figure 6: Deflections results for FEM and model with transfer.

For this test case, we are interested in comparing the deflection between the configuration which follows a transfer cycle and a reference solution: a FEM model without transfer during the simulation. The results are presented in figure 6.

The results are close between the reference and the model with transfer. The error is quite low of the order of 2%. It shows that although the method employed is pragmatic, the error is kept reasonable.

4 Conclusions and Contributions

This short paper proposes a strategy to substitute FEM sub-domains by DEM ones during the computation, in order to evaluate failure propagation, while still being as much as possible cost-effective. Pragmatic choices have been made initially, and the errors are considered reasonable, while more accurate developments could be performed for each step if required. These steps are:

- The non-overlapping coupling of FEM-DEM translational degrees of freedom using Lagrange multipliers,
- The transfer of translational displacements from FEM to DEM during the computation by means of FE interpolation functions
- Criterion-based automatization of the sub-domain substitution

The next step concerns the crack propagation evaluation in a bending V notched beam initially purely discretized with FEM (figure 7). As much as possible, the numerical results will be compared to experimental ones.

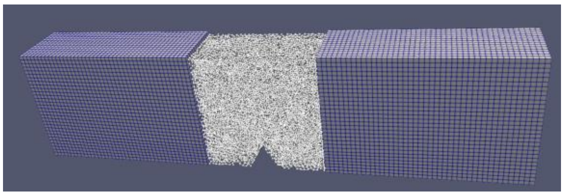

Figure 7: Three-point bending test case model.

Some of the prospects concern the application of the strategy to industrial test cases specifically, cases that are related with the cracking of the train wheel, and other future work will deal with the study of the mathematical treatments of spurious waves.

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