The Eleventh International Conference on
Engineering Computational Technology
23–25 August, 2022 | Montpellier, France

Proceedings of the Eleventh International Conference on
Engineering Computational Technology
Edited by B.H.V. Topping and P. Iványi
Civil-Comp Conferences, Volume 2, Paper 4.8
Civil-Comp Press, Edinburgh, United Kingdom, 2022, doi: 10.4203/ccc.2.4.8

Nonlinear Preconditioning of Phase-Field based Structural Topology Optimization Problems

R. Tavakoli and D. Tourret

IMDEA Materials Institute, Madrid, Spain

Abstract

The nonlinear preconditioning of phase-field based topology optimization problems is considered in the present work. In this method, the leading order solution of inner problem corresponding the phase-filed PDE is used as the nonlinear transformation of phase-field variable. The transformed problem exhibits a similar structure to the original problem, possibly with higher nonlinearity. The method is applied to the volume constrained minimum compliance topology optimization problem. Numerical experiments illustrate the superior performance of this approach compared to non-preconditioned form of problem. The method is general and easy to implement, and it can be readily extended to alternative problems.

Keywords: diffuse interface modeling, nonlinear preconditioning, phase-field modeling, topology optimization.

1 Introduction

Nowadays, structural topology optimization (TO) is a standard method during the engineering product design cycle [1]. It provides the optimal outline of structure with minimal input information. In this approach a specific amount of material is distributed in the spatial domain such that an objective functional, like the structure's compliance, is minimized subject to appropriate partial differential equations (PDEs), such as the linear elasticity equation, and design constraints, like total weight of material. The pointwise density of material is commonly considered as the design

variable in this method. Considering the variational formulation of minimum compliance TO [2], there exists an analogy between TO and phase separation with microelasticity [3]. The phase-field modelling is the most successful theoretical and computational framework to study phase transition problems [4]. The solution of topology optimization problems based on phase-field method was introduced in [5]. Later, it has been used to solve single-phase [6] and multi-phase [7] minimum compliance TO problems. In these methods, the system of PDEs corresponding to H^{-1}-gradient flow of augmented objective functional is solved numerically until the steady state condition is reached. The final form of PDEs is similar to Cahn-Hilliard equation with elastic misfit [8]. The Cahn-Hilliard dynamics naturally conserves the density field and keeps the feasibility of the corresponding volume constraint. The Cahn-Hilliard dynamics however is extremely slow, and $O(10^5)$ iterations is commonly required to reach a sufficiently mature solution, even using semi-implicit schemes [7]. Moreover, the solution of corresponding nonlinear forth-order PDE is computationally expensive. The phase-field based TO by constrained Allen-Cahn dynamics was introduced in [9] to moderate these limitations. According to [9], this dynamic is about two orders of magnitude faster than that of the Cahn-Hilliard.
The nonlinear preconditioning of phase-field models, like Cahn-Hilliard and Allen-Cahn equations, was presented in [10], where the matched asymptotic analysis of phase-field equations to their sharp interface solution was performed. Then, the nonlinear transformation of original PDEs was made using the leading order inner solution. According to [10], this transformation not only increases the convergence rate, but also permits to use coarser grid to capture the details of solution near the interface. The success of this method was confirmed by its use in phase-field modelling of alloy solidification problems [11,12].

The present work aims at applying the nonlinear preconditioning to topology optimization problems based on phase-field method.

2 Methods

The phase-field formulation of volume constrained TO problem can be stated as follows (see [2,6,7,9]):

$$\mathcal{J}(\phi, u) = \frac{1}{2}\int_\Omega \phi^p\big(C{:}\mathcal{D}(u)\big){:}\mathcal{D}(u)dx + \frac{\zeta}{4\varepsilon}\int_\Omega \phi^2(1-\phi)^2\,dx + \frac{\zeta\varepsilon}{2}\int_\Omega |\nabla\phi|^2\,dx$$

subject to:

$$\begin{aligned} \phi^p\nabla\cdot\big(C{:}\mathcal{D}(u)\big) &= 0 && in \quad \Omega \\ u(x) &= \hat{u}(x) && on \quad \Gamma_u \\ \phi^p\big(C{:}\mathcal{D}(u)\big)n &= 0 && on \quad \Gamma_f \\ \phi^p\big(C{:}\mathcal{D}(u)\big)n &= \hat{t}(x) && on \quad \Gamma_t \\ \int_\Omega \phi\,dx &= \Lambda|\Omega|, && 0 \le \phi \le 1 \end{aligned} \tag{1}$$

where Ω denotes the spatial domain, Γ_u, Γ_f and Γ_t denote the prescribed displacement, traction-free and prescribed traction boundaries, $\boldsymbol{u}$ and ϕ denote the displacement and phase fields, $\boldsymbol{C}$ denotes the elasticity tensor, $\mathcal{D}(u) \coloneqq \frac{1}{2}(\nabla u + (\nabla u)^T)$, Λ denotes the volume fraction of material in Ω, p denotes the SIMP penalization parameter, ε is

proportional to the thickness of diffuse interface and the regularization parameter ζ emphasizes the perimeter penalization term. In the regularized projected steepest descent algorithm based on Allen-Cahn dynamics, the constrained gradient flow of the above functional is decomposed into two steps. First, the L^2-gradient flow of the first two terms of objective functional subject to linear elasticity equation, volume and inequalities constraints is computed. Then, the H^1-gradient flow of the last term of objective functional is computed by solving a linear Helmholtz PDE (see [9]).
Analogous to phase transition model based on Cahn-Larche dynamics, the gradient flow of this optimization problem includes the following parabolic PDE subject to homogeneous Neumann boundary condition, together (1):

$$\frac{\partial \phi}{\partial t} = \Delta\left(\frac{\zeta}{2\varepsilon}(2\phi^3 - 3\phi^2 + \phi) - \zeta\varepsilon\Delta\phi - \frac{1}{2}p\phi^{p-1}\big(\boldsymbol{C}:\mathcal{D}(\boldsymbol{u})\big):\mathcal{D}(\boldsymbol{u})\right) \qquad (2)$$

Using the formal method of matched asymptotic, the leading order solution of ϕ in the inner region has the following explicit form

$$\phi_0(\psi) = \frac{1}{2}\left(1 + tanh\left(\frac{\psi}{2\sqrt{2}\,\varepsilon}\right)\right) \qquad (3)$$

where ψ denotes the signed distance function to the interface. Thus, following [10], we can derive the preconditioned form of our two-step gradient flow algorithm by transforming ϕ in all equations to $\phi \approx \phi_0(\psi)$. The application of this transformation to (1) and projection step is straightforward. Applying this transformation to the second step of gradient flow results in:

$$\frac{\partial \psi}{\partial t} = \zeta\varepsilon\,\Delta\psi - \frac{1}{\sqrt{2}\zeta\varepsilon}|\nabla\psi|^2 \qquad (4)$$

However, unlike the Helmholtz equation, this PDE is nonlinear. It has two major drawbacks: its numerical solution is computationally expensive, and it does not preserve the feasibility of volume and bound constraints during the second step. To cope with these limitations, we include the second, nonlinear, term of this PDE into the first step of gradient flow. In fact, the solution algorithm is identical to [9], but an additional term is included into equation (28) of [9], and we solve the problem for the regularized signed distance field, ψ. The above mentioned nonlinear transformation is used to find the density field whenever it is required.

3 Results

We applied the presented algorithm to cantilever beam test case given in [9]. Except when explicitly mentioned here, the physical and computational parameters are identical to that of [9]. To avoid the numerical singularity, the value of 10^{-9} is considered for the elastic modulus of void here. Moreover, the SIMP penalization power is fixed to 3. As a first example, we solve the problem for the void-material scenario with $\Lambda = 0.3$. Figure 1 shows the variation of compliance with iterations for non-preconditioned and preconditioned solvers. According to the plots, the convergence rate of preconditioned solver is better than that of the non-preconditioned solver and the final value of compliance is smaller than that of the original algorithm. Figure 2 illustrates the final topology after 300 iterations for this test case. The next example is identical to test case #29 in [9]. Figure 2 shows the final topologies after 500 optimisations iterations. The final values of structure's compliance are equal to 41.24 and 44.97 for the preconditioned and non-preconditioned solvers respectively. According to the plots, the final topology is more mature in the case of preconditioned solver. According to these results the convergence rate of preconditioned version of algorithm is better than the original version. It is worth noticing that the computational overhead of nonlinear preconditioning in these examples are less than 5 % of the total CPU time. These observations illustrate the value of presented preconditioning approach to solve phase-field based topology optimization problems.

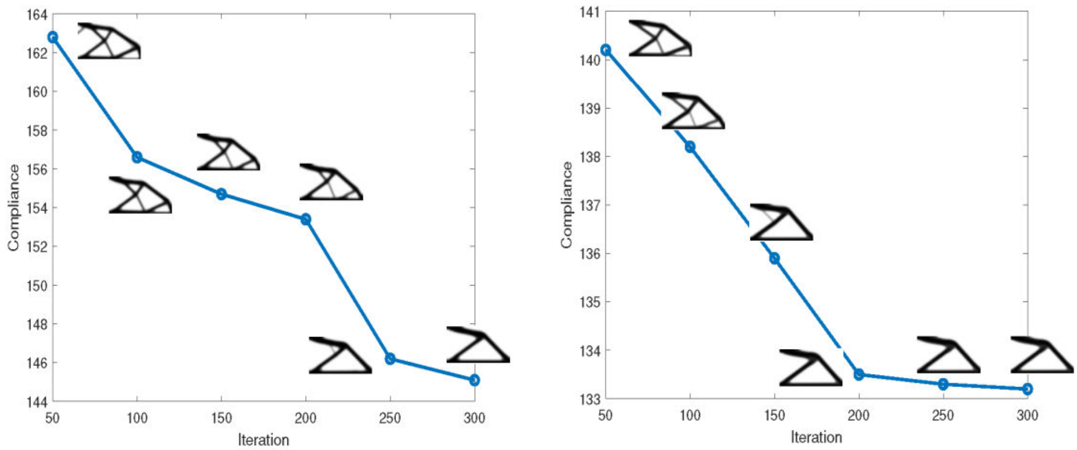

Figure 1: Topology optimization of cantilever beam: variation of compliance with iteration for single-phase material case for non-preconditioned (left) and nonlinearly preconditioned (right) phase-field solvers.

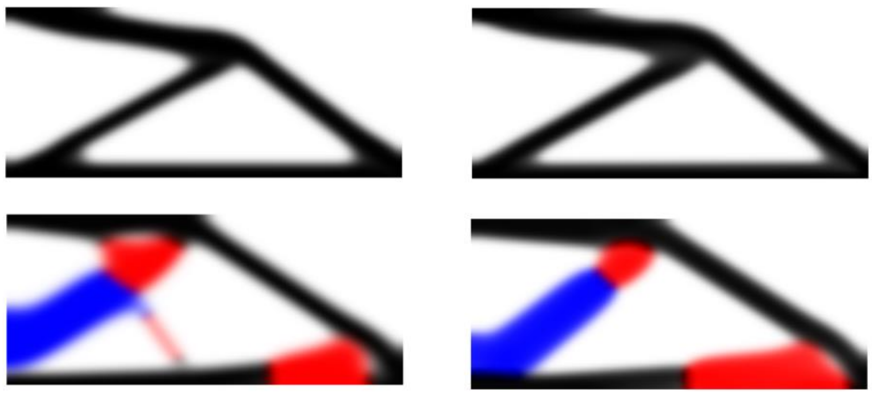

Figure 2: Topology optimization of cantilever beam: single-phase material case (top) after 300 optimization iterations and three materials case (bottom) after 500 optimization iterations, for non-preconditioned (left) and nonlinearly preconditioned (right) phase-field solver.

4 Conclusions and Contributions

The nonlinear preconditioning of diffuse interface models introduced in [10] is applied, for the first time, to phase-field based topology optimization problems. In this method, the asymptotic analysis of the diffuse interface model based on method of matched asymptotic is firstly performed. Then, the leading order solution of inner problem corresponding the phase-filed PDE is used as the nonlinear transformation of phase-field variable. The method is general and can be readily applied to different problems. The transformed problem exhibits a similar structure to the original problem. It possibly possesses higher nonlinearity in practice. The method is applied to volume constrained minimum compliance TO based on the regularized projected steepest descent algorithm. According to numerical experiments in the present work, the convergence rate of optimization solver can be improved by the application of this preconditioning approach, while the additional computational overhead is negligible compared to the total CPU time.

Systematic study on the convergence rate of this method and its grid sensitivity analysis is suggested as the scope of future works. Moreover, the application of this approach to alternative problems is recommended. There exist different ways to drive nonlinearly preconditioned version of the original problem. Further studies on these issues can be considered as scope of future works too.

Acknowledgements

This work is part of the project PCI2021-122023-2B, funded by MCIN/AEI/10.13039/501100011033 and by the European Union "NextGenerationEU"/PRTR".

References

[1] M. Bendsoe, O. Sigmund, “Topology optimization: theory, methods, and applications”, Springer, 2003.

[2] L. Blank, H. Garcke, M.H. Farshbaf-Shaker, M Hassan, V. Styles, “Relating phase field and sharp interface approaches to structural topology optimization”, ESAIM: Control, Optimisation and Calculus of Variations, 20, 1025-1058, 2014.

[3] H. Garcke, D. Jung, C. Kwak, “On asymptotic limits of Cahn-Hilliard systems with elastic misfit, Analysis, modeling and simulation of multiscale problems”, Analysis, Modeling and Simulation of Multiscale Problems, 87-111, 2006, doi:10.1007/3-540-35657-6_4.

[4] N. Provatas, K. Elder, “Phase-field methods in materials science and engineering”, John Wiley, 2011.

[5] R. Haber, C. Jog, M. Bendsoe, “A new approach to variable-topology shape design using a constraint on perimeter”, Structural Optimization, 11, 1-12, 1996.

[6] M. Wang, S. Zhou, “Phase field: a variational method for structural topology optimization”, CMES-Computer Modeling in Engineering and Sciences, 6, 547-566, 2004.

[7] S. Zhou, M. Wang, “Multimaterial structural topology optimization with a generalized Cahn- Hilliard model of multiphase transition”, Structural and Multidisciplinary Optimization, 33, 89-111, 2007.

[8] L. Reischmann, A. Peter, “Multiscale Modeling and Simulation of a Cahn--Larche System with Phase Separation on the Microscale”, SIAM Journal on Applied Mathematics, 80, 2170-2193, 2020.

[9] R. Tavakoli, “Multimaterial topology optimization by volume constrained Allen--Cahn system and regularized projected steepest descent method”, Computer Methods in Applied Mechanics and Engineering, 276, 534-565, 2014.
[10] K. Glasner, “Nonlinear preconditioning for diffuse interfaces”, Journal of Computational Physics, 174 ,695-711, 2001.
[11] D. Tourret, A. Karma, “Growth competition of columnar dendritic grains: A phase-field study”, Acta Materialia, 82, 64-83, 2015.
[12] A. Badillo, C. Beckermann, “Phase-field simulation of the columnar-to-equiaxed transition in alloy solidification”, Acta Materialia, 54, 2015-2026, 2006.