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A 3D fiber beam element based on a plastic damage model for prestressed concrete bridges

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Abstract

Damage modeling in prestressed concrete beams is a relevant issue in the assessment of existing structures, especially bridges. This work focuses on the formulation of a high-performance fiber beam element to model the nonlinear behavior of prestressed reinforced concrete beams and the progressive damaging phenomena. The proposed beam formulation relies on the force-based approach.

A fiber discretization of the cross section is used to model concrete, steel reinforcements and prestressing cables through three-dimensional constitutive laws based on damage and plasticity for concrete, and plasticity for steel. The presence in the concrete constitutive law of strain-softening requires appropriate regularization procedures.

One or more tendons are added to the fiber section as fibers where the applied prestressing is equivalent to an initial fiber strain. No additional discretization is required. This model is implemented in the OpenSees framework to carry out specific tests and demonstrate its potentialities.

Keywords: beam finite element, fiber section, damage, plasticity, regularization, prestressed concrete, cyclic analysis, nonlinear analysis.

1 Introduction

To carry out accurate analyses, two-dimensional and three-dimensional models can be used, which typically require a high computational burden, but to perform many analyses it is preferable to use more efficient finite elements [1,2].

This work focuses on the formulation of a high-performance fiber beam element to model the nonlinear behavior of prestressed reinforced concrete beams and the progressive damaging phenomena. The proposed beam formulation relies on the force-based approach which, as opposed to the classical displacement approach, allows the equilibrium to be satisfied in a strong form [3]

As known, both in the linear and nonlinear field, the advantages are significant compared to the displacement approach, resulting in a more accurate and efficient formulation. Indeed, in the latter case, sufficiently dense mesh must be used to obtain accurate results, leading to a significant increase in computational burden [4].

A fiber discretization of the cross section is used to model concrete, steel reinforcements and prestressing cables through three-dimensional constitutive laws based on damage and plasticity for concrete, and plasticity for steel [5–7]. The presence in the concrete constitutive law of strain-softening requires appropriate regularization procedures, as discussed in detail in [6].

Several approaches were proposed to introduce the effect of prestressing. As for example, in [8] the tendon is modeled by enriching the element analysis. A mixed approach is used to evaluate the section displacements, by which it is possible to introduce a bond relationship between the prestressed element and the rest of the beam.

Here, prestressing is introduced at the level of a single steel fiber. This approach follows previous work [9] and allows to operate directly on the strain components at the material fiber level rather than at the section or element level.

In this paper, one or more tendons are added to the fiber section as fibers where the applied prestressing is equivalent to an initial fiber strain. No additional discretization is required.

This model is implemented in the OpenSees framework to carry out specific tests and demonstrate its potentialities [10,11].

2 Methods

2.1 3D Beam Finite-Element Formulation

The prestressed fiber beam element is based on a classical 3D force-based approach. The element nodal forces vector is:

$$\mathbf{q} = [p_{xj} \quad m_{zi} \quad m_{zj} \quad m_{xi} \quad m_{yi} \quad m_{yj}]^T \quad (2.1)$$

where p_{xj} is the axial force, while $m_{zi} \quad m_{zj} \quad m_{xi} \quad m_{yi} \quad m_{yj}$ are the moments at nodes i and j around x , y and z axes.

The element basic displacement vector \mathbf{v} is:

$$\mathbf{v} = [u_x \quad \varphi_{zi} \quad \varphi_{zj} \quad \varphi_{xj} \quad \varphi_{yi} \quad \varphi_{yj}]^T \quad (2.2)$$

where u_{xj} is the displacement of node j while $\varphi_{zi} \varphi_{zj} \varphi_{xj} \varphi_{yi} \varphi_{yj}$ are the rotations at nodes i and j around x, y and z axes.

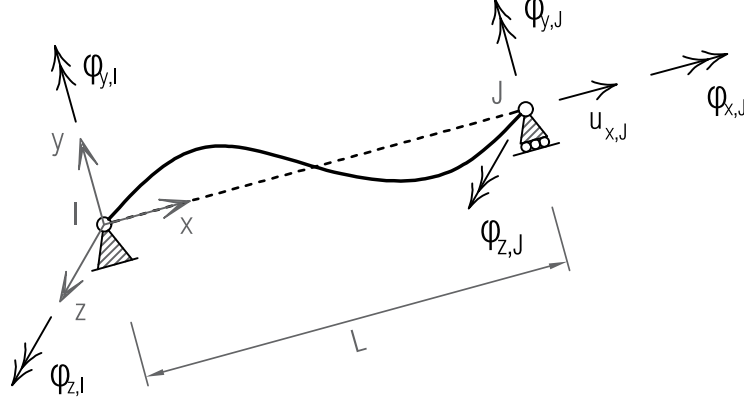


Figure 1 – 3D beam element basic configuration

At the section level, the balance equation is:

$$\mathbf{s}(x) = \mathbf{b}(x)\mathbf{q} \quad (2.3)$$

where $\mathbf{s}(x)$ contains the generalized stress components. The generalized section strains are as follows:

$$\mathbf{e}(x) = [\varepsilon_G(x) \quad \chi_y(x) \quad \chi_z(x) \quad \gamma_{xy}(x) \quad \gamma_{xz}(x) \quad \chi_x(x)]^T \quad (2.4)$$

where $\varepsilon_G(x)$ is the axial strain, $\chi_y(x)\chi_z(x)\chi_x$ are the curvatures around y, z and x axes and $\gamma_{xy}(x) \gamma_{xz}(x)$ the shear strains in the xy and xz directions. The section constitutive relationship is:

$$\mathbf{e}(x) = \mathbf{f}(x) \mathbf{s}(x) \quad (2.5)$$

where $\mathbf{f}(x)$ is the section flexibility matrix.

2.2 Section model with the prestressed cables

Each fiber strain vector $\boldsymbol{\varepsilon}(x, y, z)$ is determined from $\mathbf{e}(x)$ by means of the compatibility matrix $\bar{\boldsymbol{\alpha}}(y, z)$:

$$\boldsymbol{\varepsilon}(x, y, z) = \bar{\boldsymbol{\alpha}}(y, z) \mathbf{e}(x) + \boldsymbol{\varepsilon}_0 \quad (2.6)$$

where $\boldsymbol{\varepsilon}_0$ is a vector collecting the initial fiber strains, which accounts for prestress. The compatibility matrix is defined so that to consider the effects of the cable inclination and produces a parabolic distribution for the fiber shear strains. A classical J2 plastic model is used for cables and steel reinforcements.

2.3 3D damage-plastic constitutive law for concrete

The stresses of the concrete fibers are determined by means of a 3D constitutive law which combines a J2 Von-Mises plasticity and an isotropic damage model, as proposed in [7,12,13].

$$\boldsymbol{\sigma} = (1 - D)^2 \mathbf{C}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^P) \quad (2.7)$$

Where D is the damage variable, \mathbf{C} the constitutive matrix, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}^P$ the total and plastic strains.

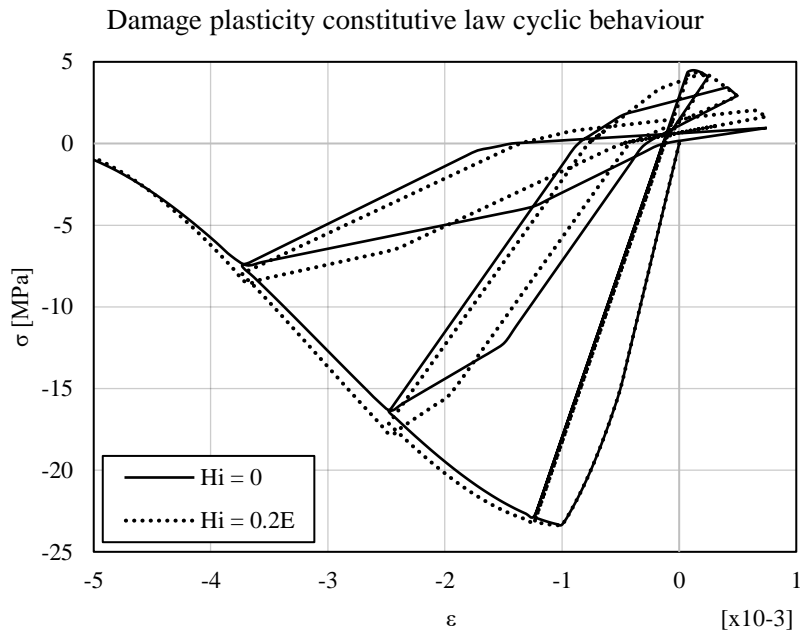


Figure 2 – Cyclic uniaxial patch test using the damage-plastic constitutive law. E is the Young's modulus, H_i the isotropic hardening.

2.4 Computational aspects

A consistent force-based solution algorithm is adopted [3]. Due to the presence of strain-softening in the concrete constitutive response, a regularization procedure like those proposed in [6,14] is adopted to avoid mesh dependency of the numerical results.

To link the 3D strain and stress fields evaluated at the fiber level to the generalized beam measures, a static condensation process is implemented as described in [4].

3 Results

The models presented were implemented in OpenSees [10] combined with program STKO [11] that contains an input and output graphical user interface.

Several benchmark tests were carried out to validate the model. First, the 3D concrete plasticity and damage model was used for reinforced concrete beams followed by tests on prestressed beams. For the sake of brevity, only the results of one reinforced concrete column and two prestressed concrete beams are reported here.

3.1 Cyclic load test on a reinforced concrete column

The response of an axially loaded column fixed at the base with height $L = 1.6$ m is studied, corresponding to a simple shelf with an applied constant axial load and an imposed symmetrical cyclic lateral displacement at the top.

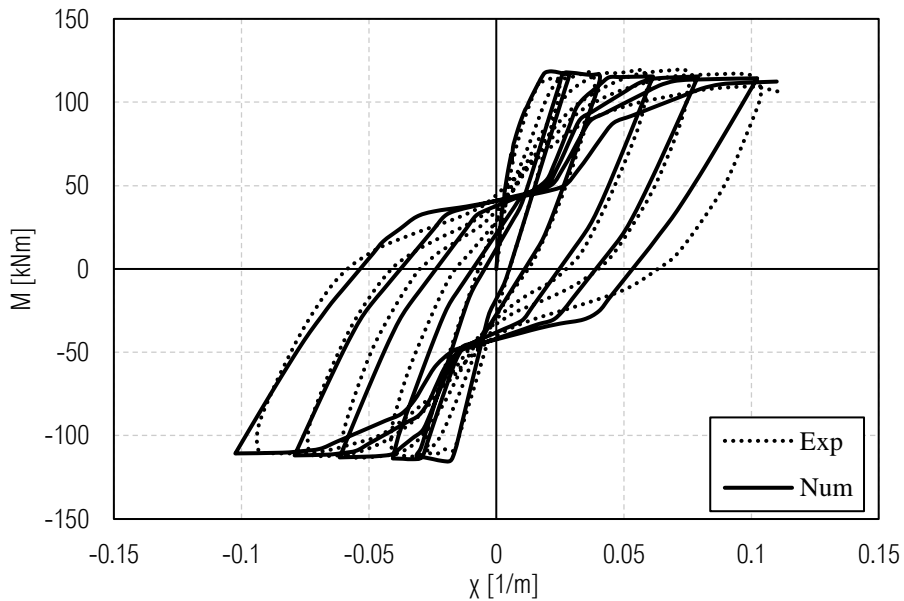


Figure 3 – Comparison of experimental and numeric results on a cyclic test

In Fig. 3 the numerical results are compared with the experimental outcomes for specimen C1 [15] obtaining a good agreement. The results of a similar test are shown in [12].

The section mesh is more refined on the section edges where the tension gradients are higher.

The regularization of the element is carried out on the Gauss-Lobatto integration scheme that, following [6] imposes a region of defined length by the user.

3.2 Pushdown analysis performed on prestressed beams

The prestressed concrete beams considered for this case were experimentally tested and described in [16,17]. Here a comparison is presented with the analytical results obtained in [18]. The two beams are similar in geometry and mechanical properties, only the prestressing and the load conditions vary.

Good agreement is observed in Fig. 4 between the global numerical and analytical responses. Additional details on the calibration of the geometric parameters and constitutive properties can be found in the related references.

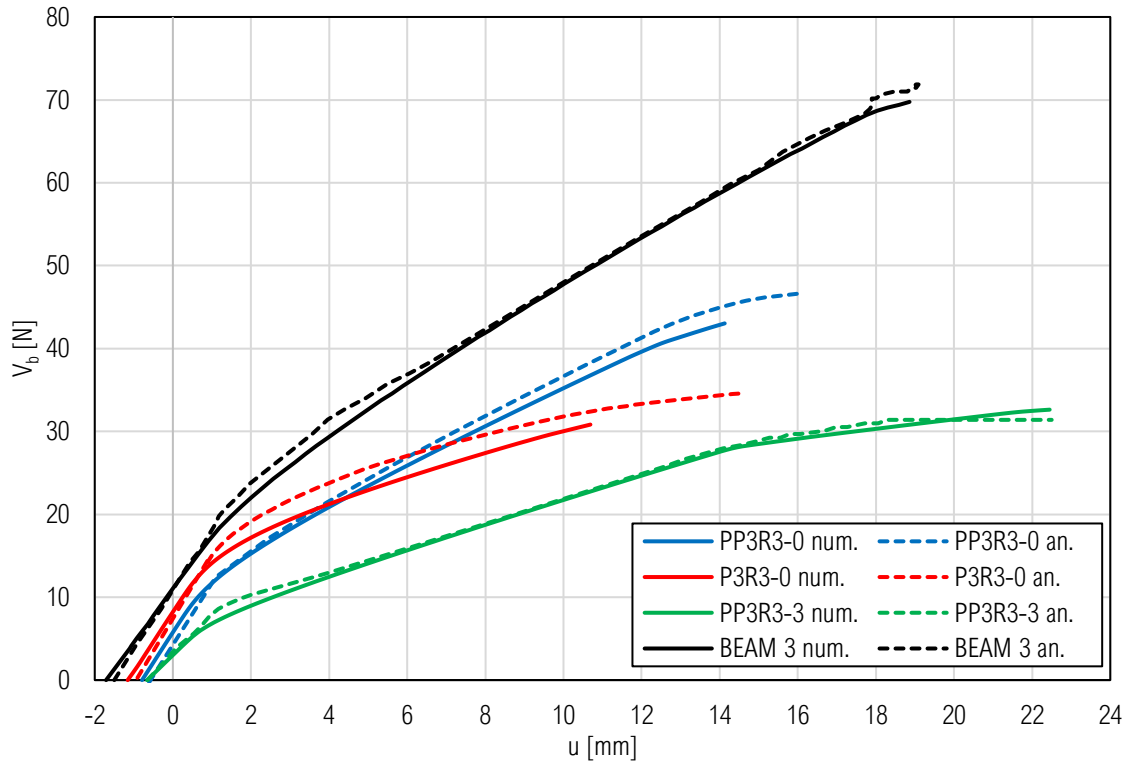


Figure 4 – Comparison of analytical and numerical results of pushdown tests on prestressed beams [18]

4 Conclusions and Contributions

This work presents a novel 3D fiber beam finite element, where the well-known force-based approach originally formulated for reinforced concrete beams is extended here to include the effects of prestressing. This element is of interest for the analysis of our aging infrastructure that comprises a large number of older prestressed concrete bridges. The main features of the new element are:

- The hardening phenomena of the steel components are considered through 3D J2 plasticity laws for both regular and prestressing reinforcements. The effective steel fiber strains are evaluated, deriving from the section deformation and the initial fiber strains applied to them.
- The concrete progressive cracking process takes place following the implemented damage-plastic model that accounts for the partial closure of the cracks which occurs in compression states following tensile states by introducing two different damage variables that evolve independently.
- The section has a flexure-shear interaction thanks to the 3D material model which returns the whole stress vector. The shear stress is evaluated in a consistent form since the shear strains along the section are interpolated to satisfy the beam boundary conditions. Also, the confinement effects can be

easily controlled in the axial and flexural behaviour, while it's partially controlled in the shear behaviour through the material parameters.

- Prestressing is introduced at the section level through initial fiber strains added to the cable fibers, which represent the elongation deriving from the external stresses.
- Every cable fiber has its own inclination at each integration point. Given the cable coordinates along the beam, two angles are derived and used to rotate the stress vector if the tension direction is not orthogonal to the section.

Since the entire strain field is computed, it is possible to determine with sufficient accuracy (in each point of the volume occupied by the structure) the actual stress-strain state and the phenomena considered.

With few elements, sections and fibers, the proposed element can be used along modern processing algorithms requiring fast-performing finite elements to study a large number of structures.

The proposed model can have multiple applications, mainly in the domain of health and safety checks of structures and infrastructures such as bridge girders and precast concrete frames.

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