A Recursive Model Updating Algorithm for Multi-element Hybrid Simulation of Structures

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Abstract

Hybrid simulation is a cost-effective method of testing structures under seismic loading that combines numerical and experimental methods through partitioning the structure into; 1) numerical substructure simulating the well-understood components of the structure, and 2) physical substructure representing the critical components of the structure. The hybrid simulation results can become biased and uncertain when only one or a limited number of potential critical components, e.g., seismic fuses, are physically tested due to laboratory or cost constraints. Furthermore, the critical components modelled in the numerical substructure are often calibrated using experimental test results of similar prototype specimens under a predefined loading protocol, which fails to consider the effects of dynamic loading characteristics to which it will be subjected in hybrid simulation. This paper proposes a new recursive model updating algorithm incorporated into the conventional seismic hybrid simulation framework to leverage the data collected in real-time from the physical specimen of one of the critical elements and integrate a new data-driven model into the numerical substructure. The data-driven model, which is being progressively updated owing to the proposed model updating algorithm, is responsible for predicting the nonlinear cyclic response of the other critical components of the system that are not physically tested. To develop the data-driven model, the parameters of the Prandtl-Ishlinskii model are first estimated using a sparse regression algorithm and then updated during the hybrid simulation using the recursive least-squares algorithm. The simulation accuracy of the model updating algorithm is assessed through nonlinear response history analysis of a two-storey steel buckling-restrained braced frame, which consists of a virtual experimental specimen (first-storey brace) and the model

updating algorithm integrated into the numerical model of the structure to predict the second-storey brace force. The results suggest that the application of the model updating algorithm in conventional seismic hybrid simulation yields a more accurate and create an unbiased seismic simulation tool that can be used to examine the seismic response of multi-storey structural systems.

Keywords: hybrid simulation, model updating, data-driven simulation, machine learning, dynamics of structures, substructuring, system identification.

1   Introduction

Hybrid simulation (HS) is a mixed numerical simulation - physical testing method of structural response assessment, which was first introduced by Koichi Takanashi et al. [1] in the early 1970s. The underlying idea of HS is to divide the structure into two computationally parallel substructures. The well-understood parts of the structure are simulated numerically using a finite element analysis program while the critical components expected to respond in the inelastic range or experience instability, e.g., seismic fuses in a seismic force-resisting system, are tested physically in the laboratory.

The results obtained from seismic HS may become biased when only one or a limited number of potential critical components are physically tested due to laboratory or financial constraints. For example, Imanpour et al. [2] physically tested only one of the two critical columns of a two-tiered steel concentrically braced frame using the pseudo-dynamic HS technique, while both columns would have buckled if they had been tested experimentally in the laboratory, which may have resulted in a distinctly different structural response. In addition, the critical components modelled in the numerical substructure of the conventional HS technique are often calibrated using experimental test results of similar prototype specimens under a predefined loading protocol, which fails to consider the effects of dynamic loading characteristics during hybrid simulation.

Recently, the model updating concept was successfully implemented into HS to leverage the wealth of real-time data collected from the physical specimen during HS towards improving the accuracy of the numerical substructure. The majority of these studies have employed Kalman filter-based system identification techniques to update the parameters of a phenomenological hysteresis model in real-time [3]–[7].

A new model updating algorithm is proposed in this study for the seismic HS of structural systems having multiple critical elements, which benefits from an adaptive data-driven model that predicts the hysteretic response of similar – but not identical – critical elements. The developed algorithm functions in two steps: 1) the least absolute shrinkage and selection operator (LASSO) algorithm is utilized to obtain a computationally-efficient and reduced-order Prandtl-Ishlinskii (PI) model in the initial (passive) training phase. This phase is triggered before HS to estimate the response of the data-driven model of the critical component based on available experimental test data; 2) the new incoming (experimental test) data as obtained from

the physical specimen is fed into the recursive least-squares (RLS) algorithm during HS to progressively improve the prediction of hysteretic response, particularly the hardening behaviour affected by the real-time dynamic loading protocol, as HS progresses. This phase is referred to as the recursive model updating (RMU).

2   Methods

The data-driven model used for simulating the hysteretic response of the critical components modelled in the numerical substructure is developed based on the Prandtl-Ishlinskii (PI) hysteresis model [8]. In principle, the PI model constructs the hysteresis memory of the model updating algorithm by expanding the input deformation signal, $x(t)$, into a higher dimensional space to convert hysteretic nonlinearity into a unique one-to-one mapping problem by means of stop operators, $E_r[.]$. Stop operator produces an elastic-perfectly plastic hysteretic response and can be expressed mathematically as:

$$y_r(0) = e_r\big(x(0)\big) \tag{1}$$

$$y_r(t) = e_r\big(x(t) - x(t_i) + y_r(t_i)\big) \; for \; t_i < t \le t_{i+1}; \; 0 \le i \le N-1 \tag{2}$$

$$e_r(s) = \min\big(r, \max(-r, s)\big) \tag{3}$$

in which $y_r(t) = E_r[x(t)]$ is the output signal of a single stop operator, which is defined using a threshold r $(r > 0)$. The relationship between the input deformation signal, $x(t)$, and the restoring force signal, $y(t)$, can be expressed by the linear superposition of multiple stop operators as:

$$\boldsymbol{y}_{n\times 1} = \boldsymbol{\Theta}_{n\times m} \cdot \boldsymbol{\Xi}_{m\times 1} = \begin{bmatrix} E_{r_1}[x(t_1)] & \cdots & E_{r_m}[x(t_1)] \\ \vdots & \ddots & \vdots \\ E_{r_1}[x(t_n)] & \cdots & E_{r_m}[x(t_n)] \end{bmatrix}_{n\times m} \cdot \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_m \end{bmatrix}_{m\times 1} \tag{4}$$

in which n and m are the number of training data points and stop operators, respectively. The thresholds can be assumed to be $r_i = i/(m+1)|x|_{max}$, $i = 1, 2, ...,m$, where $|x|_{max}$ is the maximum input signal amplitude. $\boldsymbol{\Theta}$ is called the library matrix which stacks the stop operators in multiple columns of the matrix, and $\boldsymbol{\Xi} = [\xi_1, \xi_2, ..., \xi_m]^{\mathrm{T}}$ contains the weights associated with each stop operator that is obtained by minimizing the L_1-regularized mean squared error (MSE) using the LASSO [9] regression algorithm as given:

$$minimize \frac{1}{2n} \|\boldsymbol{y} - \boldsymbol{y}'\|_2^2 + \lambda\|\boldsymbol{\Xi}\|_1 \tag{5}$$

where λ denotes the regularization parameter that controls the sparsity of the solution, which is obtained using the Akaike Information Criterion (AIC) [10], and $\boldsymbol{y'}$ is the restoring force vector obtained from the experimental data.

3

The recursive model updating (RMU) phase utilizes incoming experimental test output comprised of specimen's strain data, x_k, and stress data, y_k, in the kth step of the analysis to update the weight matrix, Ξ, recursively:

$$\Xi_k = \Xi_{k-1} + K_k[y_k - \Theta(x_k)\Xi_{k-1}] \tag{6}$$

in which the estimated parameters of the previous step, Ξ_{k-1}, are updated by a corrective term based on the difference between the most recent stress measurement, y_k, and the anticipated value of the stress obtained from the previous step, $\Theta(x_k)\Xi_{k-1}$. The corrective term is weighted by a gain matrix, K_k, that is obtained using:

$$P_k = \left(I - K_k\Theta(x_k)\right)P_{k-1}\left(I - K_k\Theta(x_k)\right)^T + K_k R_k K_k^T \tag{7}$$

$$K_k = P_{k-1}\Theta(x_k)^T(\Theta(x_k)P_{k-1}\Theta(x_k)^T + R_k)^{-1} \tag{8}$$

where P_k is the estimation-error's covariance matrix computed using Eq. (7), and R_k is the covariance of the measurement noise, which is taken as a white noise with variance of 1.

3   Results

The effectiveness of the recursive model updating algorithm proposed was verified through a virtual hybrid simulation (VHS) in which three numerical analysis programs were coupled together to evaluate the seismic response of a two-storey buckling-restrained braced frame (BRBF) as shown in Figure 1a. The prototype frame consisted of a Ductile (Type D) BRBF located in Vancouver, British Columbia, Canada, on site Class C. Gravity and seismic loadings were calculated in accordance with the 2015 National Building Code (NBC) of Canada [11] and the structural design of the BRBF was performed based on Canadian steel design standard CSA S16 [12]. Refer to [13], [14] for details of the BRBF design.

In the VHS with RMU (Figure 1b) well-understood elements of the structure including beams and columns were simulated in OpenSees [15], while, the first storey buckling-restrained brace (BRB), as one of the critical components, was modelled in ABAQUS [16], and the model updating algorithm was implemented in MATLAB [17] to predict the restoring force of the second-storey BRB. The reference VHS (Figure 1a) included BRBs modelled in ABAQUS and the rest of the BRBF in OpenSees. The communication between these software packages to send and receive deformation and force commands was established using the UT-SIM platform [18], [19]. All beams and columns were modelled using elastic beam-column elements [20] in OpenSees. The detailed model of the BRB in ABAQUS was constructed using the Voce-Chaboche multiaxial plasticity material model [21], [22] with combined isotropic-kinematic hardening parameters.
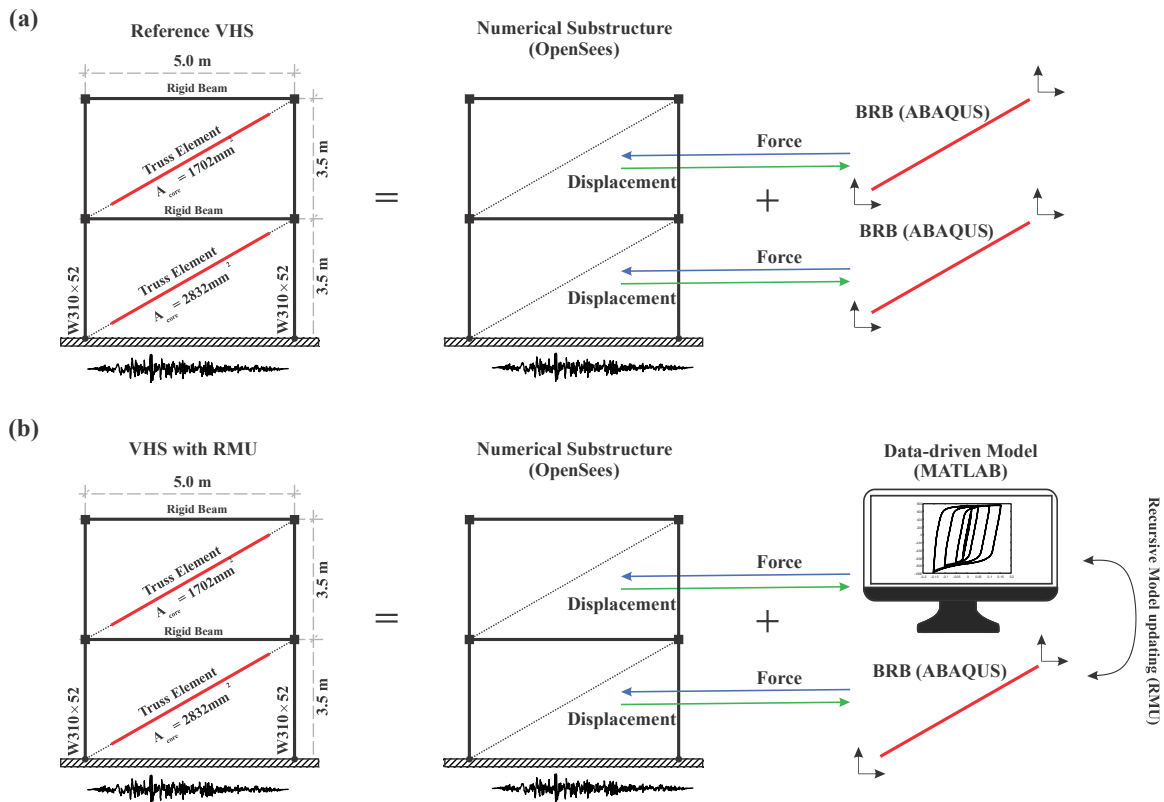
Figure 1. Virtual hybrid simulation of buckling-restrained braced frames: (a) reference VHS model, (b) VHS model equipped with recursive model updating (RMU) algorithm

The LASSO regression was performed to find the weight matrix of the PI model, Ξ_{LASSO}, using fictitious test data which was created using the pushover analysis conducted on an isolated BRB under an increasing cyclic displacement protocol shown in Figure 2a.

The result of the initial training is shown in Figure 2b, which suggests that the LASSO regression successfully captured the nonlinear cyclic behaviour of the BRB with normalized root-mean-square-error (NRMSE) of 0.40%. The RLS algorithm was then activated to perform VHS of BRBF under the 1995 Kobe, Japan–Tadoka station earthquake. The history of BRBF storey drift ratios and the hysteretic response of the BRBs at both storeys are shown in *Figure 3* as compared to their counterparts from reference VHS. The NRMSE of Storey 2 BRB hysteretic response was 2.12%, which confirms the accuracy of the RLS algorithm in predicting the response of the data-driven BRB in Storey 2 using the real-time data obtained from the numerical BRB in Storey 1.
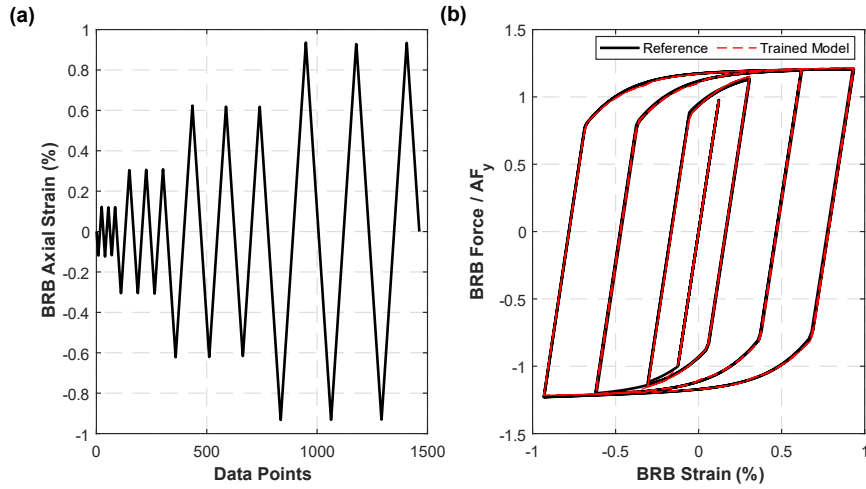
Figure 2. Initial Training of the data-driven model, (a) Input displacement protocol, (b) Training data generated by a pushover analysis of an isolated BRB vs. predicted data
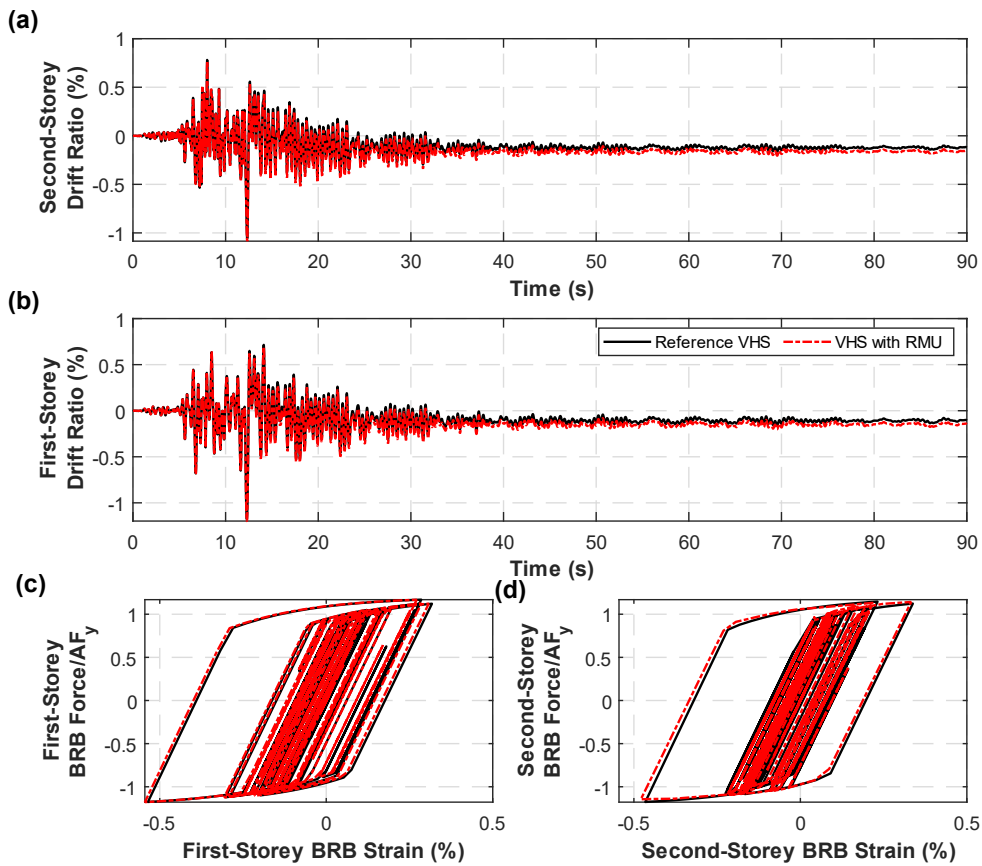

Figure 3. Virtual hybrid simulation of the BRBF under 1995 Kobe, Japan–Tadoka station earthquake, (a) history of drift ratio in Storey 2, (b) history of drift ratio in Storey 1, (c) Storey 1 BRB response, (d) Storey 2 BRB Response

4   Conclusions and Contributions

A novel recursive model updating algorithm was proposed for multi-element HS of structural systems under earthquake loading. This algorithm was developed by incorporating the PI model to reproduce nonlinear hysteretic response, LASSO algorithm for initial training, and RLS algorithm to recursively update the data-driven model that predicts the restoring force of numerical critical components similar – but not identical – to the critical component physically tested.

The proposed RMU algorithm can efficiently alleviate the uncertainties associated with the numerical simulation of the critical elements of structures in conventional HS of multi-storey seismic force-resisting systems. Furthermore, the proposed algorithm or real-time training technique can improve the prediction of the hysteretic response of the data-driven model in the nonlinear range of the material, e.g., improved cyclic hardening, because the amplitude, frequency and duration of real-time dynamic loading, e.g., earthquake accelerations, are explicitly accounted for in the simulation of the data-driven model during HS. This feature of the proposed algorithm highlights the benefit of the HS powered by the RMU algorithm over the conventional HS technique where the numerically-modelled critical components are often calibrated against experimental test data of similar prototype specimens under a predefined loading protocol, which lacks taking into consideration the influence of dynamic loading characteristics during HS.

This study verified the capability of the RMU algorithm and demonstrated its potential to overcome the limitations of seismic HS using pure numerical examples. Further verification of the proposed algorithm using small-scale and large-scale experimental test programs remains the subject of future studies.

Acknowledgements

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