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An IRF method for stochastic meshless analysis of 2D beams with lognormally varying random Young's modulus

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Abstract

The current paper proposes to use an improved response function (IRF) method for stochastic meshless analysis of a 2D beam wherein Young's modulus is assumed to vary as a lognormal random field. The meshless tool used in the present study is the element-free Galerkin method. In IRF method, the total displacement response is evaluated in each simulation by adding a deterministic part and an IRF. This displacement decomposition combined with the stiffness modelling using the second order expansion of Taylor series is substituted in the stochastic system of equations to find the expressions for the deterministic solution and the IRF. The deterministic solution evaluation is possible outside the loop for simulation and the expression for IRF involves only simple algebraic operations to be carried out inside the simulation loop. A 2D beam with cantilever boundary conditions loaded at the free end with a parabolic traction is analysed and the response moments are determined using the IRF method proposed. The first two moments of response obtained are observed to be matching well with the MCS moments even at higher values of coefficient of variation. The IRF method proposed also produces the distributions of response comparable with the distributions evaluated using MCS. The difference in response moments evaluated using IRF method and the second order perturbation method is in the acceptable limit. The computational efficiency of the IRF method is evidently better compared to MCS.

Keywords: Improved response function, Second order perturbation, Monte Carlo simulation, Lognormal random field, Shape function method, element-free Galerkin method

1 Introduction

The stochastic analysis considers the various system uncertainties for the analysis and design of structures [1] and demands the response moments and distributions. The time required is high since it involves the analysis of complex structural models. The most widely used stochastic finite element method (SFEM) [2] for the probabilistic structural analysis leads to random field to response mapping issues [3] pertaining to the inherent mesh dependency associated with the finite element method (FEM). Stochastic element-free Galerkin method (SEFGM) [3] is considered to replace SFEM since the element-free Galerkin method (EFGM) [4] is a commonly used meshless method and doesn't involve highly structured meshes for interpolating the field variables.

Monte Carlo simulation (MCS) [3], Perturbation techniques [3], Neumann expansion methods [5] etc. are some of the common stochastic methods used in SEFGM formulations. Even though it produces the complete probabilistic characteristics of response, use of MCS becomes cumbersome for the problems involving more field variables or random variables. Nonetheless, it is used as a tool for validation purpose in all the probabilistic studies. The perturbation and Neumann expansion methods produce the first two response moments with reasonable accuracy for input random fields with smaller coefficient of variation (*CV*) [3]. However, the probabilistic distributions of response cannot be evaluated using these methods.

An improved response function (IRF) method in combination with EFGM [6] is based on the decomposition of response into deterministic and stochastic parts; the stochastic part is called as IRF. Taylor series is used for modelling the stiffness. This method is computationally more efficient and produces the statistical moments of response and probabilistic distributions comparable with MCS at higher *CV* values. However, the use of IRF method has been limited to symmetrically distributed input random field in this study. The probabilistic structural analysis using the IRF method with unsymmetric random field is not studied so far. The present study suggests an extension of the IRF method for a 2D beam analysis using EFGM when the Young's modulus is unsymmetrically distributed; particularly lognormal. The results are validated using the MCS results. Second order perturbation (SOP) method is also used for comparing the results.

Stochastic meshless analysis formulations used in the present study are briefly discussed in the following section.

2 An IRF based stochastic meshless formulation for 2D beams

Young's modulus ($E(\mathbf{x})$) in the present paper is assumed as a lognormally distributed homogeneous random field [7];

$$E(\boldsymbol{x}) = C_l e^{(\eta(\boldsymbol{x}))},$$

(1)

where, $C_l = \frac{\mu_E}{\sqrt{1+CV^2}}$ and $\eta(\boldsymbol{x})$ is a stochastic field having mean zero and an exponential autocovariance $\Gamma_\eta = \log(1+\sigma_\eta{}^2)\exp\left[-\left(\frac{|\delta x|}{\lambda_x} + \frac{|\delta y|}{\lambda_y}\right)\right]$. μ_E and σ_E are the mean and the standard deviation of $E(\mathbf{x})$; $CV = \frac{\sigma_E}{\mu_E} = \sigma_\eta$ is the coefficient of variation of the input random field; δx and δy are the distances between the neighbourhood points in the *X* and *Y* directions. λ_x and λ_y are the correlation length parameters. The current study discretizes $\eta(\boldsymbol{x})$ utilizing the shape function method [8] that uses Moving least square [9] shape functions. The system of stochastic equations in EFGM [4,10] with stiffness $\overline{\boldsymbol{K}}(\boldsymbol{\eta})$ and displacement $\overline{\boldsymbol{d}}(\boldsymbol{\eta})$ is written as,

$$\overline{\boldsymbol{F}} = \overline{\boldsymbol{K}}(\boldsymbol{\eta})\overline{\boldsymbol{d}}(\boldsymbol{\eta}).$$

(2)

Direct simulations on the above equation (MCS) gives the complete probabilistic characteristics of response at the cost of computational time. Perturbation method on the other hand uses Taylor series expansion for both $\overline{\boldsymbol{K}}(\boldsymbol{\eta})$ and $\overline{\boldsymbol{d}}(\boldsymbol{\eta})$. It computes the mean and standard deviation of response in lesser time. Nonetheless, it cannot evaluate the response distributions. So, an IRF method [6] is proposed, which evaluates the response moments and the response distributions in a reasonable amount of time.

In IRF method, the total displacement response $\overline{\boldsymbol{d}}(\boldsymbol{\eta})$ is evaluated by adding the deterministic component ($\overline{\boldsymbol{d}}_{\boldsymbol{0}}$) and a stochastic component ($\overline{\boldsymbol{d}}_{\boldsymbol{IRF}}(\boldsymbol{\eta})$) during each simulation. $\overline{\boldsymbol{K}}(\boldsymbol{\eta})$ is modelled using second order Taylor series**.** The modified system of stochastic equations can be written as,

$$\overline{\boldsymbol{F}} = \left(\overline{\boldsymbol{K}}_{\boldsymbol{0}} + \overline{\boldsymbol{K}}_{,i}\eta_i + \frac{1}{2}\overline{\boldsymbol{K}}_{,ij}\eta_i\eta_j\right)\left(\overline{\boldsymbol{d}}_{\boldsymbol{0}} + \overline{\boldsymbol{d}}_{\boldsymbol{IRF}}(\boldsymbol{\eta})\right)$$

(3)

from which $\overline{\boldsymbol{d}}_{\boldsymbol{0}}$ and $\overline{\boldsymbol{d}}_{\boldsymbol{IRF}}(\boldsymbol{\eta})$ are evaluated as

$$\overline{\boldsymbol{d}}_{\boldsymbol{0}} = \overline{\boldsymbol{K}}_{\boldsymbol{0}}{}^{-\boldsymbol{1}}\overline{\boldsymbol{F}},$$

(4)

$$\overline{\boldsymbol{d}}_{\boldsymbol{IRF}}(\boldsymbol{\eta}) = \left(\overline{\boldsymbol{K}}_{\boldsymbol{0}} + \overline{\boldsymbol{K}}_{,i}\eta_i + \tfrac{1}{2}\overline{\boldsymbol{K}}_{,ij}\eta_i\eta_j\right)^{-\boldsymbol{1}}\left(-\left(\overline{\boldsymbol{K}}_{,i}\eta_i + \tfrac{1}{2}\overline{\boldsymbol{K}}_{,ij}\eta_i\eta_j\right)\overline{\boldsymbol{d}}_{\boldsymbol{0}}\right).$$

(5)

$(\ )_{\boldsymbol{0}}$, $(\ )_{,i} = \frac{\partial(\ )}{\partial\eta_i}$ and $(\ )_{,ij} = \frac{\partial^2(\ )}{\partial\eta_i\partial\eta_j}$ are the quantities calculated at the mean values of random variables. The response moments and distributions are computed using the basic probability theory [1] from the set of $\overline{\boldsymbol{d}}(\boldsymbol{\eta})$ calculated. In this method, a single time evaluation of $\overline{\boldsymbol{d}}_{\boldsymbol{0}}$ is possible outside the loop for simulation. $\overline{\boldsymbol{d}}_{\boldsymbol{IRF}}(\boldsymbol{\eta})$ can be computed easily inside the simulation loop, using simple algebraic operations on the generated random variables and the stiffness derivatives which are random variable independent and are evaluated outside the loop. This reduces the computational time considerably compared to MCS, which needs construction of $\bar{\boldsymbol{K}}(\boldsymbol{\eta})$ in (eqn (2)) at each Gauss point during each simulation. The accuracy in results is also guaranteed since the displacement approximation considers the cross dependency of random variables.

The next section deals with the numerical example of a 2D cantilever beam analysed using the proposed method.

3 Numerical Example

A 2D beam shown in Figure 1 is solved using the IRF based SEFGM. Poisson's ratio is taken as 0.3 and μ_E as 2×10^5 MPa. In EFGM analysis, a 2D linear basis, cubic spline weight function and a scaling parameter of 2 are used [4,11,12]. Meshless discretization uses 187 nodes; 54 background cells are used for numerical integration that employs four-point Gauss quadrature. Point A in Figure 1 is the reference point taken for further analysis.

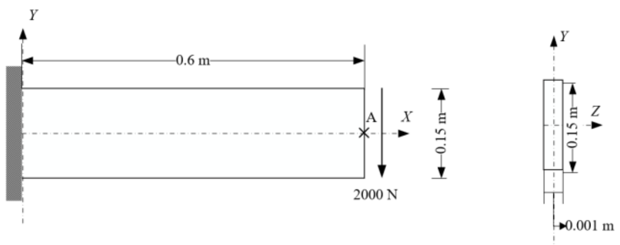

Figure 1: A 2D cantilever beam 0.6 m $\times$ 0.15 m $\times$ 0.001 m loaded at free end with a parabolic traction of 2000 N.

The stochastic analysis uses 12 number of random field discretization points. λ_x and λ_y are fixed as 0.5; *CV* values range from 1% to 30%. Response moments using MCS, SOP and IRF methods vary with *CV* as shown in Figure 2. It is seen that IRF results are comparable with MCS even at higher CV values. The IRF results are also comparable with SOP results.

Fixing *CV* and λ_y as 15% and 0.5, λ_x is varied from 0.25 to 1. The variations of response moments are plotted in Figure 3. The IRF results are comparable with MCS results independent of λ_x values. The probabilistic distributions of vertical deflection at point A using MCS and IRF methods are further potted in Figure 4 and a good agreement is evident. *CV* used here is 15% and λ_x and λ_y are 0.5.

It is observed that the IRF method requires only 0.444 times the time required for executing MCS. This is because it calculates the deterministic solution only single time outside the simulation loop and computing IRF requires only simple algebraic operations. Perturbation needs the least time; 5.45×10^{-3} times the time needed for executing MCS, but it computes only the response moments and fails to produce the response distributions.

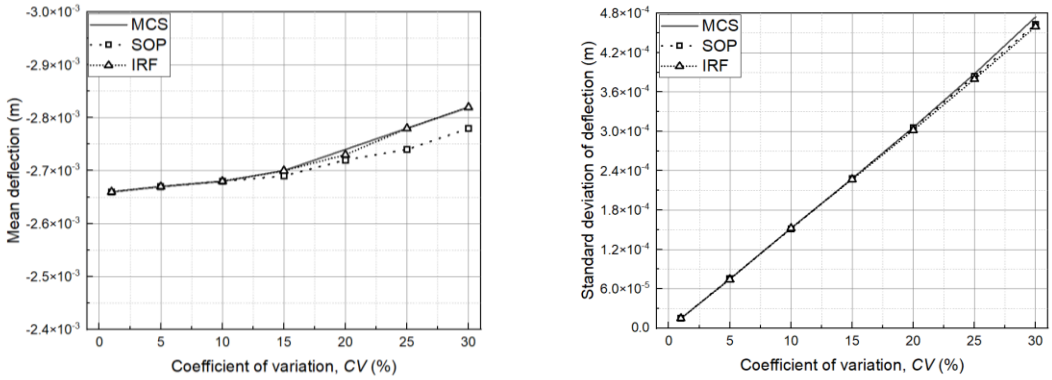

Figure 2: Variations in the first two statistical moments of vertical deflection at point A with *CV* of the input random field.

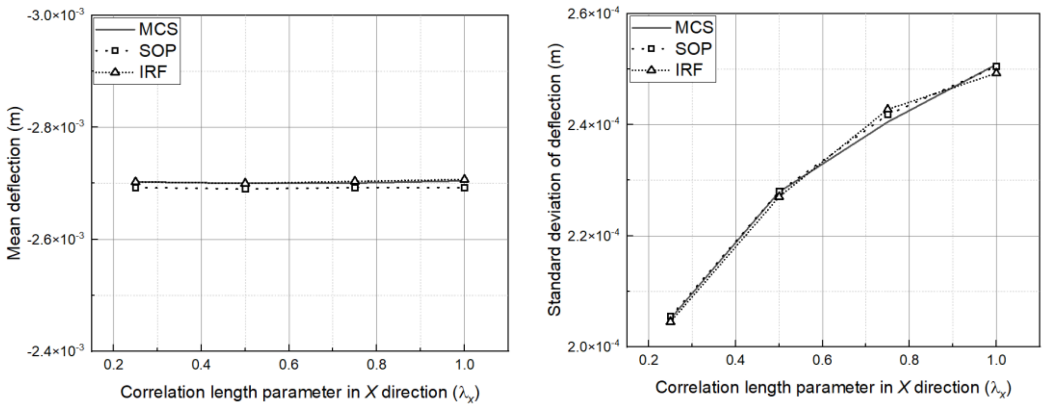

Figure 3: Variations in the first two statistical moments of vertical deflection at point A with correlation length parameter in X direction (λ_x).

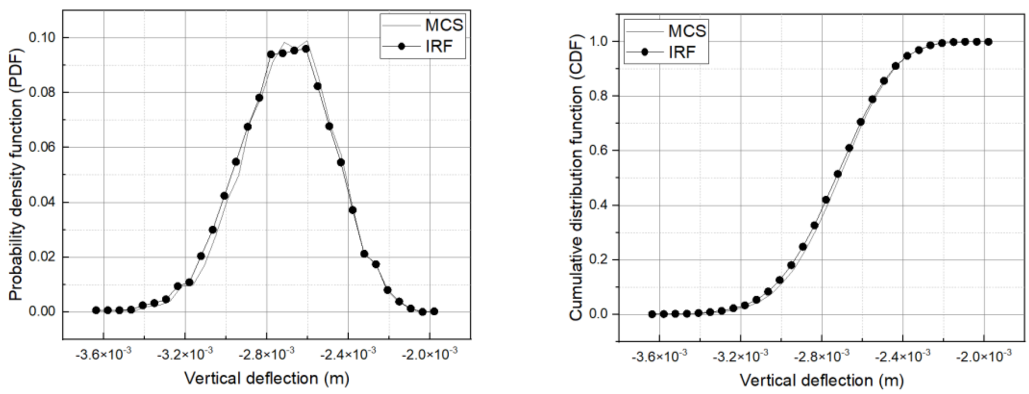

Figure 4: Probabilistic distributions of the vertical deflection at point A.

4 Conclusions

Use of an IRF for the stochastic meshless analysis is proposed in the present study for a 2D beam analysis wherein a lognormally varying random Young's modulus is assumed. In IRF method, the total displacement response during each simulation is evaluated by adding a deterministic part and an IRF. The second order expansion of Taylor series is employed for modelling the stiffness. A single time evaluation of the deterministic part is possible outside the loop for simulation. Evaluation of stochastic part or IRF needs performing simple algebraic operations during each simulation. The response moments and distributions are evaluated based on the simple probability theory from the complete set of total displacement responses obtained.

The numerical example solved in the present study is a 2D beam with cantilever boundary conditions, loaded at the free end with a parabolic traction. The mean and standard deviation of response are evidently matching with the MCS values at higher *CV* values as well. The obtained results are also found comparable with the SOP results. The statistical moments of vertical deflection evaluated using different methods used in the present study are seen comparable independent of the correlation length parameters of the input lognormal random field. A good agreement in the probability distributions of vertical deflection computed from both MCS and IRF methods is also observed. The IRF method is computationally far efficient compared to MCS.

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