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Robust Design Optimization of Structures Using Stochastic Simulation Based Approach

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Abstract

There are numerous uncertainties in the structural design, such as the randomness or variation in the loading, structural parameters, geometric parameters, operation conditions, etc. The Robust Design Optimization (RDO) methodology aims to determine an optimal solution corresponding to the insensitive system performance when subjected to these uncertainties. Available RDO approaches can effectively take into account these uncertainties. Still, accuracy and computational cost in evaluating the mean and variance (robustness measures) are prohibitive for designing complex and realistic structural systems. To obliviate this limitation, a novel Stochastic simulation-based approach is proposed in this paper. The newly developed approach is constructed based on the 'augmented optimization problem,' in which design variables are artificially considered as an uncertain parameters. Furthermore, for optimization, a two-stage approach is adopted. Firstly, the design space size is reduced by formulating an unconstraint optimization approach followed by any standard random search method (KN direct search method) to determine the optimal solution within the reduced design space. As the mean and variance frequently conflict with each other, so to obtain the Pareto optimum, a linear scalarized objective function is adopted. Three optimization problems: quadratic function and six-hump camel-back function, and 10-bar truss structure subjected to uncertain loading and uncertain material properties are solved with the proposed approach to demonstrate the efficiency of the proposed approach. The results obtained indicate that the proposed approach is as accurate as of the conventional Monte Carlo simulation approach. This paper allows the designers to design insensitive structure systems. Moreover, the proposed RDO approach is general and not limited to the civil structures only, but it can also be enforced in the design of any realistic linear/nonlinear systems.

Keywords: robust design optimization, optimization under uncertainty, uncertainty, genetic algorithm, mean-variance.

1 Introduction

In a realistic state, the structural systems are inherent in several inevitable uncertainties, such as loading, structural parameters, etc. Typically, a deterministically designed structural system may not perform as intended due to inherent uncertainties, leading to improper design. One of the most promising ways to minimize the effect of these uncertainties is the Robust Design Optimization (RDO) methodology. Robust design pioneered by Taguchi [1] aims to minimize the mean and variance [2] to ensure the insensitive system is subjected to uncertainties[3–8]. Figure 1. shows the concept of RDO. As shown, design-1 (optimal design) has the lower optimal value of performance function (*f*) as compared to design-2 (robust design) when subjected to the same variation (Δx) in the structural parameter. Since the variation in performance function is less in design-2 ($\Delta f_{\text{robust}} < \Delta f$), design-2 is considered insensitive to the uncertainties.

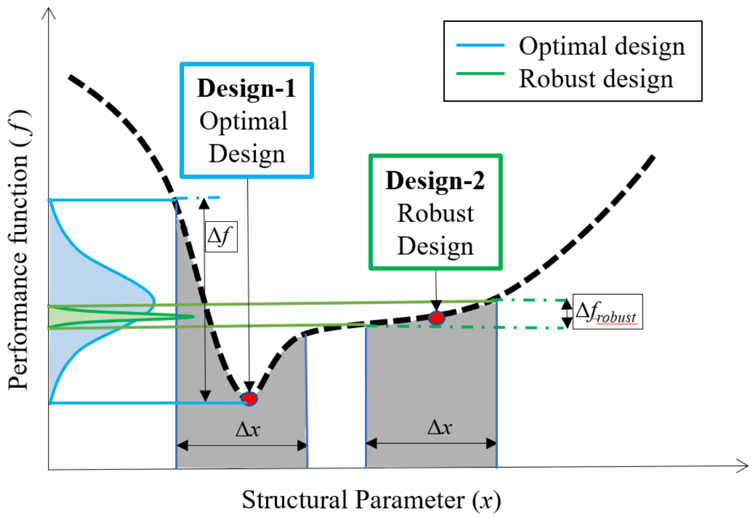

Figure 1: RDO concept.

The RDO problem can be mathematically formulated as the determination of,

$$\mathbf{x}^* = \arg\min_{\mathbf{x}\in\Phi} \left\{ \mu_f(\mathbf{x}), \sigma_f^2(\mathbf{x}) \right\}, \quad \text{subjected to} \quad \mathbf{g}_c(\mathbf{x}) \le 0, \tag{1}$$

where,

$$\mu_f(\mathbf{x}) = \mathrm{E}_\theta\left[f(\boldsymbol{\theta},\mathbf{x}) \right] = \int_\Theta f(\boldsymbol{\theta},\mathbf{x})\, p(\boldsymbol{\theta}) d\boldsymbol{\theta}, \tag{2}$$

and

$$\sigma_f^2(\mathbf{x}) = \mathrm{E}_\theta\left[\left(f(\boldsymbol{\theta},\mathbf{x}) - \mu_f(\mathbf{x}) \right)^2 \right] = \int_\Theta \left(f(\boldsymbol{\theta},\mathbf{x}) - \mu_f(\mathbf{x}) \right)^2 p(\boldsymbol{\theta}) d\boldsymbol{\theta}, \tag{3}$$

denotes the mean and variance of the performance function $f(\boldsymbol{\theta}, \mathbf{x}): R^{n_\theta \times n_x} \to R$, respectively. $\mathrm{E}_\theta[\cdot]$ represents expectation with respect to Probability Distribution Function (PDF) for **θ.** The constraint vector is characterized by $\mathbf{g}_c(\mathbf{x})$. Usually, mean and variance frequently conflict with each other, so to obtain the Pareto optimum, a linear scalarized performance function is adopted [9]. In case of many uncertain parameters and implicit performance, the evaluation of the stochastic integrals Equations (2) & (3) is a challenging task. Methods in literature to evaluate these integrals can be categorized as analytical, approximation-based, and simulation-based. Estimating Equation (2) analytical is possible only for limited cases; therefore, several approximation-based methods are developed. The well-known approximation approaches include Taylor Series Expansion (TSE) based approach [6,10–15] and Dimension Reduction (DR) method [16]. The accuracy of the TSE methods is infeasible due to the limitation of evaluating the high-order derivates. In contrast, the DR method does not apply to problems with a large number of uncertain parameters [17]. Metamodels [18] are efficient, but models themselves involve several approximations, and thus, achieving higher accuracy is infeasible. Simulation-based approaches are applicable to the complex high dimensional problem, but high accuracy necessitates a high computational cost. Due to this reason, the simulation-based approach is prohibitive.

Thus, an effective and efficient novel stochastic simulation-based approach is proposed in this paper. The results are compared with the conventional Monte Carlo Simulation approach and indicate the proposed approach's accuracy and effectiveness.

2 Methods

Consider any structural performance function $f(\boldsymbol{\theta}, \mathbf{x}): R^{n_\theta \times n_x} \to R$. Initially discussed in [19], the augmented formulation artificially considers the design parameters as uncertain. In this framework, an auxiliary PDF is defined as

$$\pi(\boldsymbol{\theta}, \mathbf{x}) = \frac{f(\boldsymbol{\theta}, \mathbf{x})\, p(\boldsymbol{\theta}, \mathbf{x})}{\mathrm{E}_{\theta,x}[f(\boldsymbol{\theta}, \mathbf{x})]}, \tag{4}$$

where,

$$p(\boldsymbol{\theta}, \mathbf{x}) = p(\boldsymbol{\theta} \mid \mathbf{x})\, p(\mathbf{x}). \tag{5}$$

In this setting, the mean of performance function $\mathrm{E}_\theta\left[f(\boldsymbol{\theta}, \mathbf{x})\right]$ is given as,

$$\mathrm{E}_\theta\left[f(\boldsymbol{\theta}, \mathbf{x})\right] = \frac{\pi(\mathbf{x})}{p(\mathbf{x})} \mathrm{E}_{\theta,x}[f(\boldsymbol{\theta}, \mathbf{x})], \tag{6}$$

and

$$\mathrm{E}_{\theta,x}[f(\boldsymbol{\theta}, \mathbf{x})] = \int_\Phi \int_\Theta f(\boldsymbol{\theta}, \mathbf{x})\, p(\boldsymbol{\theta}, \mathbf{x})\, d\boldsymbol{\theta}\, d\mathbf{x}. \tag{7}$$

where the marginal PDF $\pi(\mathbf{x})$ is equal to

$$\pi(\mathbf{x}) = \int_\Theta \pi(\boldsymbol{\theta}, \mathbf{x})\, d\boldsymbol{\theta}. \tag{8}$$

In this setting, since $\mathrm{E}_{\theta,x}[f(\boldsymbol{\theta},\mathbf{x})]$ is a normalizing constant, the minimization is equivalent to minimization of $J(\mathbf{x})$, given as,

$$J(\mathbf{x}) = \frac{\pi(\mathbf{x})}{p(\mathbf{x})}. \tag{9}$$

Note that minimization of $J(\mathbf{x})$ is equivalent to minimize $\pi(\mathbf{x})$, for simplicity $p(\mathbf{x})$ is considered to be uniformly distributed. The $\pi(\mathbf{x})$ is simulated as explained in [19]. Similarly, in the setting of augmented formulation, the variance of performance function is given as,

$$h(\boldsymbol{\theta},\mathbf{x}) = \left(f(\boldsymbol{\theta},\mathbf{x}) - \mu_f(\mathbf{x}) \right)^2. \tag{10}$$

The auxiliary PDF $\pi(\boldsymbol{\theta},\mathbf{x})$ is equal to

$$\pi(\boldsymbol{\theta},\mathbf{x}) = \frac{h(\boldsymbol{\theta},\mathbf{x})\, p(\boldsymbol{\theta},\mathbf{x})}{\mathrm{E}_{\theta,x}[h(\boldsymbol{\theta},\mathbf{x})]}, \tag{11}$$

where,

$$p(\boldsymbol{\theta},\mathbf{x}) = p(\boldsymbol{\theta})\, p(\mathbf{x}), \tag{12}$$

and

$$\mathrm{E}_{\theta}\left[h(\boldsymbol{\theta},\mathbf{x})\right] = \frac{\pi(\mathbf{x})}{p(\mathbf{x})} \mathrm{E}_{\theta,x}[h(\boldsymbol{\theta},\mathbf{x})] \tag{13}$$

where the marginal PDF $\pi(\mathbf{x})$ is equal to

$$\pi(\mathbf{x}) = \int_{\Theta} \pi(\boldsymbol{\theta},\mathbf{x})\, d\boldsymbol{\theta}. \tag{14}$$

In this setting, since $\mathrm{E}_{\theta,x}[h(\boldsymbol{\theta},\mathbf{x})]$ is a normalizing constant, the minimization is equivalent to minimization of $J(\mathbf{x})$, given as,

$$J(\mathbf{x}) = \frac{\pi(\mathbf{x})}{p(\mathbf{x})}. \tag{15}$$

Thus, the RDO optimization can be reformulated as the determination of

$$\mathbf{x}^* = \arg\min_{\mathbf{x}\in\Phi} \mathrm{E}_{\theta}\left[\alpha \frac{f(\boldsymbol{\theta},\mathbf{x})}{\tilde{\mu}_f} + (1-\alpha) \frac{\left(f(\boldsymbol{\theta},\mathbf{x}) - \mu_f(\mathbf{x}) \right)^2}{\tilde{\sigma}_f^2} \right], \tag{16}$$

where $\tilde{\mu}_f$ and $\tilde{\sigma}_f^2$ are obtained from solving the optimization problem of minimizing the mean and variance of the structural response, respectively. The weighting factor $\alpha \in [0,1]$ represents the relative importance of the two objective functions.

An excellent approach to reduce the design space is adopted from [19]. As proposed in [20], the average value of $J(\mathbf{x})$ given as,

$$H(I_k) = \frac{1}{V_{I_k}} \int_{I_k} J(\mathbf{x})\, d\mathbf{x} = \frac{V_{\hat{I}_{k-1}}}{V_{I_k}} \int_{I_k} \pi(\mathbf{x})\, d\mathbf{x} \tag{17}$$

is minimized. Where V_{I_k} and $V_{\hat{I}_{k-1}}$ denote the volume of the set I_k and $\hat{I}_{k-1}$, respectively such that $I_k \subset \hat{I}_{k-1}$ and $\hat{I}_{k-1}$ is the optimal subset identified at the (*k*-1)*th* iteration such that $\hat{I}_{k-1} \subset \cdots \subset \hat{I}_1 \subset \Phi$. Based on the samples distributed according to $\pi(\boldsymbol{\theta}, \mathbf{x})$ belonging to subset $\hat{I}_{k-1}$, an estimate of $H(I_k)$ is given as,

$$\bar{H}(I_k) = \frac{N_{I_k} / V_{I_k}}{N_{\hat{I}_{k-1}} / V_{\hat{I}_{k-1}}}, \tag{18}$$

where N_{I_k} and $N_{\hat{I}_{k-1}}$ denote the number of samples from $\pi(\boldsymbol{\varphi})$ belonging to the sets I_k and $\hat{I}_{k-1}$, respectively. Some predefined shape of the admissible subset is adopted (hyper-rectangle). Later, the standard random search (KN search) approach is adopted to determine the optimal solution within the above-identified subset. More detail can be found in [20].

3 Results

Example-3.1: Quadratic test function

The quadratic function considered is expressed as

$$f(\theta, x) = (2 - x)(0.1 - \theta - 0.1x) + 0.3, \tag{19}$$

where x is the design variable $\Phi = [0, 4]$, and θ is the uncertain parameter uniformly distributed $[-0.2, 0.2]$.

Figure 2. shows the mean and variance minimization results evaluated from the MCS approach using 100,000 sample, respectively. Figure 3. illustrates the upper and lower limits of design parameter obtained from the proposed approach (25 independent runs) for mean and variance minimization, respectively. It can be observed that with the increase in generation the optimal result is converged. The results are in good agreement with the MCS approach.

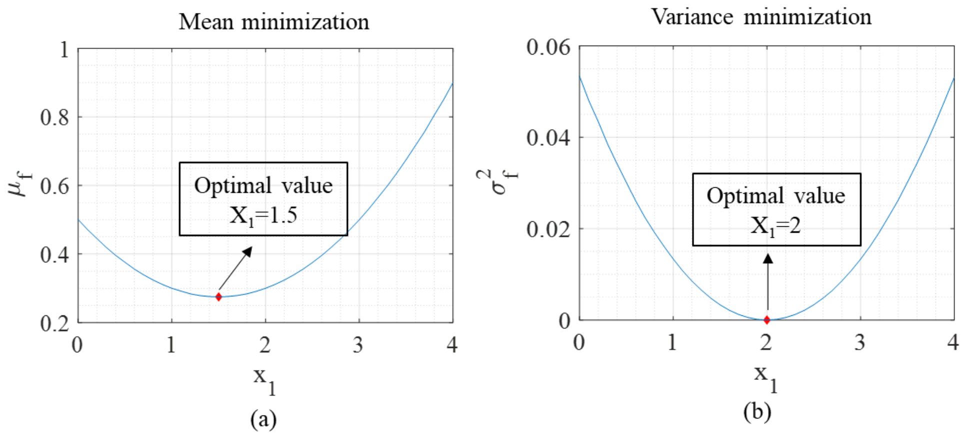

Figure 2: (a) Mean minimization versus design variable x_1, (b) Variance minimization versus design variable x_1.

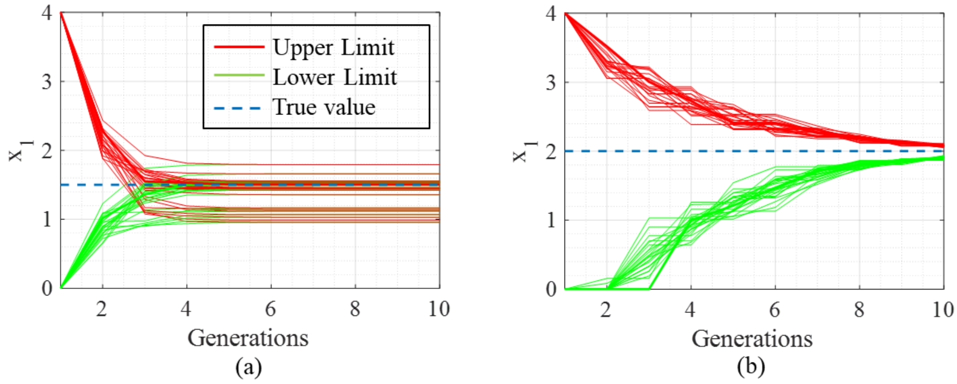

Figure 3: Limits of design variable for (a) mean minimization and (b) variance minimization.

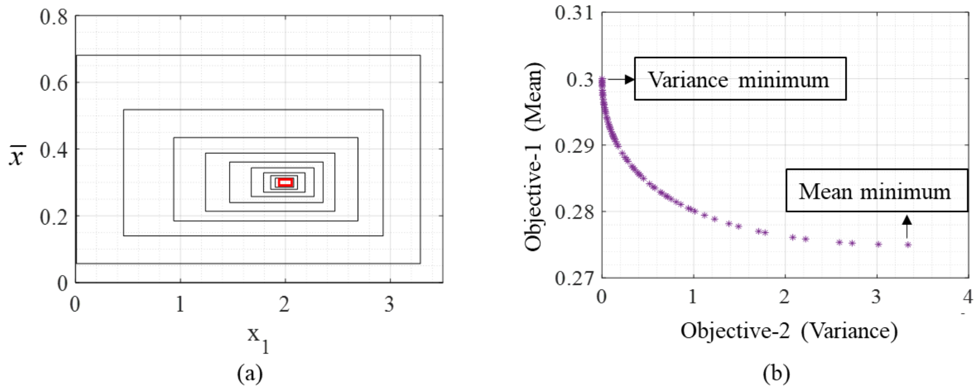

Figure 4: (a) subsets of reduced design space for variance minimization, (b) Pareto front.

Figure 4. (a) shows the iteratively identified reduced design space. It can be observed that the smallest design space (red colour) accurately includes the optimal solution. Figure 4. (b) show the Pareto front. As expected, mean and variance minimization lies on the extreme ends.

Example-3.2: Six-hump camel-back function

A multiple minima six-hump camel-back function is considered, given as

$$f(\theta, x) = 4x^2 - 22x^4 + \frac{x^6}{3} + x\theta - 4\theta^2 + 4\theta^4, \tag{20}$$

where x is the design variable $\Phi = [-2, 2]$, and θ is the uncertain parameter uniformly distributed $[-1, 1]$.

Figure 5. shows the mean and variance minimization results evaluated from the MCS approach using 100,000 samples. Figure 6. illustrates the upper and lower limits of design parameters obtained from the proposed approach (25 independent runs) corresponding to mean and variance minimization. It can be observed that results are well matched with MCS.

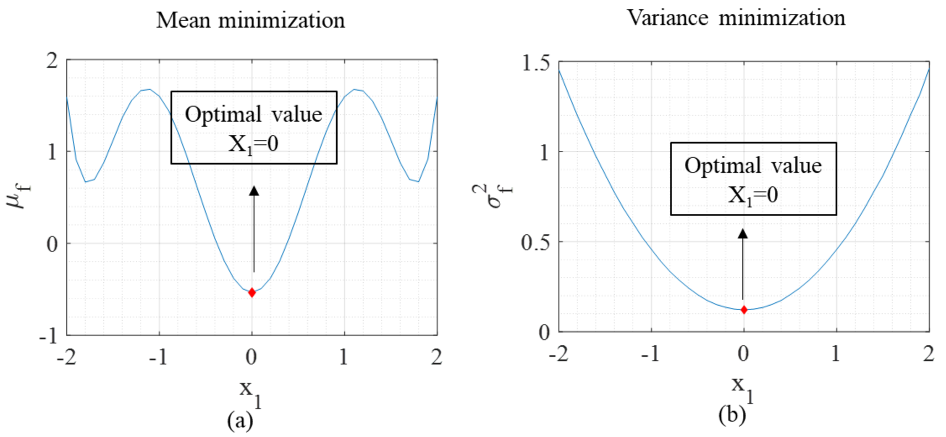

Figure 5: (a) Mean minimization versus design variable x_1, (b) Variance minimization versus design variable x_1.

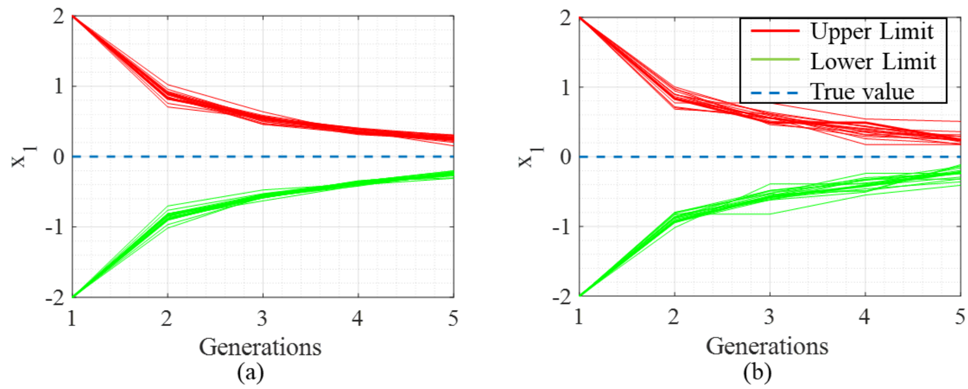

Figure 6: Limits of design variable for (a) mean minimization and (b) variance minimization.

Example-3.3: 10-bar truss structure

Ten-bar truss structure is shown in Figure 7.

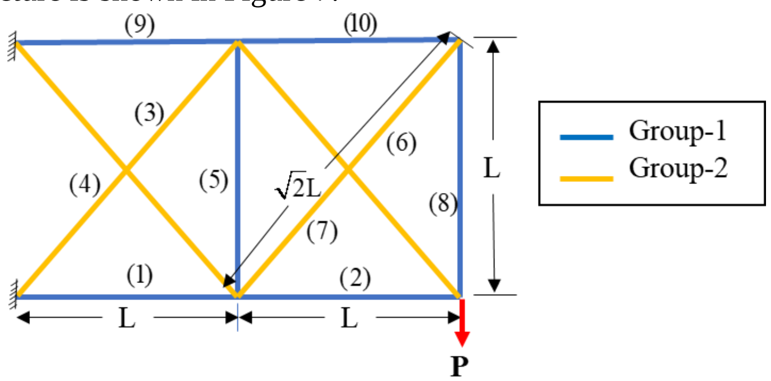

Figure 7: Ten-bar truss structure.

The end node of the truss is subjected to a random vertical load *P* normally distributed with a mean value of 100kN and standard deviations of 20 kN. The Young's modulus

(E) for the truss is uncertain and normally distributed with mean and standard deviation of 100N/m^2 & 80 N/m^2 and 30 N/m^2 &10 N/m^2, respectively, for the two groups. The cross-sectional areas of the two groups A_1 & A_2 are considered as design variables. Compliance (defined as the inner product of the applied load vector and the nodal displacement vector) is the performance function. A volume constraint $V \leq 500$ is considered. In this setting, uncertain parameters $\boldsymbol{\theta} = [E_1, E_2, P]$ and $\mathbf{x} = [A_1, A_2]$.

Table 1 represents the results obtained from solving the mean, variance, and combination of mean and variance minimization problem, respectively. It can be observed that the results obtained are in good agreement with that of MCS results. This indicates that the proposed approach (PA) is accurate.

Case α	A_1 (mm^2)		A_2 (mm^2)		$\mu_g(\mathrm{x}^*)$ (Nm)		$\sigma_g^2(\mathbf{x}^*)$ (Nm)^2	
	PA	MCS	PA	MCS	PA	MCS	PA	MCS
1	55.6	55.0	29.4	30.0	13.4	13.9	215.9	216.6
0.5	60.8	60.5	23.9	23.6	20.5	20.4	186.1	186.4
0	64.2	64.0	20.3	20.5	28.1	28.0	172.5	172.4

Table 1: Results of the ten-bars truss structure.

4 Conclusions and Contributions

This paper attempts to contribute a novel stochastic simulation-based optimization approach to perform the robust design optimization of the structures. Issues with the available approximate, analytical, and simulation-based methods are also highlighted. The proposed approach is constructed on the concept of the augmented formulation. The two-stage optimization approach is adopted to obtain the desired accuracy. For optimization, firstly, the size of the design space is reduced by using the concept of stochastic subset optimization. Then direct search optimization is performed to determine the optimal design in the reduced design space. The effectiveness of the proposed approach is illustrated with the help of three well-known optimization problems, including (1) quadratic function, (2) six-hum camel-back function, and (3) ten-bar truss structure. Comparisons between the conventional Monte Carlo Simulation approach and the proposed approach are performed. The results obtained indicate that the proposed approach is as accurate as of the conventional Monte Carlo simulation approach. This paper allows the designers to design insensitive structure systems. Moreover, the proposed RDO approach is general and not limited to the civil structures only, but it can also be enforced in the design of any realistic linear/nonlinear systems. It should be noted that this study focuses on unconstrained optimization and could be extended to constrained optimization. Further, research efforts will focus on the issues and applications of engineering design in practice.

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