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Beam model for thermal buckling of thin-walled functionally graded box-beam

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Abstract

The paper presents the beam finite element model for thermal buckling analysis of thin-walled functionally graded (FG) box-beams. The model is based on Euler-Bernoulli-Navier bending theory and Vlasov torsion theory. The cross-sectional displacement field includes the effects of warping torsion and large rotations. Material properties are assumed to be graded across the wall thickness and considered as a function of temperature. Three cases of the temperature distribution across the thickness of the cross-section walls are considered, which are uniform, linear and nonlinear. The critical buckling temperature and post-buckling behavior of FG box-beams under thermal loading with different values of power law index *p* for different types of boundary conditions, clamped-clamped (CC), clamped- simply supported (CS), and simply supported (SS), are given to investigate the effects of the power-law index on structural behavior. Nonlinear stability analysis is performed to obtain the thermal load versus displacement curves. The accuracy and reliability of the beam model are verified by comparing it with previous research results and several benchmark examples.

Keywords: thermal buckling, post-buckling, temperature distribution, thin-walled box-beam, functionally graded material, numerical analysis.

1 Introduction

Functionally graded thin-walled beams and structures are used in many engineering applications due to their very good strength-to-weight ratio, improved thermal resistance and high fracture toughness. On the other hand, due to their slenderness, these structures exhibit very complex structural behaviour and susceptibility to buckling failure. Many papers have addressed the behaviour of FGM box-beams subjected to a variety of loading and placed in a high temperature environment, but only some of them are cited here [1-12].

When the temperature changes, the longitudinal fibres tend to extend in the axial direction. If the extension is prevented, axial stresses occur in the longitudinal fibres, which can lead to the buckling. For this reason, the loss of stability is defined by the critical buckling temperature.

Beam models are efficient and computationally cheap in the application of analysis of slender components. This is the reason why beam models have been extensively used and developed in the last decades. The classical beam models are those of Euler-Bernoulli and Timoshenko, but many refined models have also been developed.

In this paper, a beam model for thermal buckling of FG thin-walled sandwich box-beams is discussed. The model is based on Euler-Bernoulli-Navier bending theory and Vlasov torsion theory, assuming large displacement and small strains.. The equilibrium equations of the finite elements are developed by an updated Lagrangian formulation. Material properties are assumed to be graded across the wall thickness. Three cases of the temperature rise over wall thickness are considered, which are uniform, linear and nonlinear. Numerical results are obtained for FG box beams with different boundary conditions and temperature distributions to investigate the effects of the power-law index on the critical buckling temperature and post-buckling response.

The main objective of the paper is the thermal buckling analysis of thin-walled beam structures considering FG temperature-independent materials. The analysis is based on the numerical model developed by the authors [13,14] and verified by benchmark examples and research results.

2 Methods

The box beam under consideration of length l, width b, height h and wall thickness t is shown in Fig. 1.

Assume that the beam is made of a functionally graded material. The material properties are assumed to vary continuously through the wall thickness from a metal-rich inner surface to a ceramic-rich outer surface according to power-law as defined in [15]:

$$P(n,T)=[P_o(T)-P_i(T)]\cdot V_c(n)+P_i(T) \tag{1}$$

where P represents the effective material property such as Young's modulus E, shear modulus G, coefficient of thermal expansion α and thermal conductivity κ, respectively. The subscripts i and o represent the inner and outer surface constituents while V_c is the volume fraction of the ceramic phase. In this work, the material properties are assumed temperature-independent.

$$V_c = (0{,}5+n/t)^p \tag{2}$$

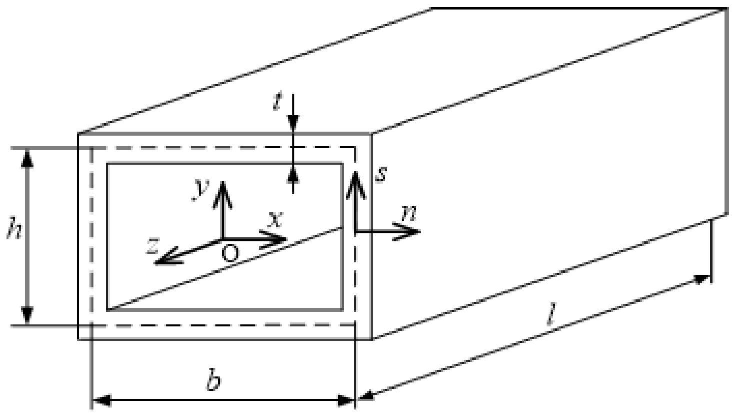

Figure 1. Cross-section of a thin-walled FG box beam

The stress-strain relation in terms of generalized Hooke's law can be written as follows:

$$\begin{gathered}\sigma_z = E\,(n,T)\cdot[\varepsilon_z - \alpha\,(n,T)\cdot\Delta T],\\ \tau_{zs} = G\,(n,T)\cdot\gamma_{zs}\end{gathered} \tag{3}$$

where σ_z and τ_{zs} are stress components, ε_z and γ_{zs} are strain components; n, s denote the flange normal and transverse directions, while z is parallel to the beam axis; ΔT is a temperature change, T_0 is the reference temperature.

The beam is subjected to a uniform, linear and nonlinear temperature distribution over the beam wall thickness. In the case of uniform temperature rise, if the axial beam displacements are prevented and the reference temperature of the beam is T_0, the temperature can be raised to $T = T_0 + \Delta T$ so that the beam buckles. With linear and nonlinear temperature rise across the thickness, the temperature at the inner and outer surfaces can rise differently.

3 Results

A thin-walled FG sandwich box beam with the length l = 8 m, height h = 0.2 m, width b = 0.1 m and wall thickness t = 0.005 m is carried out. The FG material properties are assumed as follows: Alumina (E_c = 380 GPa, $\alpha_c = 7.2\cdot10^{-6}$ 1/C^0) and Aluminium (E_m = 70 GPa, $\alpha_m = 2.3\cdot10^{-5}$ 1/C^0). The Poisson's ratio is assumed to be constant ν = 0.3. The beam is subjected to uniform, linear and nonlinear distributed temperature change.

The thermal buckling of the beam was analysed for three different boundary conditions (clamped-clamped, simply supported and combination) and different values of the power-law index p. The results of the author's beam model were verified with a numerical model based on solid and shell finite elements. FG material was simulated by homogeneous layers. The solid model was created using the FEMAP programme and the numerical analyses were performed using the MSC solver. The shell model was created with Ansys. Table 1 shows a comparison of the critical temperatures obtained with the beam, shell and solid model for the double clamped beam and the different power-law values p subjected to uniform temperature rise.

p	**0**	**0,2**	**0,5**	**1**	**5**	**10**
Beam model	166,812	126,72	106,008	92,18	69,273	62,884
Shell model	164,88	124,86	104,21	90,58	69,658	64,697
Solid model	165,043	126,856	106,509	93,309	72,351	66,885

Table 1 Critical temperatures for clamped-clamped box beam; uniform temperature distribution

Figure 2 shows a graphical comparison of the critical temperatures for the simply supported beam and vaious power-law indices. The temperature increases at the inner surfaces and is distributed linearly over the wall thickness. The comparison of the results shows good agreement and lower values for the critical temperatures in the beam model, which could be due to different simulations of the FG material.

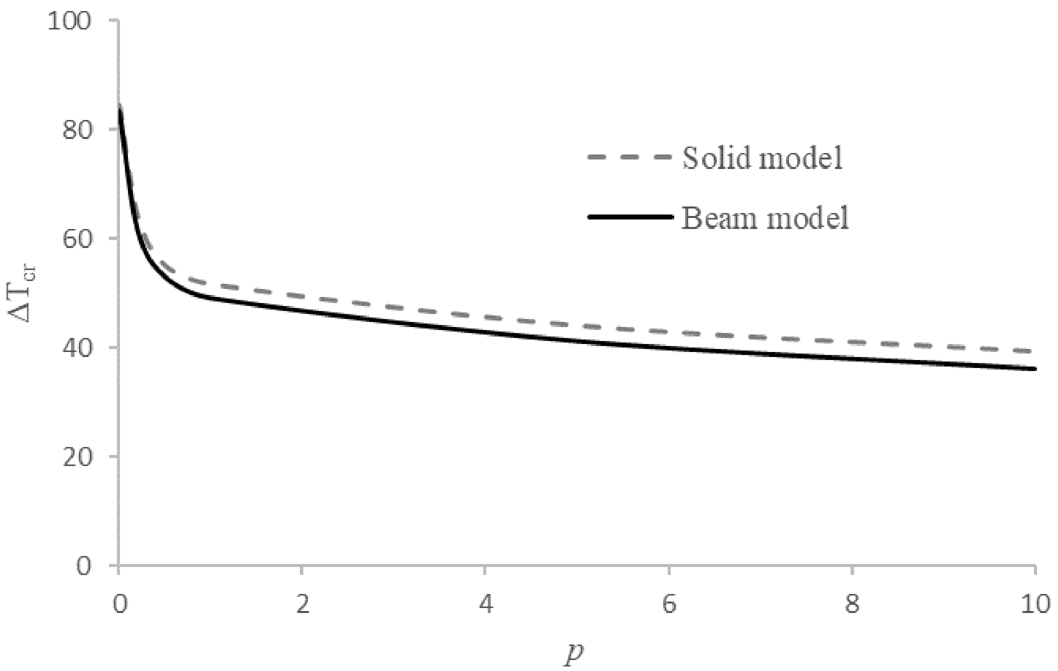

Figure 2 Critical temperatures for simply supported box beam; linear temperature distribution

To initiate buckling, a lateral perturbation force $\Delta F = 0.001F$ is applied incrementally in the x-direction at the mid-span for C-C and S-S and at $0{,}7l$, where the largest displacement are expected, for C-S.

4 Conclusions and Contributions

To evaluate the effects of the power-law index on the critical buckling temperature and post-buckling behaviour of FGM box beams, an improved beam finite element code, developed over years is proposed.

Three cases of boundary conditions are considered: clamped-clamped, clamped-simply supported and simply supported at both ends. The analysis of thermal buckling under three types of thermal loading is presented.

The critical buckling temperature decreases with increasing the power-law index p and is lower with uniform temperature rise. This relationship is recognised for all boundaries considered. As expected, the beam clamped at both ends has the highest thermal buckling resistance and the beam with simply supported ends has the lowest.

The authors' further research activities in this area focus on the extension of a numerical model to simulate the thermal buckling of FG beam-type structures and frames subjected to different temperature distributions across the wall thickness and along the beam axis taking into account temperature-dependent material properties.

Acknowledgements

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