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Across-Partition Contact Analysis with Domain Decomposition

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Abstract

The integration of the node-to-surface contact element into parallel analysis with across-partition contact boundaries can offer significant reduction in computational demands for large-scale contact simulations. In the context of contact analyses with parallel computation , dual-interface-based domain decomposition methods are adopted in the current work for their considerable flexibility without degradation of accuracy and convergence behaviour. However, nonlinearity in contact problems still poses great difficulties for dual-interface-based domain decomposition methods. In addition, the *a priori* unpredictability of the contact regions and contact states complicate the across-partition contact enforcement. The current work purposed a dual-interface-based domain decomposition method emphasising across-partition contact coupling. A pair of fully decomposed node-to-surface contact element are proposed to discretise the across-partition contact boundaries. The assumption of small incremental displacements is adopted, which a) avoids the excessive coupling between the decomposed master and slave, b) significantly reduces the communication overhead, and c) facilitates a flexible across-partition adaptive analysis.

Keywords: finite element, contact analysis, node-to-segment contact, parallel analysis, domain decomposition method, dual Lagrange multipliers.

1 Introduction

The ever-growing size and complexity of finite element problems involving contact has necessitated the use of parallel computation. The non-overlapping domain decomposition method, on which the parallel strategy of the current work is based, is the prevalent solution method for the parallel realisation of finite element problems. In this method, an originally large domain is physically decomposed into smaller non-overlapping subdomains coupled at the interfaces along their partitioning boundaries. The compatibility and equilibrium conditions between the subdomains are enforced at these interfaces, where the contributions from all associated subdomain boundaries are assembled. Conventionally, only the coupling between the displacement degrees of freedom (DoFs) of a fixed pair of subdomain boundaries is required, such that the definition of the interface remains unchanged during the analysis. However, this approach is associated with several shortcomings when modelling contact between the subdomains.

In parallel contact analyses, especially those involving large displacements, it is often beneficial to allow for contact coupling between the partitioning boundaries of subdomains in order to enable greater flexibility in problem decomposition and hence achieve better load balancing in the parallelisation. However, the inherent nonlinearity and *a priori* unpredictability in contact problems pose some difficulties, which mainly arise from: 1) the search for potential contact regions on the partitioning boundaries; 2) the prediction of the contact state during iteration; and 3) the nonlinear contact coupling between the partitioning boundaries. This paper aims to address the difficulties faced in the latter two processes, and the resulting method provides an adaptable framework for the implementation of general contact search algorithms. In this work, node-to-surface contact elements are adopted for the discretisation of the partitioning boundaries in contact, due to their versatility and widespread use.

Depending on the parameters adopted for the partitioning interface, non-overlapping domain decomposition methods can be mainly classified into three types: primal-interface-based, dual-interface-based, and penalty-method-based. Compared to the primal-interface-based method [1, 2], which enforces direct connections between the displacement fields of the partitioning boundaries and the interface, the dual-interface-based method [3] employs a Lagrangian multiplier (LM) field, which facilitates a flexible alteration of the contact boundaries. Compared to penalty-method-based domain decomposition [4], the dual-interface-based method maintains a high level of accuracy and does not suffer from convergence difficulties. The current work aims to develop an efficient parallel strategy involving node-to-surface contact coupling between the partitioning boundaries of separate subdomains based on the dual-interface-based domain decomposition method.

2 Methods

Figure 1 shows a conventional nonlinear 9-noded node-to-surface contact element involved in large-displacement sliding. Based on the nonlinear kinematic relationship,

the closest projection of the slave node onto the master facet as well as the corresponding normal are functions of the nodal displacements of both the master and slave counterparts. This relationship accordingly leads to a non-zero coupling stiffness between the DoFs of the master facet nodes and the slave node, the result of which is that the independent elimination of boundary displacement DOFs within each subdomain becomes unachievable. In order to bypass this difficulty, an assumption of small incremental displacements is made, specifying that the closest projection of a slave node on a master facet and the corresponding normal do not vary significantly over one time- or load-step. Given this assumption, the normal gap function g_N between the slave node and master facet is:

$$g_N = \boldsymbol{n^p}^T \left(\boldsymbol{X_s} - \bar{\boldsymbol{X}}_{\boldsymbol{m}}(\xi^p, \eta^p) \right) \tag{1}$$

where (ξ^p, η^p) and $\boldsymbol{n^p}$ denote the natural coordinates of the closest projection of the slave node on the master facet and the outward normal vector upon this point, respectively, both of which are evaluated at the last known configuration, whereas $\boldsymbol{X_s}$ and $\bar{\boldsymbol{X}}_{\boldsymbol{m}}(\xi^p, \eta^p)$ denote the global coordinates of the slave nodes and the closest projection, respectively.

A static contact potential energy Π_c can be defined upon (1) to enforce a frictionless contact constraint using the LM method:

$$\Pi_c = \begin{cases} \lambda g_N, & \text{in contact state} \\ \frac{1}{2}\lambda^2, & \text{release state} \end{cases} \tag{2}$$

The LM is treated as an additional DoF. The first and second partial derivatives of Π_c for the in-contact state would give the contact element force vector and tangent stiffness matrix respectively, which can then be respectively assembled into the overall resistance forces vector and stiffness matrix associated with the DoFs on the partitioning boundaries of the coupled subdomains. In the derivatives, both the master and slave entities are only coupled with the LM and not directly with each other. Accordingly, when the node-to-surface contact element is decomposed across the partition into its master and slave counterparts, they will share the LM as the only inter-counterpart DoF, as illustrated in Figure 2.

In each of the subdomains, an in-contact state is initially assumed for all across-partition contact elements. The checking of the contact state can be carried out directly at the partitioning interface after assembling contributions from all subdomain boundaries. If the in-contact state is confirmed, no special treatment is required. If a release state is predicted instead, the LM is restrained to zero, which will accordingly result in zero contact forces at the corresponding nodal DoFs, as required for a release state.

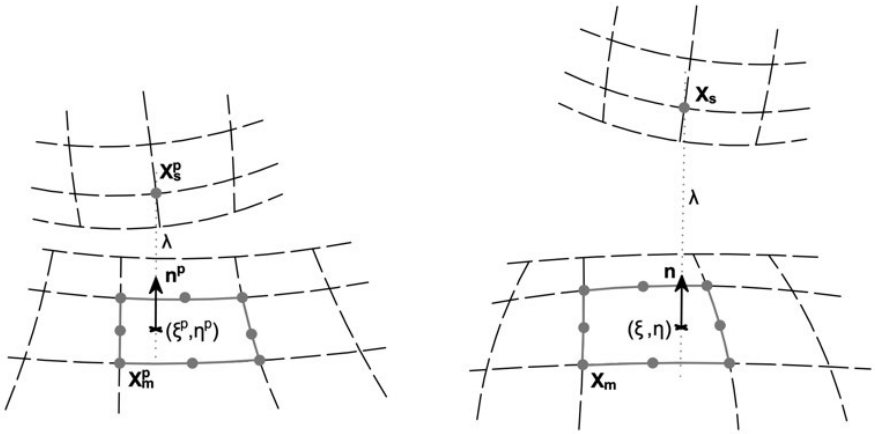

Figure 1: Initial and current configuration of a nonlinear node-to-surface contact element.

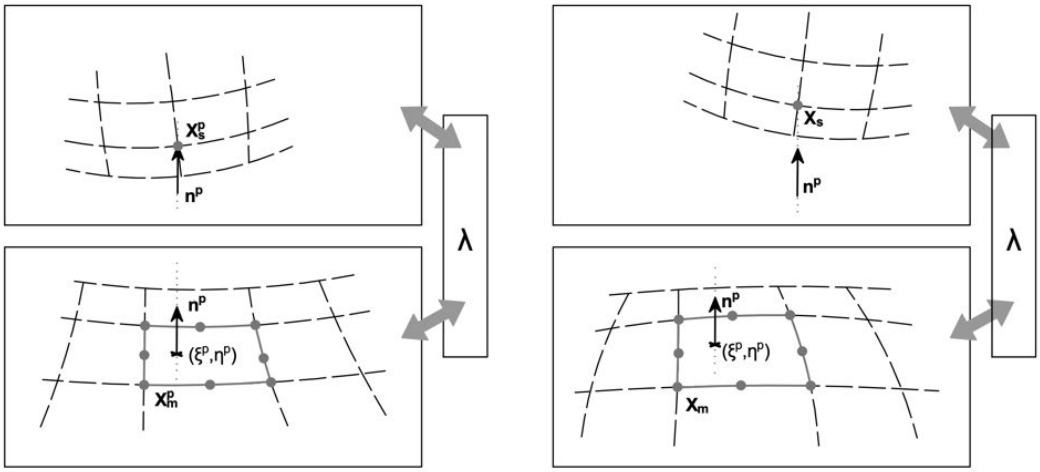

Figure 2: Decomposed slave node and master facet in separate subdomains, coupled via the LM on the partitioning interface.

3 Results

The proposed method is realised in ADAPTIC [5] and is implemented with an adaptive tracking algorithm. An example problem [6] is set up as shown in Figure 3 to assess the proposed parallel contact algorithm. Two half-rings, mirrored vertically with some vertical overlap, approach each other in the horizontal direction and eventually press onto one another. The upper half-ring is designated as the slave body and the lower as the master. A horizontal displacement equal to dx = 6m is applied over 20, 50 and 100 steps for the parallel analysis. The nonlinear monolithic analysis is only run using 20 and 100 steps, with the results of the latter adopted as the benchmark for the calculation of the relative error.

The relative error in strain energy from each analysis is presented in Figure 4. Because this problem involves a continual and significant change of the closest projection and corresponding normal, and because these are updated only at the end

of each incremental step, a notable error is observed in the 20-step parallel analysis, especially during the middle phase of the loading regime. The 100-step parallel analysis has a comparable level of accuracy to the 20-step monolithic analysis. Nevertheless, these results show that the inaccuracies of the parallel analysis also do not accumulate over the loading steps, due to the step-wise updating of the closest projection and normals.

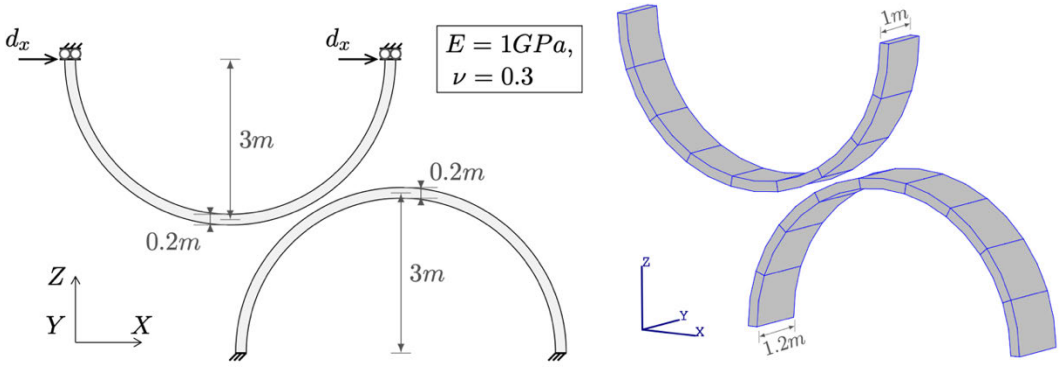

Figure 3: Two half-rings problem.

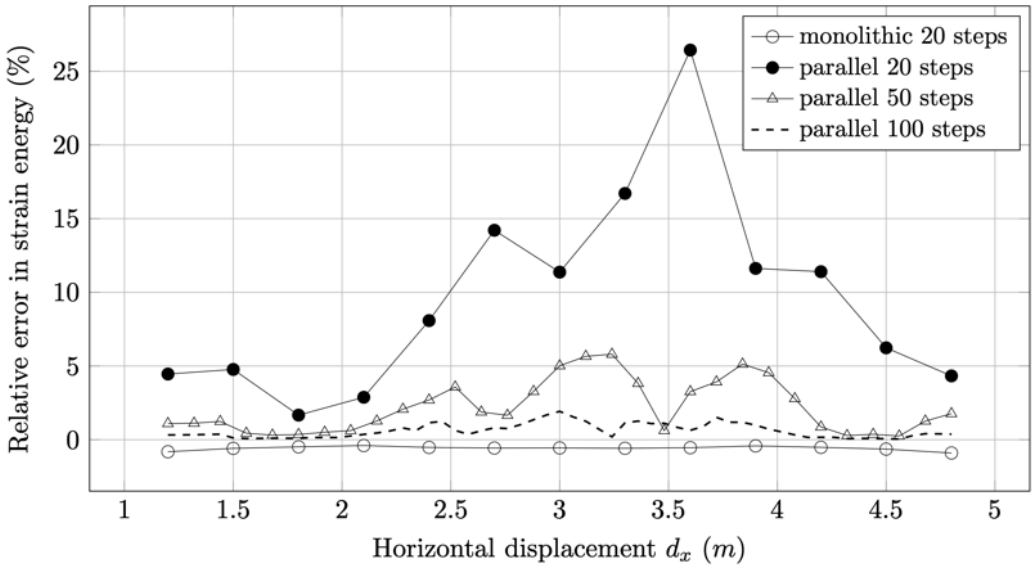

Figure 4: Relative error in strain energy compared to monolithic analysis with 100 load steps.

4 Conclusions and Contributions

The current work proposes a parallel strategy involving node-to-surface contact coupling between the partitioning boundaries of separate subdomains which is developed upon the dual-interface-based domain decomposition method.

An assumption of small incremental displacements is adopted for across-partition contact. This results in a linearised contact formulation, which bypasses the excessive coupling of the displacement DoFs between the slave node and master facet of the contact pair. Both the slave nodes and master facets are directly linked only to the corresponding LMs on the partitioning interface, where the contact state is predicted. This indirect coupling between the slave node and the master facet facilitates a flexible across-partition adaptive analysis. Furthermore, with this assumption, the across-partition transfer of information regarding the relative positions of the master and slave counterparts only needs to occur at the end of each incremental step. Thus, the communication overhead between the parallel processors is significantly reduced.

Because the slave node projection and the corresponding normal are updated at the end of each incremental step, the inaccuracies introduced due to the assumption of small incremental displacements can be reduced by decreasing the step size, and any errors should not be cumulative over the incremental solution procedure provided a sufficiently small step size is adopted. Although the time-saving effect of parallelisation is diminished by the extra requirement of the smaller step size, for analysis which inherently requires small incremental steps, such as dynamic analysis, no further constraint on the step size is required and a considerable improvement in computational efficiency is expected. Therefore, the proposed method is effective for large-scale contact problems requiring parallel analysis, especially for dynamic problems.

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