



Proceedings of the Eleventh International Conference on
Engineering Computational Technology
Edited by B.H.V. Topping and P. Iványi
Civil-Comp Conferences, Volume 2, Paper 6.4
Civil-Comp Press, Edinburgh, United Kingdom, 2022, doi: 10.4203/ccc.2.6.4
©Civil-Comp Ltd, Edinburgh, UK, 2022

Big-data based concrete mix proportion optimization and application development

B.Chen^{1,2}, H.Zhou³, D.C. Xia⁴, and S.H. Huang^{1,2}

- ¹ **Zhejiang University of Water Resources and Electric Power, Hangzhou, China**
- ² **Key Laboratory for Technology in Rural Water Management of Zhejiang Province, Hangzhou, China**
- ³ **Zhejiang Institute of Hydraulics & Estuary, Hangzhou, China**
- ⁴ **China No. 12 Water Conservancy and Water Power Engineering Bureau, Hangzhou, China**

Abstract

Based on big data technology, according to the nonlinear relationship between mix proportion and performance of concrete with multi cementitious materials, the mix proportion optimization method of concrete with multi cementitious materials is proposed. Firstly, 1443 sets of mixed samples were collected for correlation analysis, and the prediction abilities of linear regression, BP artificial neural network and support vector machine (SVM) were compared. The prediction model of concrete strength and workability based on support vector machine was selected. Secondly, the nonlinear optimization model of concrete mix proportion is established by using particle swarm optimization (PSO) algorithm and artificial bee colony algorithm (ABC). Finally, a series of concrete mix proportions are designed and tested to verify the effectiveness of the method. Furthermore, the concrete quality and cost control system (Compos) is developed to facilitate the application of this method.

Keywords: concrete; mixture optimization, artificial neural network, support vector machine, particle swarm optimization, artificial bee colony algorithm.

1 Introduction

Concrete is a relatively complex material which is composed of cement, water, sand, gravel and / or other raw materials. The best concrete mixture should be the lowest

cost but meet all performance requirements. For simple four component concrete, Bolomey rule has been widely used, and it shows that the ratio of cement to water(C/W) is directly proportional to the compressive strength of concrete. Other concrete properties, such as workability and durability, are mainly adjusted through experience and test. With the continuous development of concrete technology over the past decades, the composition of concrete materials has become increasingly complicated. Many types of admixtures and additives, such as fly ash, slag, silica powder, water-reducing agent, and air-entraining agent, are introduced. Such introduction makes a mixture design to be increasingly complex, and the defects of the traditional method are gradually becoming apparent.

At present, in the mix design of concrete with multi cementitious materials, the main strategy is to improve the traditional methods, such as ACI211.1-91 (R2009) and JGJ55-2011. Some other methods have been proposed, such as the mix design method of high performance concrete recommended by P.K.Mehta and P.C.Aïtcin [1], which is believed that when the volume ratio of cement slurry to aggregate is 35:65, the concrete will achieve the best working performance and mechanical properties, and the recommended mix design procedure is put forward. The Total Calculation Method, recommended by J.K.Chen and D.M.Wang[2], further establishes the relationship between cement slurry volume and water binder ratio, cement slurry volume and aggregate volume. On this basis, the water consumption and sand production rate are calculated to complete the mix design.

The absolute volume method is another mixed proportion design method with considerable use. H.T.Le[3] designed a self-compacting HPC mixture through a method based on Funk–Dinger tight packing theory.

All of the above methods are empirical experimental methods with certain theoretical assumptions, and the results are relatively feasible, but not ideal. By introducing system analysis and mathematical programming into concrete mix proportion design, the optimal mix proportion in mathematics can be obtained. In this respect, J.P.Cannon and G.P.Krishna Murti[4] first used the simplex method of linear programming, which opened up a new way for mix proportion design. With the increasing types of raw materials, B.Chen[5] found that there was a nonlinear relationship among raw materials, mix proportion and concrete performance, through the correlation analysis of 1078 sets of mix proportion trial mixing records. Therefore, a multi-objective optimization model of concrete mix proportion was established by using stepwise regression analysis and complex method.

In recent years, with the rapid development of big data and artificial intelligence technology, it provides new ways and opportunities for theoretical research and solving engineering problems. The so-called big data refers to the research method that does not use the shortcut of random analysis (sampling survey), but uses all data for analysis and processing. It has the characteristics of 5V, namely volume (large), velocity (high), variety (diversity), value (low value density) and veracity (authenticity). Its statistical results are more accurate and closer to the facts, which provide a new way for the scientific research of concrete. Y.C.Yeh[6] established a

concrete mix proportion optimization model based on nonlinear programming and genetic algorithm. A.Habibi and J.Ghomashi[7] developed the optimal mixture design method of self-compacting concrete by using sequential quadratic programming. E.M.Golafshania and A.Behnood[8] used biogeography-based programming estimating the optimal mix design of silica fume concrete. Z.N.Amin[9] proposed a dynamic cost optimization method for concrete mix design. The feasibility of intelligent optimization of concrete mix proportion is verified.

2 Prediction model of concrete performance

An accurate prediction model of concrete performance is necessary to optimize the concrete mix proportion. Most prediction models are based on the regression analysis of experimental data. In the past 20 years, the author has collected 7002 sets of the concrete mixture records from numerous construction projects in different regions and compared various prediction methods, including linear regression analysis, nonlinear stepwise regression analysis, and ANN model. The SVM method is determined to have the highest accuracy and efficiency (S.M.Gupta[10], B.Chen[11]). The following subsection is a brief introduction to this method.

2.1 Statistical Learning Theory (SLT) and Support Vector Regression (SVR)

For a regression problem, the data can be divided into two parts, namely, training and verification sets. For the training sample set,

$$D = \{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}, y_i \in R$$

The mathematical essence of its fitting (modelling) is to select an appropriate function $f(x)$ from the function set, thereby minimizing the value of the risk function, that is,

$$R[f] = \int (y - f(x))^2 P(x, y) dx dy \quad (1)$$

However, the probability distribution function $P(x, y)$ is unknown; thus, $R(f)$ cannot be directly calculated. Traditional statistical mathematics assumes that the abovementioned risk function can be replaced with empirical risk function $R_{emp}[f]$, that is,

$$R_{emp}[f] = \frac{1}{n} \sum_{i=1}^n (y - f(x_i))^2 \quad (2)$$

According to the law of large numbers, Formula (2) can only be established when the sample number n tends to be infinite, and the set of functions is sufficiently small. The minimum fitting error in terms of least squares is considered the optimum criterion of modeling. Therefore, the prediction capability of the algorithm with fitting capability is not strong but weak due to overfitting, as depicted in Figure 1. This algorithm is a fitting/prediction process using ANN, with an error-tracking strategy.

To solve this problem, SLT replaces $R_{emp}[f]$ with $R_h[f]$, (called “structural risk function”), and $R_h[f]$ can be minimized using the following functions:

$$\min_{S_h} \left\{ R_{emp}[f] + \sqrt{\frac{h[\ln(2n/h)+1]-\ln(\delta/4)}{n}} \right\} \quad (3)$$

where n is the number of training samples; S_h is the VC dimension space structure; and h is the dimension of the VC space, which is the measurement of the complexity of function set or learning capability. $1 - \delta$ is a parameter to characterize calculation reliability.

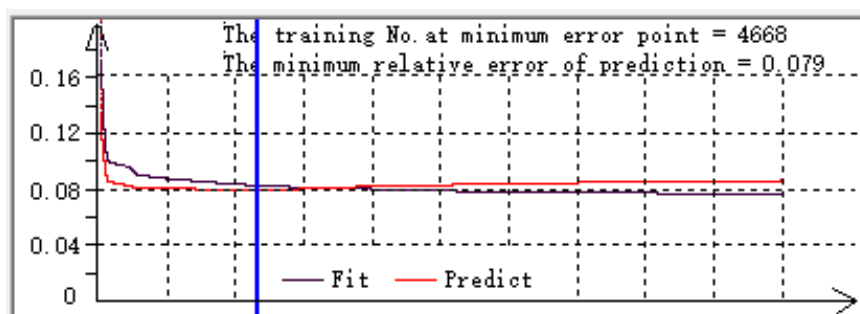


Figure 1: Overfitting of concrete performance

SLT requires the pursuit of fitting accuracy on the premise of controlling the upper bound of fitting capability marked by the VC dimension (to limit overfitting). Three methods are used to control the VC dimension, as presented as follows:

- (1) Enlarging the interval between two types of sample points in a feature space.
- (2) Reducing the distribution range of the two types of sample points in the feature space.
- (3) Reducing the dimensions of the feature space.

The third method is the only means of control overfitting, but the new theory emphasizes that the first two methods can ensure that the operation of a high-dimensional feature space still has low VC dimension, thus ensuring that the overfitting is restricted.

SVM is an implementation method of the abovementioned SLT. The basic idea is to map the sample space to a high-dimensional feature space by a nonlinear mapping. The algorithm for finding the optimal linear regression hyperplane is reduced to solve a convex programming problem with convex constraints, and the global optimal solution can be obtained. Simultaneously, the SVM model transforms the inner product operation in a high-dimensional space into the kernel function operation in the original space by defining the kernel function. Let the sample set be

$$D = \{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}, y_i \in R$$

The regression function is expressed as the following linear equation:

$$y(x) = (w \cdot \phi(x)) + b \quad (4)$$

where w is the weight vector, b is the bias term, and $\phi(x)$ is the nonlinear mapping from the input space to the output space. All training samples are fitted with linear functions without error under the accuracy of ε . Considering the existence of allowable fitting errors, two non-negative relaxation variables ξ_i and ξ_i^* are introduced, and the constraints are presented as follows:

$$\begin{cases} y_i - w\phi(x_i) - b \leq \varepsilon + \xi_i \\ \phi(x_i)w + b - y_i \leq \varepsilon + \xi_i^* \\ i = 1, 2, \dots, n \end{cases}$$

The Lagrange multiplier method is used to solve the abovementioned programming problems. The optimal regression function is obtained by finding the minimum extremum of the following functions.

$$\phi(w, \xi_i^*, \xi_i) = \frac{1}{2} \|w\|^2 + C(\sum_{i=1}^l \xi_i + \sum_{i=1}^l \xi_i^*),$$

where C is the set point for the penalty factor, which controls the degree of penalty for samples exceeding error ε .

Using duality theory, the abovementioned optimization problem can be transformed into

$$\begin{aligned} \max & \left\{ -\frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*) K(x_i, x_j) - \varepsilon \sum_{i=1}^n (\alpha_i - \alpha_i^*) + \sum_{i=1}^n y_i (\alpha_i - \alpha_i^*) \right\} \\ \text{s. t. } & \sum_{i=1}^n (\alpha_i - \alpha_i^*) = 0 \quad (\alpha_i, \alpha_i^* \in [0, c]) \end{aligned} \quad (5)$$

where $K(x_i, x_j) = \phi(x_i)\phi(x_j)$ is the kernel function. The undetermined Lagrange coefficients, α_i and α_i^* , can be obtained by solving Formula (5) with SMO algorithm, and the regression function $f(x)$ is

$$f(x) = \sum_{i=1}^n (\alpha_i - \alpha_i^*) K(x_i, x) + b$$

In the SVR application, selecting a kernel function significantly influences the regression results. Four types of kernels, which are related to the existing algorithms, are commonly used.

- (1) Linear kernel function, that is, $K(x_1, x_2) = (x_1, x_2)$.
- (2) Polynomial kernel function, that is, $K(x, x_i) = [(x^T x_i) + 1]^q$.
- (3) Radial basis kernel function (RBF), that is, $K(x, x_i) = \exp\left\{-\frac{\|x-x_i\|^2}{\sigma^2}\right\}$.
- (4) Sigmoid kernel function, such as $K(x, x_i) = \tanh(v(x^T x_i) + c)$.

For different applications, appropriate kernels must be selected. However, for most problems, the RBF kernels have improved fitting and prediction results and relatively high computing speed. When SVM is used in classification and regression modeling, selecting penalty factor C and parameters of kernel function become the key, which can typically be selected through a certain scope of cross validation.

Further questions on SVM were discussed by N.V.Vladimi [12].

2.2 Fitting results for 7- and 28-day cube compressive strength and initial slump of concrete using SVM

A total of 1443 sets of records (specimens) are selected from a database of the concrete mixture, which contains raw materials limited to water, cement, natural sand, gravel, fly ash, slag, and water reducer. The test results of concrete performance include at least 7-day strength, 28-day strength, and initial slump. Among them, 722 sets are used for training, and the other 721 sets are used for prediction.

On the basis of numerous trial calculations of the author and the introduction of related literature, the RBF is adopted. The model parameters are selected by a fivefold cross validation. The search interval of parameters is 2–5–25, and the step size is 1. That is, the fitting samples are divided into five equal parts; among them, four equal parts are selected for fitting, and the fifth equal part is used for prediction. This process rotates, and then the optimal C value and kernel function parameters are selected. The predicting group uses the abovementioned parameters for prediction.

Figure 2 presents the values of 28-day compressive strengths predicted using the SVM model versus the experimental results for training and predicting datasets. In this figure, the data points are sufficiently concentrated.

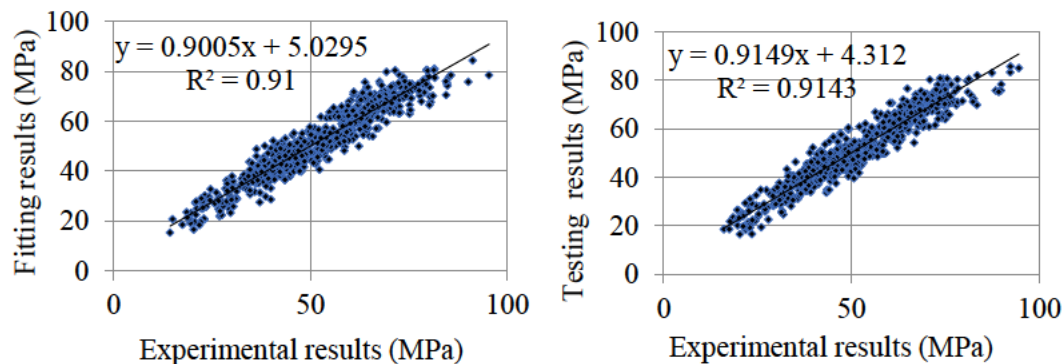


Figure 2: Predicted values of the SVM model versus experimental results for training and testing data

All the fitting results of concrete performance are demonstrated in Figure 3–Figure 5, and the prediction results are exhibited in Figure 6–Figure 8. A comparison of the fitting and prediction results of linear regression, ANN, and SVM is summarized in Table 1.

In Figure 3–Figure 8 and Table 1, the following conclusions can be drawn:

(1) The prediction accuracy is clearly higher in the nonlinear model than in the linear model.

ANN and SVM demonstrate relatively stronger fitting capability and higher prediction accuracy than the other methods. In fact, the SVM has the fastest computing speed and highest stability.

(2) The prediction accuracy is higher in of strength than in slump, and the prediction accuracy of 28-day compressive strength is the highest. The average relative error of 28-day strength is less than 8%, which can fully satisfy the accuracy requirement of the mix design.

The prediction error of slump is relatively high, which can be adjusted by changing the variety and dosage of additives.

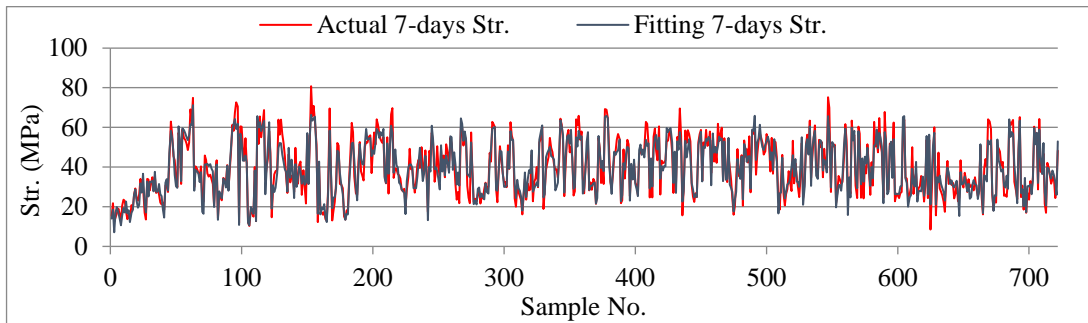


Figure 3: Fitting results of concrete 7-day Str.

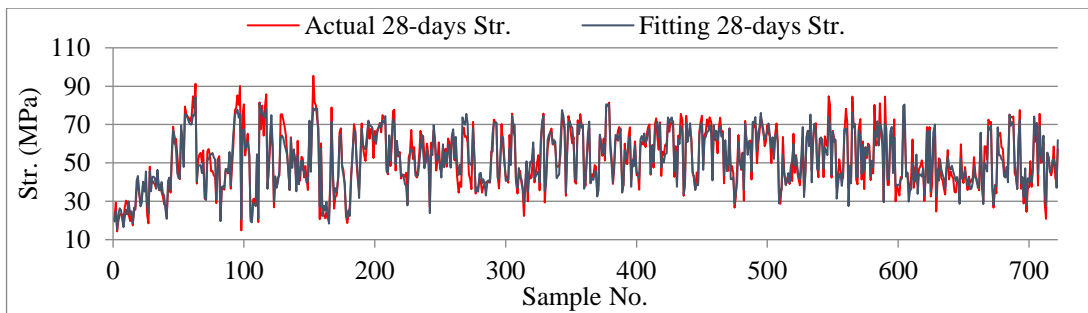


Figure 4: Fitting results of concrete 28-day Str.

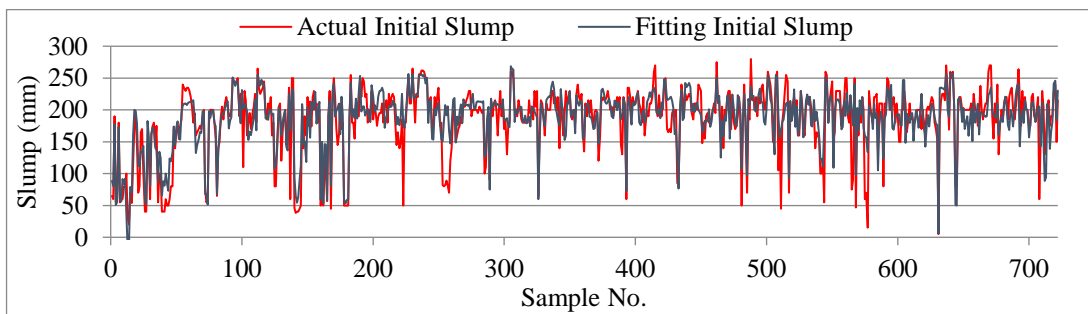


Figure 5: Fitting results of concrete initial slump

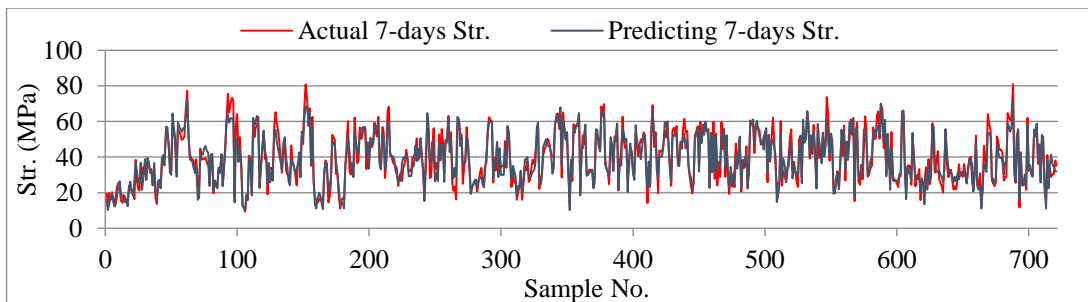


Figure 6: Predicting results of concrete 7-day Str.

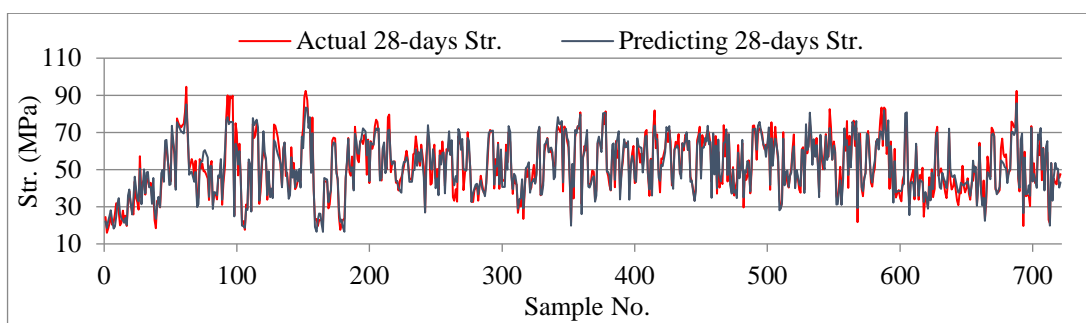


Figure 7: Predicting results of concrete 28-day Str.

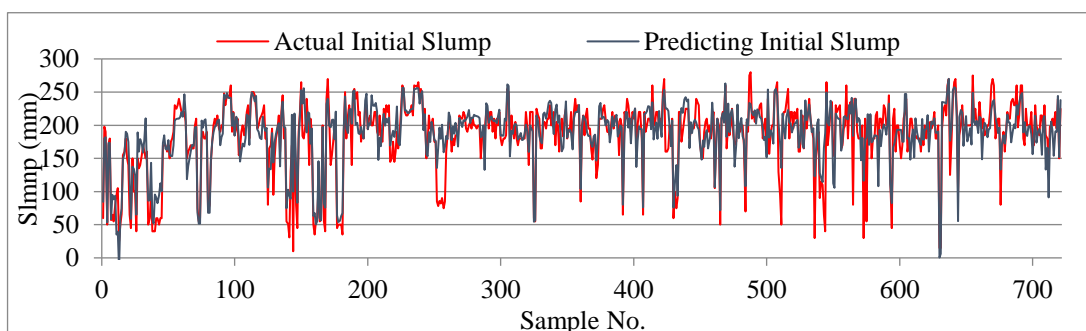


Figure 8: Predicting results of concrete initial slump

Concrete performance	Linear regression		BP ANN		SVM	
	Fitting	Predicting	Fitting	Predicting	Fitting	Predicting
7-day Str.	11.65	11.01	10.1	10.06	10.71	10.42
28-day Str.	7.91	7.81	7.49	7.61	7.62	7.43
Initial slump	19.86	20.34	18.22	17.95	17	17.14

(The verified influencing factors of concrete strength are 3-day and 28-day strengths of cement, maximum particle size and crushing index of gravel, water requirement ratio of fly ash, 28-day activity index of slag, and air content of concrete. The verified influencing factors of an initial slump are sand fineness modulus, maximum particle size of gravel, water requirement ratio of fly ash, 7-day activity index of slag, and air content of concrete.)

Table 1: Contrast of fitting and predicting mean relative errors using linear regression, ANN and SVM(%)

3 Optimizing model of concrete mix proportion

Several methods, such as Monte Carlo techniques and Genetic algorithm (GA), can be used for nonlinear optimization of the concrete mixture. In recent years, some new AI methods have been proposed, such as the Particle Swarm Optimization (PSO, recommended by J.Kennedy and R.Eberhar[13]) and the Artificial Bee Colony algorithm(ABC, recommended by D.Karaboga[14]), which may have stronger global search capability and higher parameter stability. The following is a brief introduction of PSO.

3.1 Particle swarm optimization (PSO)

PSO simulates the behavior of bird flocking. The following scenario is assumed: a group of birds is randomly searching for food in an area. Only one piece of food in the area is being searched. All the birds do not know the location of the food. However, these birds know the distance of the food in each iteration. Therefore, the optimal strategy for locating the food must be determined. The effective strategy is to follow the bird that is nearest to the food.

PSO learns from the scenario and uses it to solve the optimization problems. In PSO, every single solution is a “bird” in the search space, called “particle.” All of the particles have fitness values, which are evaluated by the fitness function to be optimized and have velocities that direct the flying of the particles. The particles fly through the problem space by following the current optimum particles.

PSO is initialized with a group of random particles (solutions) and then searches for optima by updating generations. However, in contrast to GA, PSO has no evolution operators, such as crossover and mutation. In every iteration, each particle is updated by following two “best” values. The first one is the best solution (fitness) it has achieved thus far (The fitness value is also stored). This value is called p_{best} . Another “best” value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called g_{best} . When a particle takes part in the population as its topological neighbors, the best value is a local best and is called l_{best} .

After finding the two best values, the particle updates its velocity and positions with the following equations:

$$v[] = wv[] + c_1 \text{rand}_1() (p_{best}[] - present[]) + c_2 \text{rand}_2() (g_{best}[] - present[]) \quad (6a)$$

$$present[] = present[] + v[] \quad (6b)$$

where w is the inertia weight, $v[]$ is the particle velocity, and $present[]$ is the current particle (solution). p_{best} and g_{best} are defined as stated before. $\text{rand}()$ is a random number between (0,1). c_1 and c_2 are learning factors (also called acceleration constant), typically $c_1 = c_2 = 2$.

When we examine Formula (1), we find that its right side is composed of three parts. The first part is inertia or momentum, which reflects the movement habits of particles and represents the tendency of particles to maintain their previous velocities. The second part is cognition, which reflects the memory or remembrance of particles’ own historical experience and represents the tendency of particles to approach their best position in history. The third part is social, which reflects the group historical experience of cooperation and knowledge sharing among particles and represents the trend of particles approaching the optimal historical position of groups or neighbourhoods.

3.2 Optimization model of concrete mix proportion based on PSO

The PSO-based optimization process of concrete mix proportion is presented as follows:

Step1: Initialize particle swarm, including population size N , initial position X_i , and velocity V_i of each particle. For example, the population is set to 20 birds (concrete mixtures) that are chasing food. The initial positions of the 20 birds (mixtures), which are selected by random casting points, satisfy all design requirements of concrete performance. X_i and V_i are two D -dimensional vectors:

$$X_i = (x_{i1}, x_{i2}, \dots, x_{iD}), i = 1, 2, \dots, N$$

$$V_i = (v_{i1}, v_{i2}, \dots, v_{iD}), i = 1, 2, \dots, N$$

where x_{i1} to x_{iD} are the dosages of raw materials in per unit of concrete.

Step2: Calculate the fitness of each particle. The lowest unit cost of concrete is typically used as the optimization objective. The unit price of various raw materials is C

$$C = (c_1, c_2, \dots, c_D)$$

Accordingly,

$$F_{it}[i] = X_i \cdot C$$

Step3: For each particle, compare its fitness value $F_{it}[i]$ with the individual optimal value $p_{best}(i)$. If $F_{it}[i] > p_{best}(i)$, then replace $p_{best}(i)$ with $F_{it}[i]$.

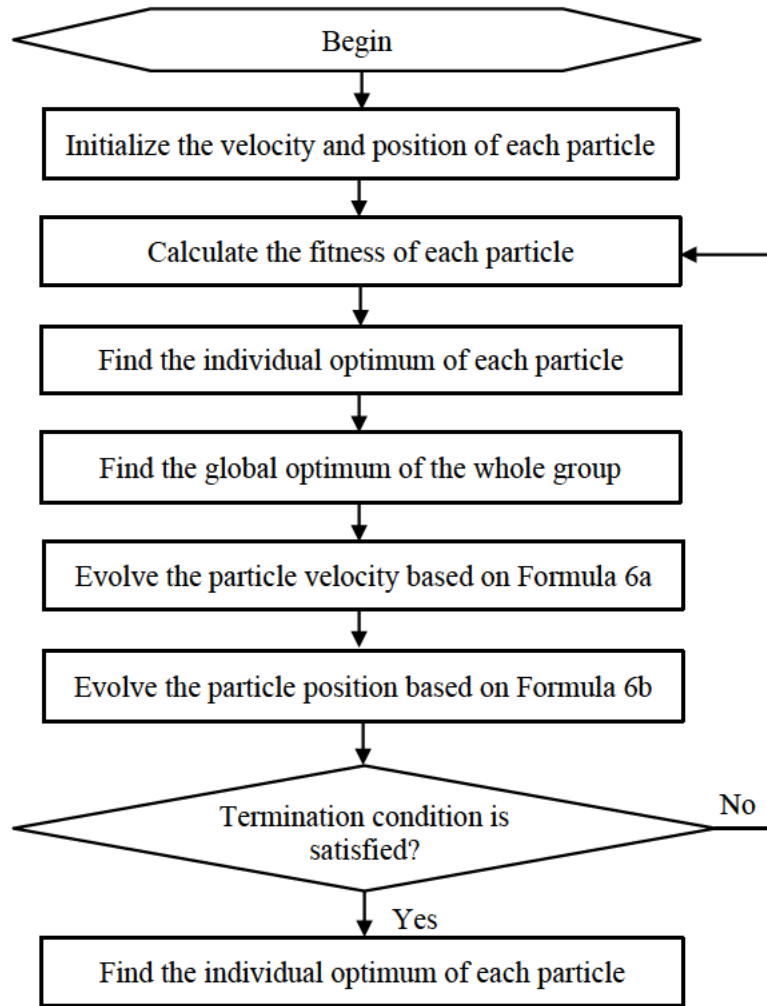


Figure 9: PSO-based concrete mix proportion optimization processes

Step4: For each particle, compare its fitness value $F_{it}[i]$ with the global optimal value $g_{best}(i)$. If $F_{it}[i] > g_{best}(i)$, then replace $g_{best}(i)$ with $F_{it}[i]$.

Step5: Update the velocity V_i and position X_i of particles in accordance with Formulas 6-a and 6-b.

Step6: If the end condition is satisfied (the error is adequately small or the maximum number of cycles is reached), then end the process; otherwise, return to Step2.

The flowchart of the PSO-based concrete mix optimization algorithm is illustrated in Figure 9.

4. Experimental verification

The test objective is to verify whether the strength and slump of the optimized mix proportion meet the design requirements.

A precast member factory designed a series of concrete mix proportion, the design strength grades are C25, C30, C35, and C40. The 7-day strength of concrete shall not be less than 75% of the 28-day design strength, and the initial slump of concrete shall be greater than 170 mm.

The raw materials used in this experiment are as follows:

Cement: ordinary Portland cement, 3-day flexural and compressive strength are 5.2MPa and 23.5MPa respectively; 28- day flexural and compressive strength are 8.1mpa and 49.3MPa, 315 yuan/ton. Fine aggregate: natural sand with fineness modulus of 3.39, 70 yuan/ton. Coarse aggregate: 5-40mm continuous graded crushed stone with crushing value of 7.5, 58 yuan/ton. Fly ash: water demand rate is 103%, 110 yuan/ton. Mineral powder: 7 days activity index is 84.2%; 28 days activity index is 98.5%, 270 yuan/ton. Water reducing agent: water reducing rate is 17.8%, solid content is 30.7%, 4850 yuan/ton. In this test, a total of 24 concrete strength test blocks of C25, C30, C35, and C40 were produced, which were used in the experiment.

The optimization results are shown in Table 2 and Table 3.

The concrete test blocks of each strength grade have be made according to the mix proportion in Table 4. The manufacturing process of the test block is shown in Figure 10 and Figure 11.



Figure 10: Concrete test block



Figure11: Concrete compression test

f_{cu}	Material consumption (kg)							Concrete performance			Price (CNY)
	Cement	Water	Dand	Gravel	Fly ash	Slag	Water reducer	Slump	7-day intensity	28-day intensity	
C25	256	186	712	1075	118	20	3.9	190	24.9	37.1	222.0
C30	276	186	683	1072	128	21	4.3	195	28.7	41.5	229.3
C35	316	186	664	1065	136	0	4.5	195	32.4	45.1	236.5
C40	338	186	664	1065	74	44	4.6	190	37.4	49.9	248.8

Table 2: Optimal concrete mix proportion using linear optimization algorithm colony algorithm

f_{cu}	Material consumption (kg)							Concrete performance			Price (CNY)
	Cement	Water	Sand	Gravel	Fly ash	Slag	Water reducer	Slump	7-day intensity	28-day intensity	
C25	207	183	796	992	113	56	3.8	220	24.9	36.4	216.8
C30	241	174	732	1034	123	48	4.2	220	28.6	41.3	225.8
C35	275	182	720	1015	129	26	4.3	220	32.4	44.7	230.3
C40	273	182	686	1043	142	59	4.7	220	37.4	50.5	240.8

Table 3: Optimized concrete mix proportion using support vector machine and artificial bee

The test results are shown in Table 4. It can be seen from Table 4 that there are errors between the concrete strength test and prediction results, and the errors are close to the average error range calculated in Table 1 and meet the requirements of concrete strength.

According to the contrast of test and model prediction results, it can be find that most of the model fitting and prediction results of concrete mechanical properties are better, which can reduce the time and cost for concrete production.

f_{cu}	7-day intensity /MPa	Actual value /MPa	relative error /%	28-day intensity /MPa	Actual value /MPa	relative error /%	design Slump / mm	Actual value /mm	relative error /%
C25	24.9	21.3	-14.4	36.4	30	-17.6	220	170	-36.3
C30	29.3	27.5	-6.1	41.2	38.6	-6.3	220	170	-36.4
C35	32.4	28.1	-13.3	44.7	39.5	-11.6	220	170	-40.9
C40	37.4	35	-6.4	50.5	50.8	0.6	220	170	-40.9

Table 4: The test results for optimized concrete mix proportion

5. Conclusions

In this research, the concrete mixture optimization method based on big data and artificial intelligence is presented and validated. The experimental verification shows that this method is effective and reliable.

An important advantage of the optimization method is that all the concrete design indexes, including different performances at various ages, such as flexural strength and durability, can be listed as limitations of the optimization model provided that the prediction accuracy is adequate.

The author has also developed a concrete quality and cost control system (Compos), as demonstrated in Figure 12, which includes several other functions in addition to mix proportion optimization, such as hydration heat calculation and temperature field analysis.

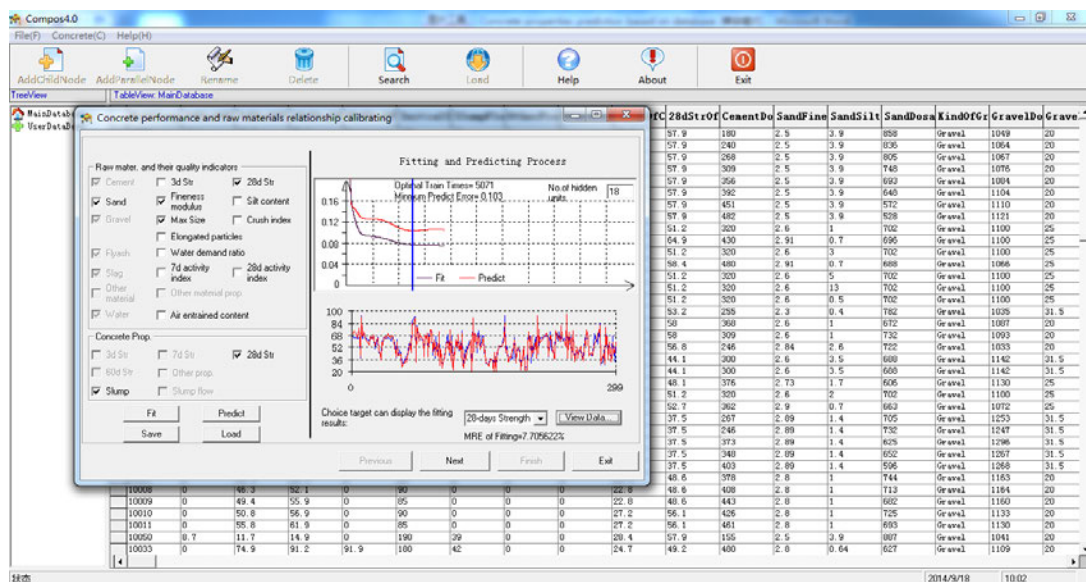


Figure 12: Interface of concrete quality and cost control system (Compos)

The production of concrete is a relatively extensive process, and the performances of concrete are affected by many factors, such as raw materials' quality, mixture, and production mode curing condition. Nevertheless, given that the prediction model is sufficiently accurate, the optimization method can be used. Therefore, the key is still to obtain the prediction model with sufficient accuracy. In this research, the average prediction error of strength is more than 7%, and the prediction error of the initial slump is even more than 15%. A large error indicates a significant safety factor that is required in the mix proportion design. Other influencing factors, such as mixing method, mixing time, and concrete curing condition, should be considered to improve the prediction accuracy of concrete performance[15]. Adequate mixture samples are necessary, and these samples must cover a wide range of materials' dosage and qualities. The distribution of concrete properties is equally important. For example, concrete strength must be evenly distributed over the 10–100 MPa range; in particular, it must not be concentrated only on a relatively narrow band.

References

- [1] P.K. Mehta, P.C. Aïtcin, “Principles underlying production of high-performance concrete”, *Cement Concrete & Aggregates*, 12(2), 70-78, 1990.
- [2] J.Chen, D.M.Wang, “New mix design method for HPC-Overall calculation method”, *Journal of the Chinese Ceramic Society*, 28(2), 194-198, 2000.
- [3] H.T. Le, M.Müller, K.Siewert, H.M.Ludwig, “The mix design for self-compacting high performance concrete containing various mineral admixtures.” *Materials & Design*, 72(26), 51-62, 2015.
- [4] J.P. Cannon, G.P.Krishna Murti, “Concrete optimized mix proportioning.” *Cement and Concrete Research*, 1(4), 353-366, 1971.
- [5] B.Chen, Q.Mao, J.Q. Gao, Z.Y.Hu, “Concrete properties prediction based on database.” *Computers and concrete*, 16(3), 343-356, 2015.
- [6] I.C.Yeh, “Computer-aided design for optimum concrete mixtures.” *Cement & Concrete Composites*, 29(3), 193-202, 2007.
- [7] A.Habibi, J.Ghomashi, “Development of an optimum mix design method for self-compacting concrete based on experimental results.” *Construction and Building Materials*, 168, 113–123, 2018.
- [8] E.M.Golafshania, A.Behnood, “Estimating the optimal mix design of silica fume concrete using biogeography-based programming.” *Cement and Concrete Composites*, 96 , 95-105, 2019.
- [9] Z.N.Amin, G.R.Tadayonfar, E.N.Hamid, “Dynamic Cost Optimization Method of Concrete Mix Design.” *Materials Today: Proceedings*, 5, 4669–4677,2018.
- [10] S.M.Gupta, “Support Vector Machines based Modeling of Concrete Strength.” *International Journal of Electrical and Computer Engineering*, 3(5), 312 - 318,2008.
- [11] B.Chen, X.M.Guo, G.H.Liu, “Prediction of concrete properties based on rough sets and support vector machine method.” *Journal of Hydroelectric Engineering*, 30(6), 251-257, 2011.
- [12] N.V.Vladimi, “The Nature of Statistical Learning Theory”, 2nd ed. New Yark: Spring-Verlag, 1999.
- [13] J.Kennedy, R.Eberhar, “Particle Swarm Optimization.” *Proceedings of IEEE International Conference on Neural Networks*, Vol.4, 1942-1948, 1995.
- [14] D.Karaboga, “An idea based on honey bee swarm for numerical optimization.” *Technical Report TR06*, Erciyes University, Engineering Faculty, Computer Engineering Department, 2005.
- [15] V.L.Mirian, L.O.Juan, M.L.Isabel, V.B.Pablo, “Analytical and genetic programming model of compressive strength of eco concretes by NDT according to curing temperature.” *Construction & Building Materials*, 144(30), 195-206, 2017.