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Braced twist-free Cubic Framework characterized as tensegrities structures

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Abstract

Bar and joint frameworks have served as models of the engineering structures. The rigidity of bar-joint structures has received much attention in geometry, mathematics, engineering, and material science. The primary purpose is to find an efficient algorithm for deciding infinitesimal rigidity in cable braced three-dimensional Skeletal framework of Cubic Structure. Using the bar-joint structure's symmetry to determine the rigidity is a problem of long-standing interest in kinematics, statics, and optimization. The algorithm has applications in robotics as an actuator-controlled mechanism and in material science as meta-materials and reconfigurable materials. The rigidity and mobility of bar and joint framework have served as valuable models of the structure of metals, crystal states of matter, and biological systems. The simple preliminary analysis gives input to a more complicated consideration of a three-dimensional frame structure than the mentioned disciplines in building science. Lattices, as repetitive objects, are helpful as preliminary structures of design. We considered the Cubic tiling as a bar-joint framework (the edges correspond to bars, the vertices correspond to ball joints). They are mechanisms. Applying some further bracing elements such as Cable, Strut, or Rod (CSR) along the diagonals of the faces of the Twist-free Cubic Tiling Framework (TCTF), the framework could move (disregarding the rigid-body like motions) or will be rigid. We give a characterization of the rigidity of TCTF using CSR bracing elements. We present a description of the system's motions if we insert CSR bracing elements. The given model describes the rigidity of the repetitive frameworks that produce smaller dimensional tensegrity structures as a configuration space that are solvable efficiently as the original structure.

Keywords: bar-joint framework, cable braced structure, infinitesimal rigidity, configuration space, tensegrities, optimization, computation complexity

1 Introduction

Bar and joint frameworks have served as valuable models of the engineering structures [1-8]. The rigidity of bar-joint structures has received much attention in geometry, mathematics, engineering, and material science [9-18]. The symmetry of the bar-joint structure to determine its rigidity is a problem of long-standing interest in kinematics, statics, and optimization [1-18]. The rigidity and mobility of bar and joint framework have served as models of the structure of metals, crystal states of matter [5], and biological systems [19]. Lattices, and tesselations as repetitive objects, are helpful as preliminary design structures [1-4, 6, 7, 9-13, 17, 18]. The analysis of simple initial consideration gives valuable input for a more detailed review of a three-dimensional frame structure besides the mentioned disciplines naturally in building science [1-7, 9-13, 15-18]. Some conditions will be presented for the rigidity of the Cubic framework by applying diagonals as bracing elements. The primary purpose is to find an efficient algorithm for deciding infinitesimal rigidity in cable braced three-dimensional Skeletal framework of Cubic Structure. The algorithm has applications in robotics as some actuator-controlled mechanism and in material science as meta-materials and reconfigurable materials.

We considered the Cubic tiling as a bar-joint framework (the edges correspond to bars, the vertices correspond to ball joints). They are mechanisms not with only pure shear movement; in this paper, we consider only shear motion as in [6,7, 16-18], disregarding the twisting that generally is implied by some further constraint as in [20-23]. Applying some bracing elements such as Cable, Strut, or Rod (CSR) along the diagonals of the faces of the Twist-free Cubic Tiling Framework (TCTF), the framework could move (disregarding the rigid-body like motions) or will be rigid. We give a reasonable assumption for the rigidity of TCTF using CSR bracing elements and using CSR as bracing elements as described in [24-28]. We present a good characterization of the mobility or just the infinitesimal motion of the TCTF if we insert some CSR diagonals of its faces as bracing elements.

The given model describes the rigidity and motions of the repetitive cubic frameworks that produce low complexity problems that are solvable more efficiently than the original structures.

2 Methods

2.1 Characterization of the stiffness of the CSR braced TCTF.

Definition 2.1: A framework is rigid if any continuous motion of the joints that keeps the length of every bar fixed also keeps the distance fixed between every pair of joints. The concept of rigidity and infinitesimal rigidity are closely related. In 3-dimensional space, the infinitesimal motion of a bar-joint framework $F(p(t))$ is mapped with respect to time.

Generally, the rigidity of a given *F(p)* can be computed as the rank of its rigidity matrix that describes the connections among the structure joints [3, 5, 27]. The complexity of this computing is $O(n^9)$ using Gaussian elimination. We consider a better characterization that decreases the complexity of computing.
Defining an equivalence relation between the bars of the Special Cubic Tiling: Let the opposite bars of the same square of a cube be equivalent. The bars in the same equivalence class Ei are parallel to vector $\boldsymbol{v_i}$, representing this equivalence class, as shown in Figure 1. We can see four equivalence classes, and the bars in the same class are colored with the same color as their representation vectors. The bars move parallel to each other and parallel to $\boldsymbol{v_i}$ if they can move at al. We distinguish the bars as bracing elements from the constituent elements as bars of the structure (the components of the framework with joints); therefore, the previous ones are denoted by rods.

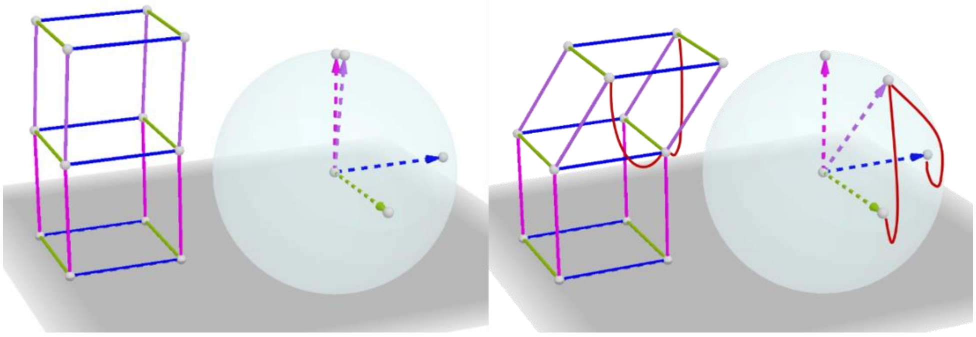

Figure 1: The left photo shows a $1\times1\times2$ Special Cubic Tiling framework without any bracing elements and its Spherical bracing framework on its right side. On the right image, we can see a possible displacement of the same framework with two cables as bracing elements and its Spherical bracing framework on the right side.

How to use CSR braces to make TCTF stiff? The used braces as cables, along with diagonals, form triangles with the two bars that are from different equivalence classes. One of the two cables along the different diagonals of a square of the TCTF corresponds to a cable. In contrast, the other corresponds to a truss in its Spherical bracing framework as a tensegrity structure.
The 1×1×2 TCTF of the left picture of Fig 1 consists of joints colored by grey and bars colored by different colors as its equivalence classes. The structure is not rigid yet; we have to use additional bracing elements to increase the number of constraints for stiffness. In the right photo, we can see the top two red-colored cables on the second floor in the two vertical walls. On the right of the pictures are the TCTF's Spherical bracing frameworks as tensegrity structures.

3 Results

3.1 The rigidity of CSR braced TCTF.

Even if several CSR were involved in the bracing, the degree of freedom of the system would not reduce due to the same constraints, and the bracing framework would not change. The structure is not rigid yet; we have to use further bracing elements that increase the number of controls for stiffness of the Spherical Bracing Framework. Our model for rigidity of the repetitive cubic frameworks produces smaller dimensional tensegrity structures as a configuration space [13] for the motion of the CSR braced TCTF. The rank of the rigidity matrix of the Spherical bracing framework is determined efficiently as the original structure.

A CSR braced TCTF is infinitesimally rigid if and only if its Spherical Bracing Framework is infinitesimally rigid.

There is a one-to-one correspondence between the motion of the representation vectors, i.e., the Spherical Bracing Framework, and the motion of the bars in the equivalence classes, i.e., the bars of the TCTF.

As a consequence of this correspondence, we find an efficient algorithm for deciding the infinitesimal rigidity of three-dimensional CSR braced TCTF. In the case of the $n\times n\times n$ cubic framework, using our model, the bracing framework consists of $3n+1$ pieces of joint. Hence the time complexity is $O(n^3)$ instead of $O(n^9)$.

3.2 Applications

3.2.1 Three-palmed "baseball glove" controlled by its bracing framework

The construction of TCTF was discussed in [6, 7]; the structure is theoretical. In [14] present the theoretical and made a prototype, and [21, 23] offers some CAD models. Each of these structures is suitable for constructing the mechanism in Fig. 2, which can be a candidate as a catch mechanism in robotics.

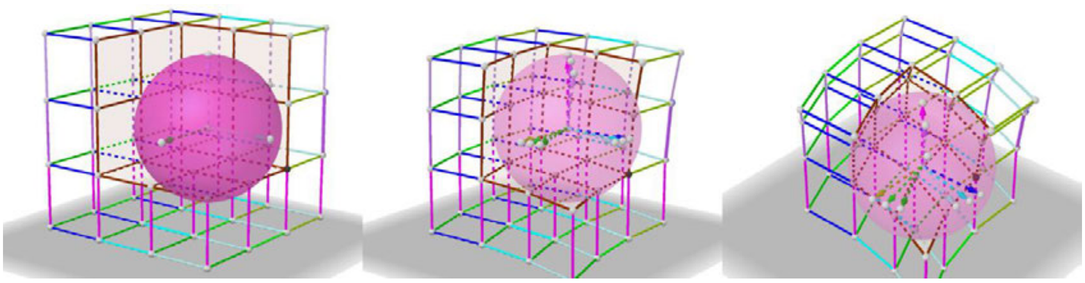

Figure 2: On the left picture, a three-palmed "baseball glove" as a gripper from rod braced TCTF, and its bracing framework is visualized. In the further pictures, we can see displacements as the gripper's vertical palms hug around a ball as its spherical bracing framework. We change the transparency of the sphere for the visualization of the displacements of the nine representation vectors.

The rod's length as bracing elements is changed (actuators) in the interest of controllable motions. The bracing elements as actuators are not visualized in the figure. The CSR braced TCTF frameworks could also be used to construct a similar mechanism.

The applications in structural engineering will be discussed in the presentation.

Another promising application is that the convex hull volume of the Cubes as the rhombohedron could change widely during the motion, similarly modifying the bracing elements' length.

4 Conclusions and Contributions

This paper considered the infinitesimal rigidity of the Twist-free Cubic Tiling Framework braced by Cables, Struts, and Rods.

- We present a necessary, and sufficient connection between the infinitesimal rigidity of the CSR braced ZTSF and its Bracing framework infinitesimal rigidity.
- As a consequence of the correspondence between the CSR braced ZTSF and its Bracing framework, we find an efficient algorithm for deciding the infinitesimal rigidity of three-dimensional CSR braced TCTF. For example, in the case of a $100\times100\times100$ cubic tiling framework, the number of steps (counting the rank of the rigidity matrix) decreased from 10^{18} to $300^3=2,7\cdot10^7$.
- The description of the deformation of the frameworks leads to a better understanding of the motion of the Cubic structure.

The above results may have the following practical advantages for professionals working with structures.

- The researchers could provide new results which lead to a better understanding of the motion of the cubic structure-based buildings and mechanisms.
- The structural designer or software designer using the characterization of the rigidity of the CSR braced three-dimensional grid-like bar-joint framework by tensegrity structure result could decrease the complexity of the stability analysis.
- Our results provide valuable inputs to further optimization methods in topology optimization and the design of actuators' controlled mechanisms.

We present a new result that characterizes the motion of the cable, strut, or rod braced bar-joint cubic building. Provides new conditions for the rigidity of the three-dimensional Cubic structure applied some further diagonal CSR brace.

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