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Open-pit slope design using a Dirichlet-to-Neumann Finite Element Method

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Abstract

Given the sustained mineral-deposits ore-grade decrease, it becomes necessary to reach greater depths when extracting ore by open-pit mining. Steeper slope angles are thus likely to be required, leading to geomechanical instabilities. In order to determine excavation stability, mathematical modelling and numerical simulation are often used to compute the rock-mass stress-state, to which some stability criterion needs to be added. A problem with this approach is that the volume surrounding the excavation has no clear borders and in practice it might be regarded as an unbounded region. Then, it is necessary to use advanced methods capable of dealing efficiently with this difficulty. In this work, a Dirichlet-to-Neumann Finite Element Method (DtN-FEM) procedure is applied to calculate displacements and stresses in open-pit slopes under geostatic stress conditions. which was previously devised by the authors to numerically treat this kind of problems where the surrounding domain is semi-infinite. Its efficiency makes possible to simulate, in a short amount of time, multiple open-pit slope configurations. Therefore, the potentiality or this method for open-pit slope design is investigated. A regular open-pit slope geometry is assumed, parameterised by the overall-slope and bench-face angles. Multiple geometrically admissible slopes are explored, and their stability is assessed by using the computed stress-field and the Mohr-Coulomb failure criterion. Regions of stability and instability are thus explored in the parametric space, opening the way for a new and flexible designing tool for open-pit slopes and related problems.

Keywords: Dirichlet-to-Neumann map, finite elements methods, open-pit, slope design.

1 Introduction

Due to the sustained ore grade decrease in mineral deposits, mine depth is increasing, leading to steeper slopes. A likely consequence of this is geomechanical instability, with results in multiple adverse effects. A possible control measure is a precise knowledge of the rock-mass stress-state surrounding the mine. The use of numerical simulation can significantly help to achieve this purpose.

In general, when applying numerical methods to calculate stresses in a mine or excavation, a computational domain needs to be established, which corresponds to the spatial region where the computations will be done. However, it is not clear a priori how large that region should be. In practice, all the surrounding rock-mass can be regarded as unbounded, but computers cannot store infinite domains. A common heuristic approach to overcome this difficulty is to employ a huge domain, typically a rectangular box, with its external boundaries far away enough from the excavation so that they have minimal effect on the results. The discretisation of such a domain requires too many points, making the numerical method performance inefficient and even inaccurate, mainly due to abuses in boundary conditions.

A powerful and specially adapted numerical technique to solve problems formulated in unbounded domains is the Dirichlet-to-Neumann Finite Element Method (DtN-FEM), where finite elements are used in combination with the DtN map defined over an artificial boundary. The main advantage of this approach is that the DtN map provides exact boundary conditions, ensuring continuity of the solution and its derivatives across the artificial boundary, which results in a method with high accuracy and using considerably fewer discretisation points.

In a relatively recent work [1], the authors presented and validated a DtN-FEM for axisymmetric problems in the elastic half-space, based on a semi-analytical approximation of the DtN map, also developed by the authors in a previous paper [2]. The method exhibited an excellent performance in terms of flexibility, speed, precision, and robustness. This work aims at exploiting the advantages of this DtN-FEM by solving a problem in open-pit slope design which requires to simulate several scenarios. A homogeneous, isotropic, and fully fractured rock-mass is mined with an open-pit having regular axisymmetric slope, fixed number of benches and overall slope height as well, whereas the overall-slope angle and bench-face angle are variable parameters. Regions of geomechanical stability/instability are thus determined in the parametric space by exploring Mohr-Coulomb failure criterion given the geometry-induced stress-field [3].

2 Methods

The DtN-FEM is based on a semi-analytical approximation of the DtN map for the elastic half-space in the axisymmetric case. Figure 1 shows a typical geometry, where Ω is the computational domain, Γ_p is the open-pit boundary and Γ_R is the semi-spherical artificial boundary (R stands for the radius). The coupling between the DtN map -expressed in series form- and the FEM scheme is done directly on the discretised variational formulation of the boundary-value problem, specifically on the integral

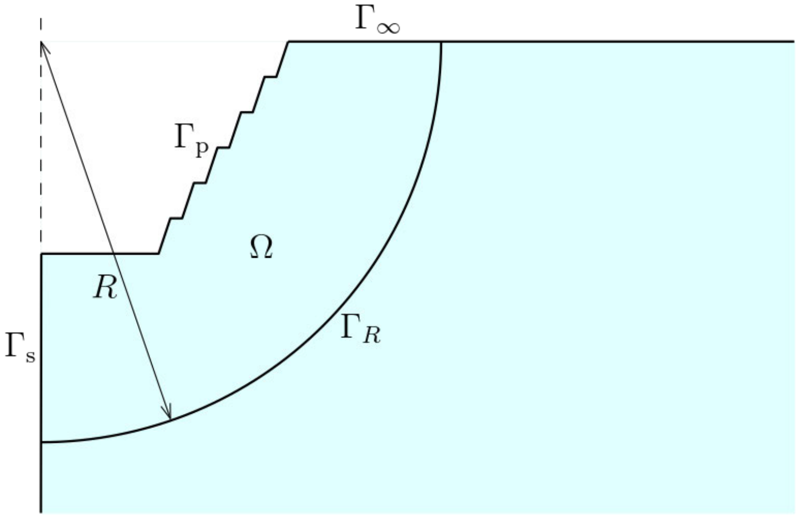

Figure 1: Computational domain.

terms on Γ_R. The DtN-FEM computes numerically the displacement vector $\boldsymbol{u}$, the strain tensor $\boldsymbol{\varepsilon}$, and the stress tensor $\boldsymbol{\sigma}$ in Ω. A full description of the method can be found in [1].

To characterise the stability of a particular open-pit geometry, we employ the Mohr-Coulomb failure criterion [4], given a computed stress-state characterised by the principal stresses $\sigma_1 \geq \sigma_2 \geq \sigma_3$. The criterion relates shear stress τ as a function of normal stress σ, describing conditions where isotropic materials are prone to fail given that most of them have a limited range of shear stresses that can sustain. Figure 2 shows graphically the failure criterion, where S_0 is the shear strength and φ is the friction angle. A particular stress-state is represented by a Mohr semicircle, defined by σ_1 and σ_3. We propose the distance γ as depicted in Figure 2 as a simpler alternative for stability assessment: the greater the distance the more stable the stress-state in a particular point of Ω, whereas a negative distance represents an unstable stress-state.
To apply this method to open-pit slope design, a regular parameterised geometry of open-pit is assumed, which is schematically described in Figure 3. The main geometrical parameters are the overall slope height H, the number of benches n, the bench length a, the berm width b, the overall slope angle β and the bench face angle α. Angles α and β must satisfy $\alpha > \beta$. The bench height is given by H/n. Additionally, we consider the horizonal distances from the z-axis to the first bench d, and to the last bench L, as indicated in Figure 3. Multiple possible combinations of the angles α and β are considered, keeping the rest of the parameters fixed. The open-pit slopes resulting of each combination are numerically solved and their stability is assessed, in order to determine regions of stability and instability in the (α, β)-space.

3 Results

First, the solution of a particular case of open pit is presented, which was forced by geostatic initial stress. Figure 4 shows the meshed computational domain (left) and the computed stability indicator γ (right). It is observed that γ is positive in almost the entire domain, however, it has a clear minimum value at the lowest point of the first bench, which is negative. Hence, that point corresponds to a failure point.

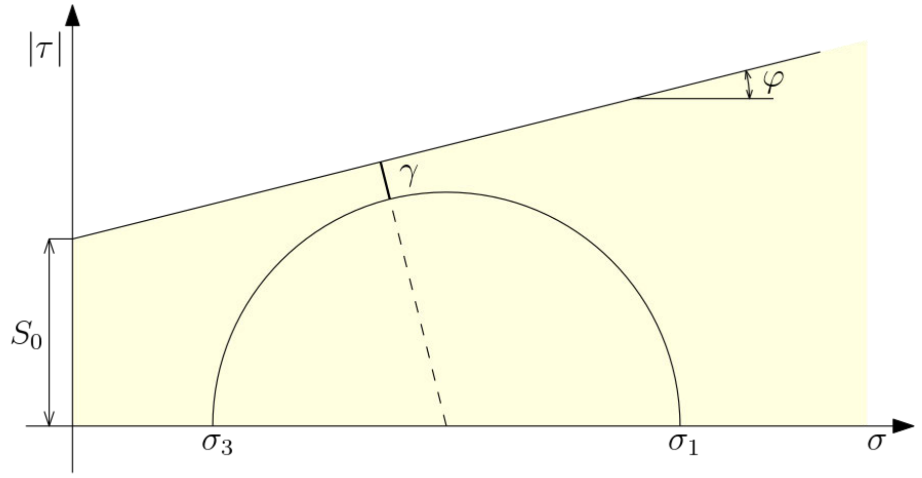

Figure 2: Mohr-Coulomb failure criterion diagram.

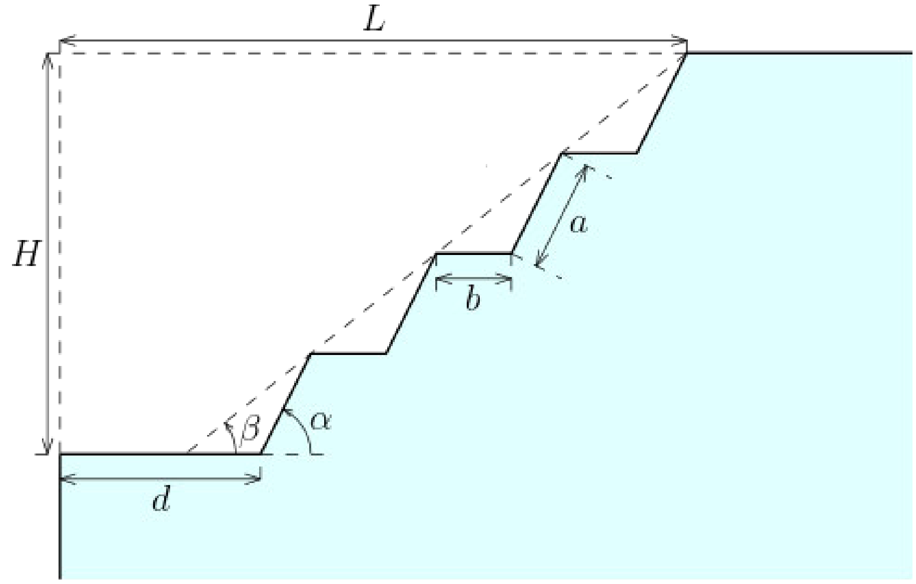

Figure 3: Parameterised geometry of axisymmetric open-pit.

Then, multiple possible combinations of angles α and β were considered. The resulting open-pit slope configurations were numerically solved. In all cases we assume an open pit with an overall height H = 1200 m and n = 12 benches. The horizontal distance d was assumed fixed. We considered α and β ranges equal to $60° \leq \alpha < 90°$ and $30° \leq \beta \leq \min(75°, \alpha - 10°)$, which were discretised in equal steps $\Delta\alpha = \Delta\beta = 1°$. The resulting number of (α, β)-combination is 1055. Furthermore, a fixed truncation radius R was assumed. Three (α, β) parameter-space explorations were carried out for three values of inherent shear strength, namely S_0 = 20, 30, 40 MPa, keeping φ fixed and equal to 30°. For each value of S_0 and for every admissible combination of α and β, γ was calculated, and its minimum was evaluated, which in all the cases occurred at the lowest bench bottom. When this minimum is positive, none of the points should fail and the open-pit slope configuration is stable. However, if the minimum is negative, then there are points that might fail and the open-pit slope

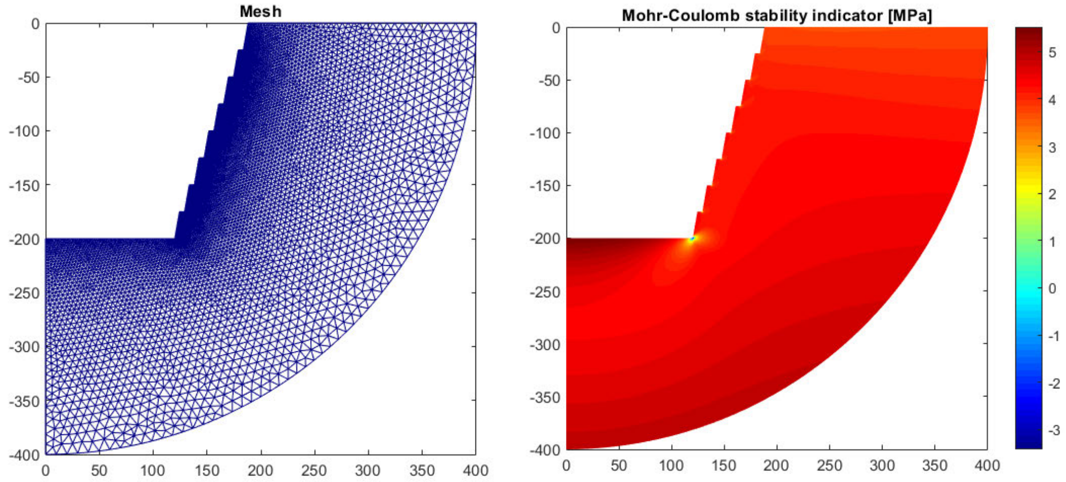

Figure 4: Numerical example of open-pit: meshed computational domain (left) and computed stability indicator (right).

configuration is unstable. Minimum-γ plots as functions of α and β are presented in Figures 5, 6 and 7 for S_0 = 20, 30, 40 MPa respectively.

It is observed that the stability of a particular configuration depends mainly on the overall slope angle β. As might be expected a priori, stable configurations (green) are associated with low overall slope angles. Even for steep bench face angles, the configuration may be stable if β is low enough. On the other hand, higher overall slope angles lead to unstable configurations (red). The $\gamma = 0$ contour line location is shown in blue in the three cases. This line defines a stability boundary in the (α, β) plane, dividing it into a stable region on the left and an unstable region on the right.

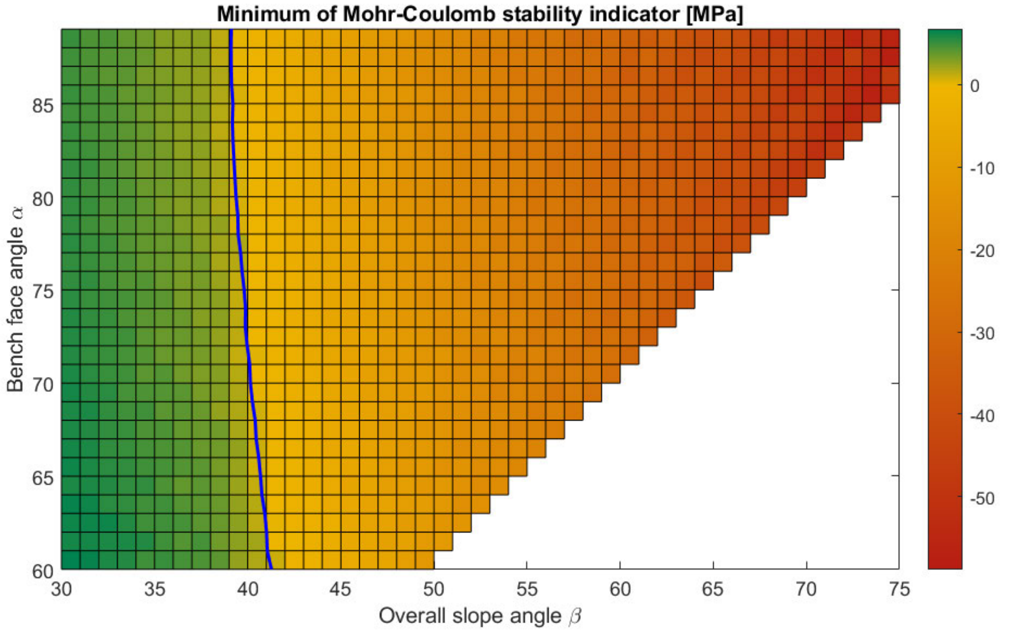

Figure 5: Minimum stability indicator in function of (α, β) for S_0 = 20 MPa.

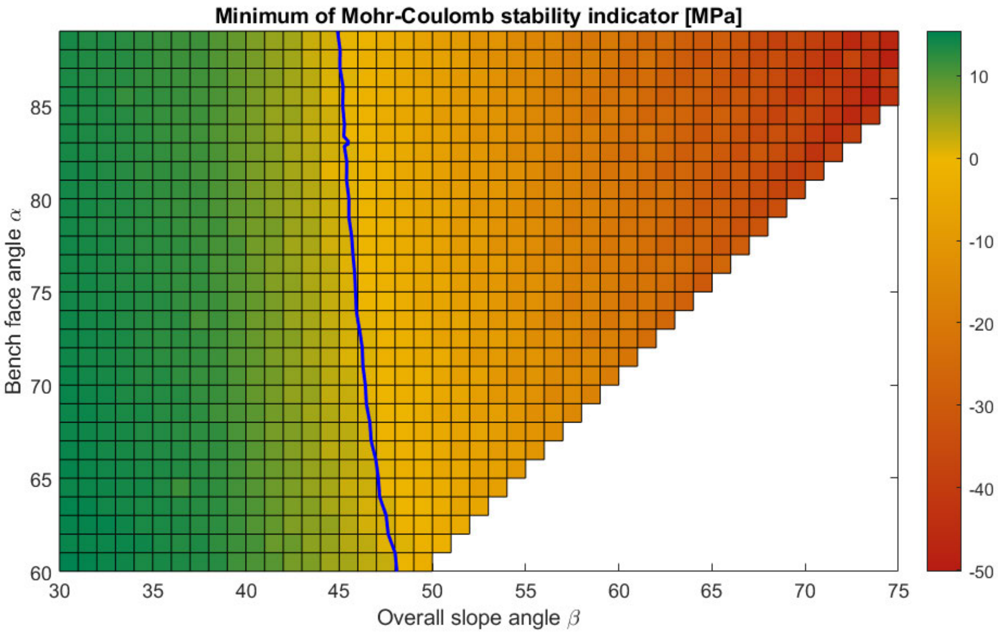

Figure 6: Minimum stability indicator in function of (α, β) for S_0 = 30 MPa.

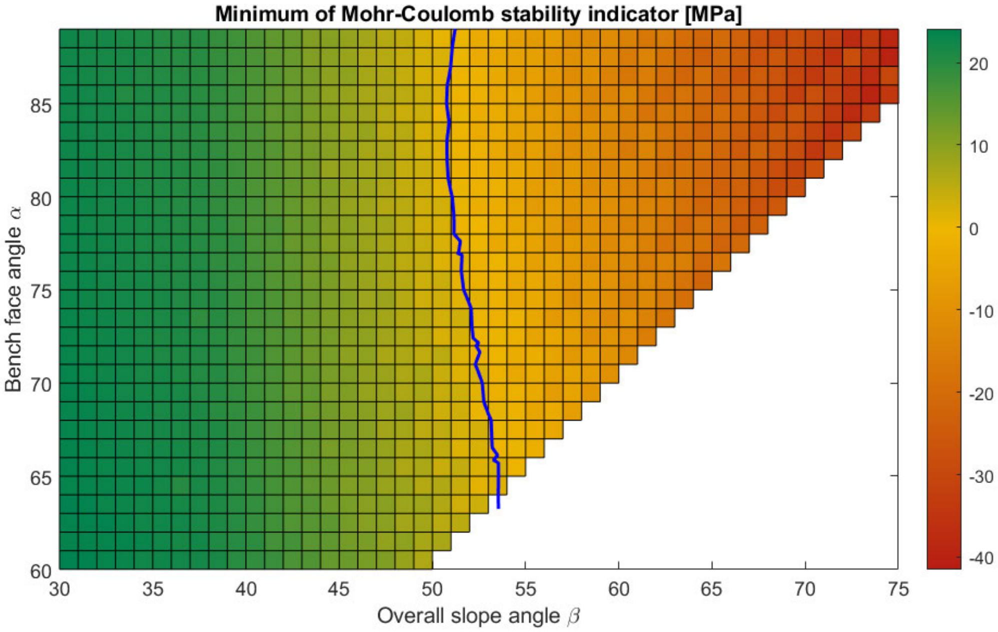

Figure 7: Minimum stability indicator in function of (α, β) for S_0 = 40 MPa.

4 Conclusions and Contributions

A DtN-FEM approach for semi-infinite elastic domains has been applied to study open-pit stability in the axisymmetric case. The method is highly accurate, flexible, and efficient, which allowed us to solve multiple open-pit slope configurations in a short amount of time. The open-pit stability was assessed by computing an indicator based upon the Mohr-Coulomb failure criterion, which is positive when the open-pit slope configuration is stable and negative otherwise. Other stability measures exist, and they might be explored as well, for instance the Hoek-Brown failure criteria in case of a fractured rock-mass. A regular open-pit geometry was considered, parameterised by the bench face angle α and the overall slope angle β, where α and β must satisfy some criteria to be considered geometrically admissible. A particular case of open-pit geometry was simulated to illustrate how the method works. It was found that the bench bottom points are more likely to be unstable, in particular the lowest bench bottom point.

The space of parameters (α, β) was explored for three values of rock-mass shear strength S_0. The stability indicator was evaluated in every domain point and its minimum value was calculated. When this minimum is negative, there are points that might fail and the entire configuration is deemed unstable. Hence, regions of stability and instability were determined in the (α, β) space. It was found that the open-pit slope stability depends mainly on the overall slope angle. There are slope angle values β_1 and β_2, with $\beta_1 < \beta_2$, such that for $\beta < \beta_2$ the open-pit slope is stable, for $\beta > \beta_2$ is unstable, and for $\beta_1 \leq \beta \leq \beta_2$ the stability depends on the bench angle α. Furthermore, the stability region size depends on the shear strength S_0: the larger the S_0 value, the larger the β_1 and β_2 angles and the larger the stability region size as well, that is, more stable configurations exist in the (α, β)-plane.

Given the flexibility of the method, more complicated geometries might be studied, with great precision, future work should focus on dropping the axisymmetry to include 3D geometries with variable properties, fractured geomaterials and time-dependence, especially the case of complex blasting sequences and induced seismicity forcing.

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