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Shear deformable beam model for stability analysis of beam-type structures with composite thin-walled cross sections

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Abstract

This paper presents a shear deformable numerical model for nonlinear stability analysis of beam-type structures. The incremental equilibrium equations for a straight thin-walled beam element are derived within the framework of updated Lagrangian formulation and the nonlinear displacement field of cross-sections, which accounts for the restrained warping and the large rotations effects. Shear deformation effects are accounted for the composite cross-section considering bending-bending and bending-warping torsion coupling effects. Cross-section properties are calculated based on the reference modulus, with ability to model different material configurations. A numerical algorithm for calculation of the geometric properties of composite cross-section is developed. Various material configurations are considered. Several benchmark examples are demonstrated for verification purposes. The obtained results indicate that the proposed model can be classified as shear locking free one.

Keywords: thin-walled, composite cross-section, beam model, buckling, large displacement, nonlinear stability analysis.

1 Introduction

As load-bearing composite structures generally contain slender beam structural elements of thin-walled cross-section, the response of such optimized structures to the effect of external loading is much more complex and their increased tendency to lose the stability of the deformation form and the appearance of buckling is particularly pronounced [1-3]. The occurrence of instability in beam structures can be manifested in the pure flexural, pure torsional, torsional-flexural or lateral deformation form. Therefore, in the optimal design of the structure, special attention should be paid to the exact determination of the limit state of stability of deformation forms, that is, the buckling strength. Analytical solutions are only available for simpler cases [4], and therefore the development and application of numerical solutions is imposed as a necessity. Geometric nonlinear analyses of composite beam structures with the influence of shear deformations are presented in [5-8]. In these works, the authors also include bending-bending and bending-warping torsion coupling shear deformation effects occurring for the asymmetric cross-section where the principal bending and principal shear axes do not coincide.

In the authors' last paper [8] the composite frames with semi-rigid connections were presented using the geometrically nonlinear beam element considering shear deformation effects. In that paper only unidirectional orthotropic composite structures were considered. The purpose of this work is large displacement nonlinear analysis of thin-walled beam type structures considering shear deformation effects and material inhomogeneity in the form of a composite cross-section. The analysis will be completely based on the numerical model developed by the authors, where the results will be compared to the results obtained by the other relevant sources.

2 Methods

To include the shear deformation effects in the formulation, Timoshenko's theory for non-uniform bending and modified Vlasov's theory for non-uniform or warping torsion are applied. Furthermore, in this work, an improved shear-deformable beam formulation is presented by taking into account the bending-bending and bending-warping torsion coupling shear deformation effects [5-9] occurring for the asymmetric cross-section where the principal bending and principal shear axes do not coincide [10]. The beam member is assumed to be prismatic and straight while external loads are supposed as conservative and static.

The element geometric stiffness is derived using the updated Lagrangian (UL) incremental formulation [11,12] and the non-linear displacement field of a cross-section, which includes the second-order displacement terms due to large rotation effects. In such a way, the incremental geometric potential of the semitangential moment is obtained for the internal bending and torsion moments respectively, thereby ensuring the moment equilibrium conditions to be preserved at the frame joint to which beam members of different space orientations are connected [13,14]. After adopting a cubic interpolation for the deflections and twist rotation, and an interdependent quadratic interpolation for the slopes and warping parameter that includes shear-deformable effects, a locking-free beam element is obtained. Such an element is also known as a super convergent element, and the reduced integration technique is not needed to avoid the shear-locking effect [15]. In terms of the incremental-iteration scheme, the generalised displacement control method [14] is utilized, while the nodal orientation updating at the end of iteration is brought out applying the transformation rule valid for semitangential rotations [16,17]. In the force recovery phase, conventional approach is adopted [17,18].

To account for material inhomogeneity in the form of the composite cross-section, separate numerical model is used for the calculation of cross-sectional properties. Cross-sectional properties are weighted by the reference modulus [10]. Since the model does not include the coupling between normal and shear, it is only applicable for the analysis of cross-ply laminates, functionally graded materials and similar materials in which there is not coupling between normal and shear.

3 Results

A computer program called THINWALL v.17 was developed on the basis of the finite element procedure including all the procedures mentioned in the previous section. The program has capabilities to deal with the linearised and nonlinear stability analyses. When performing the nonlinear stability analyses, the generalised displacement control method is used and a small perturbation load should be introduced to initiate the occurrence of buckling. In this way, the stability problems are investigated using the load-deformation approach, by which the structural behaviour throughout the entire range of loading interest, including the pre- and post-buckling phases, is evaluated by plotting the loading of the structure as a function of deformation. Such approach provides information more reliably for real structures and loading conditions then the eigenvalue approach.

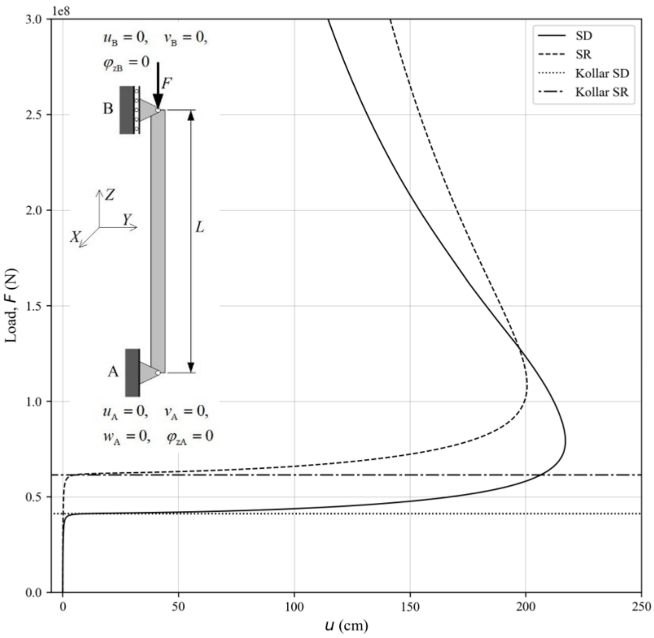

Figure 1: Load-deflection curves for simply supported I-profile column, $[0°/0°]_S$ stacking sequence.

To analyse the influence of the shear effect on the stability behaviour of the analysed structural members, two model comparisons are performed. First one ignores the shear deformability effects entirely and in the presented results is labelled by ‘SR’. Second model includes shear deformation effects on the basis of the procedure shown in this paper, and in the presented results is labelled by ‘SD’. The analysed material is graphite-epoxy (AS4/3501) whose properties are E_1 = 144 GPa, E_2 = 9.65 GPa, G_{12} = 4.14 GPa, v_{12} = 0.3.

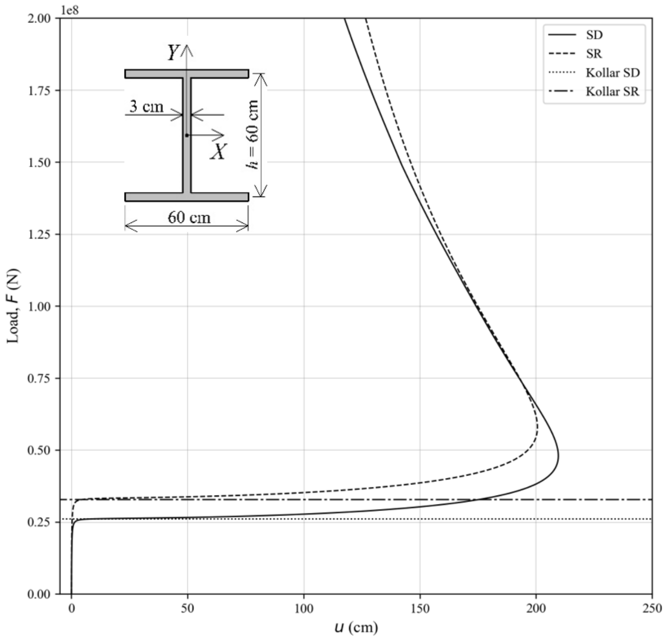

Figure 2: Load-deflection curves for simply supported I-profile column, $[0°/90°]_S$ stacking sequence.

Figure 1 shows simply supported column of length L = 500 cm, under an axial force F. Analysed cross-section shape can be observed in Figure 2, with the properties given in Table 1.

	A^* (cm^2)	I_x^* ($\times 10^3$ cm^4)	I_y^* ($\times 10^3$ cm^4)	I_t^* (cm^4)	I_ω^* ($\times 10^6$ cm^4)
$[0°/0°]_S$	540	378.270	108.135	1620	97.322
$[0°/90°]_S$	540	378.447	108.224	1620	97.401
	$\bar{Q}_{11R}$ (GPa)	$\bar{Q}_{66R}$ (GPa)	K_x	K_y	K_ω
$[0°/0°]_S$	144	4.14	1.7955	3.3748	0.006
$[0°/90°]_S$	76.825	4.14	1.7926	3.3716	0.006

Table 1: Cross-section properties.

For the symmetric I-profile the column buckling occurs in a pure flexural mode. Table 2 show the convergence study. Five different mesh sizes are used, and the obtained results are compared with the analytical solution reported in [19]. In Figure 1 and in Figure 2, nonlinear response of I-profile column can be observed. Small perturbation force is applied, acting in the *X*-direction. In Figure 1 and in Figure 2 load-deflection curves are given for the $[0°/0°]_S$ and $[0°/90°]_S$ stacking sequence, respectively. In both figures, deflection in the *X*-direction of the centroid at the mid-span of the column is shown.

Number of elements:		2	4	8	16	32	Analytical solution [19]
$[0°/0°]_S$	SR	61.94	61.51	61.48	61.47	61.47	61.47
	SD	43.11	41.63	41.27	41.18	41.16	41.12
$[0°/90°]_S$	SR	33.07	32.84	32.82	32.82	32.82	32.98
	SD	26.95	26.21	26.04	26.00	25.99	26.06

Table 2: Buckling load convergence for I-profile, load values are in MN.

4 Conclusions and Contributions

A refined shear-deformable beam formulation for geometrically nonlinear stability analysis of composite semi-rigid frames has been introduced. The shear deformation effects have been taken into account due to the non-uniform bending and torsion of thin-walled beams with the asymmetric cross-section. Benchmark example has been presented to verify the model. The importance of introducing the shear deformations into the formulation is apparent from the significant reduction of the stability strength. The model is able to control the warping restraints at nodes either globally by preventing the warping for all beam elements connected at the common joint, or locally, by using the warping transformation parameters representing the local warping restraint conditions at the node of the particular beam element. The shear-locking testing has also been performed running the model for different mesh configurations, where it can be observed that shear locking doesn't occur. Different material configurations have been analysed and their influence on the critical load has been presented and verified by selected example.

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