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Derivative-free Topology Optimisation via Explicit Level Set Parameterisation and Trust Region Strategy Optimiser

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Abstract

This work investigates the use of explicit level set parameterisation for topology optimisation using a metamodel-based trust region strategy optimiser. The explicit level set parameterisation consists of building a uniform Design of Experiments using a Permutation Genetic Algorithm, followed by building the Level Set Function using Kriging. Through decoupling the parameterisation from the simulation physics, the use of sensitivity data becomes optional thus enabling computationally complex disciplines (where sensitivity data is not available, e.g. crashworthiness, electromagnetics) to be included. This is achieved through the use of a sequence of approximations to the functions of the original optimisation problems based on a trust region strategy. The method is demonstrated on a benchmark 2D topology optimisation problem to examine the effectiveness of the technique.
Keywords: topology optimisation, explicit level set, derivative-free, trust region strategy, design of experiments, kriging.

1 Introduction

This work investigates the use of explicit level set parameterisation for topology optimisation using a metamodel-based trust region strategy optimiser. The explicit level set parameterisation consists of building a uniform Design of Experiments using a Permutation Genetic Algorithm, followed by building the Level Set Function using Kriging. Through decoupling the parameterisation from the simulation physics, the use of sensitivity data becomes optional thus enabling computationally complex disciplines (where sensitivity data is not available, e.g. crashworthiness, electromagnetics) to be included. This is achieved through the use of a sequence of approximations to the functions of the original optimisation problems based on a trust region strategy. The method is demonstrated on a benchmark 2D topology optimisation problem to examine the effectiveness of the technique.

Metamodel-based optimisation is a methodology used to reduce the computational cost and numerical noise in optimisation problems that require a large number of complex simulations. It is particularly advantageous for problems where sensitivity data is not available and when the response functions have significant numerical noise. The quality of a metamodel is dependent on several factors: the reliability and accuracy of the response data, the effectiveness of the Design of Experiments (DoE) for gathering information for metamodel building, the size of the domain in which approximations are built relative to the entire design domain, the simulation data accuracy and the number of DoE points used to build the model [1-3].

One strategy to achieve a higher quality and more reliable metamodel is to investigate a sub-domain of the design space and employ a strategy to iteratively update the size and location of this region - known as trust region strategies. The Multipoint Approximation Method (MAM), also referred to as the Mid-range Approximation Method, is a trust region strategy metamodel-based optimiser. The heritage of the MAM dates back to [4-6] and has been continuously developed to include new features [7-9]. The MAM algorithm arrives at a solution by iteratively solving approximated sub-problems in trust regions that translate and resize as the search progresses. Each iteration builds an approximated model from the simulation response data, solves the optimisation problem, employs the trust region strategy to update the trust region's size and location, and begins the next iteration by populating the updated trust region with a DoE -- with adaptions to prevent the algorithm becoming stuck in non-converging loops or converging to local minima.

2 Methods

The proposed methodology is an Explicit Level Set representation for topology optimisation -- where the function values obtained at the Design of Experiment (DoE) points are used to build a metamodel that represents the Level Set Function (LSF). The parameterisation process involves an initial DoE within the LSF design space, that remains the same for all sets of design variables, and a metamodel build with respect to each set of design variables.

Capturing the LSF efficiently is paramount to this methodology -- as the number of DoE points used represents the number of design variables within the optimisation process (MAM). Thus, an effective space-filling DoE should be established. To achieve this, this work uses a DoE obtained by a permutation Genetic Algorithm (GA) [10]. The permutation GA is coupled with an Optimum Latin Hypercube DoE. This principle refers to the distribution of DoE points in each dimension being separated by uniform intervals, with only one DoE point positioned at each interval.

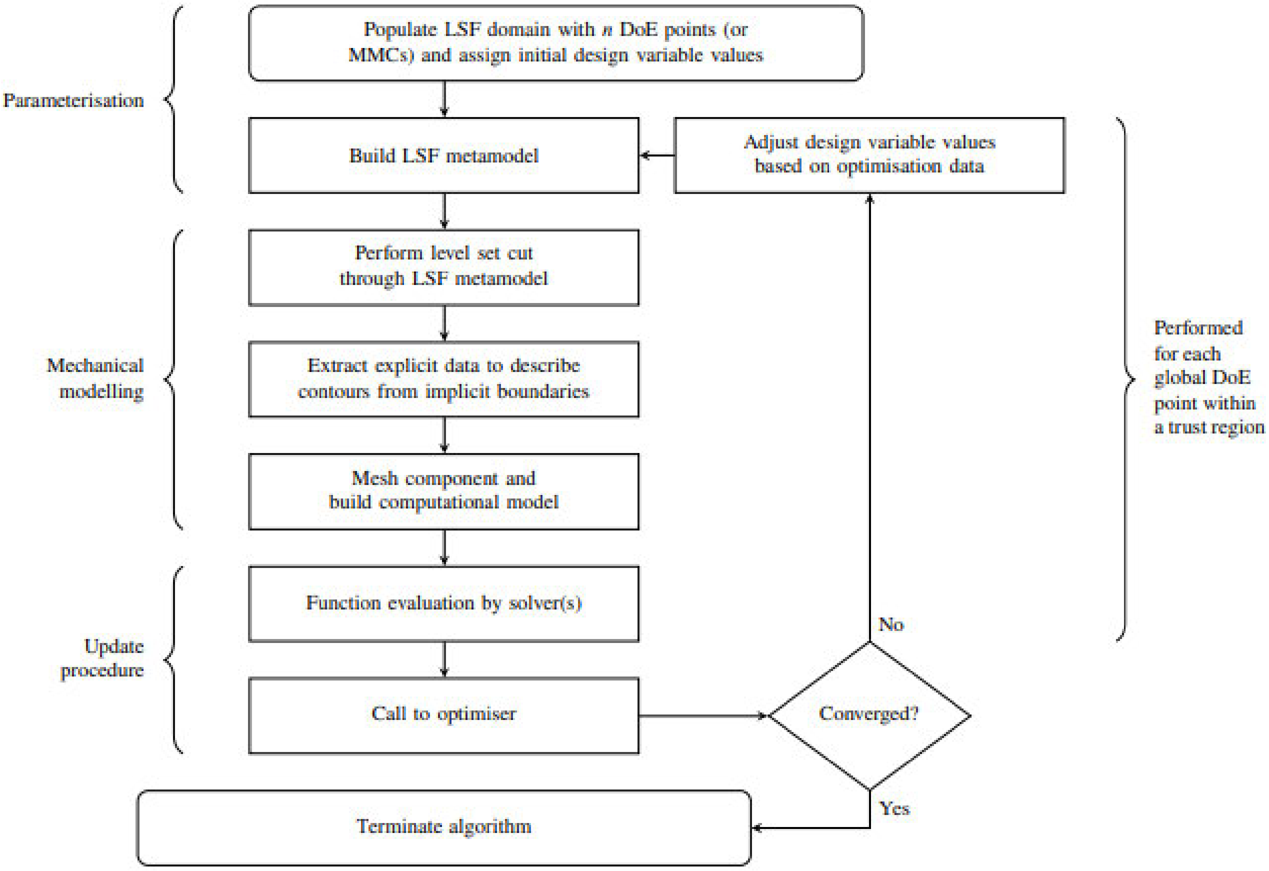

Figure 1 - Explicit Level Set Method algorithm for topology optimisation, illustrating the three primary segments of the framework and implementation within a trust region strategy, where n denotes the number of design variables

The LSF metamodel is built using the Ordinary Kriging method [11-12] -- a spatial correlation-based technique that builds an interpolating metamodel. Kriging acts as an exact interpolator, which is attractive in deterministic simulation [3,13]. It can be noted that Kriging loses it's computational efficiency when approaching problems with several thousand DoE points [14], however, such number of design variables is far beyond the scope of the proposed methodology.

The mesh is built using a conforming discretisation approach. The contour lines of the LSF are identified and the mesh is fitted to conform to these contours. The loads and boundary conditions are then imposed on the nodes of the mesh that covers those regions. Loads applied as a pressure are coupled with a stipulation that material must cover the entire region where the pressure is applied.

The trust region strategy is the definitive decision-making process of the MAM algorithm. The trust region strategy decides: the termination or continuation of the optimisation loop; the transformation and translation of the trust region; and the reuse of metamodels. This utilises information from the metamodel quality, trust region size, optimal point (location and direction) and DoE points to drive the decisions made by the trust region strategy at each iteration.

3 Results

The methodology proposed in this paper was applied to a two-dimensional linear static structural topology optimisation problem -- Michell one-load. The problem formulation is to minimise the compliance (i.e. achieve maximum global stiffness), with a constraint on the volume fraction set to be 50%. The optimisation problem can be expressed as

The Michell one load benchmark case for topology optimisation simulates a centrally loaded beam via imposing boundary conditions that act as a line of symmetry. It is an effective problem for testing topology optimisation techniques, as it requires the material to be connected and in contact with both constraints in order to satisfy static equilibrium conditions. The material used for the benchmark is steel ASTM A-36, in accordance with the benchmark data for topology optimisation performed by [15].

The results presented in Figure 3 are for with 20, 40 and 80 design variable optimisation problems and follow the convergence criteria. The 20-design variable study demonstrates the effectiveness of the technique without the requirement for large numbers of design variables. The result converged the fasted, in 32 iterations, but provided the largest objective function compared to the higher design variable studies.

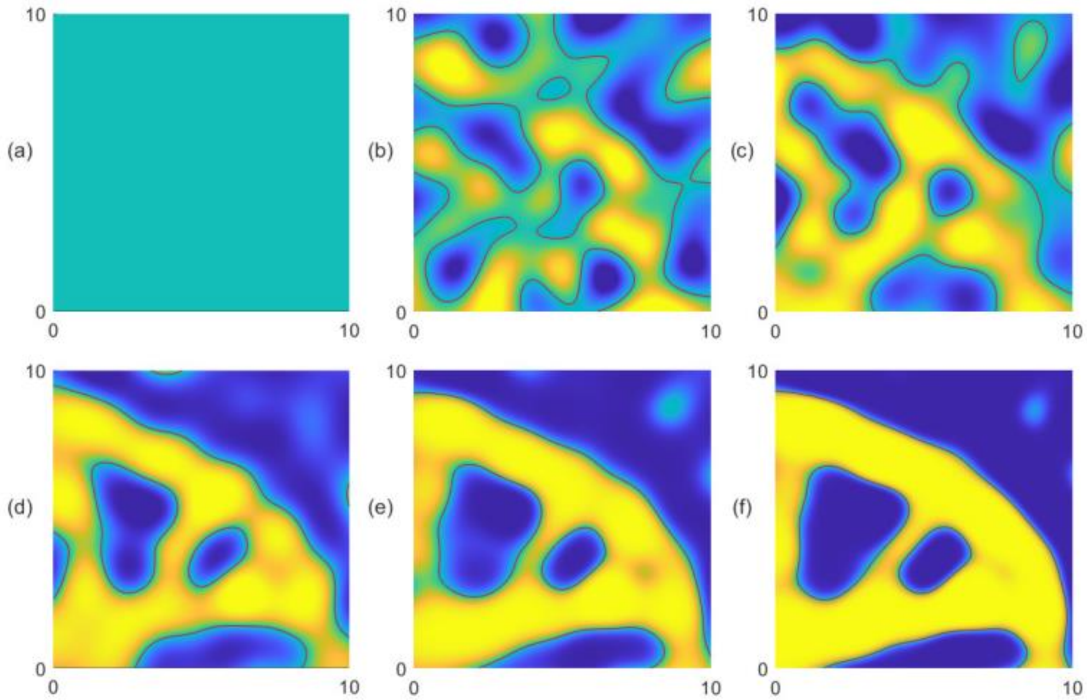

Figure 2 - Progression of the Level Set Function through the optimisation process for the 80 design variable study

In the 40-design variable study, the MAM converged after 40 iterations, reaching a final compliance of 0.268 at the volume constraint upper limit, having started with an initial value of 0.143 when the domain is entirely filled with material. The optimisation process was started from the centre of the design space, which results in having a volume fraction of 100% as all design variables are at the value of the level set cut. This results in having the largest possible amount of material available to resist the load applied, thus providing the lowest possible compliance for this problem.

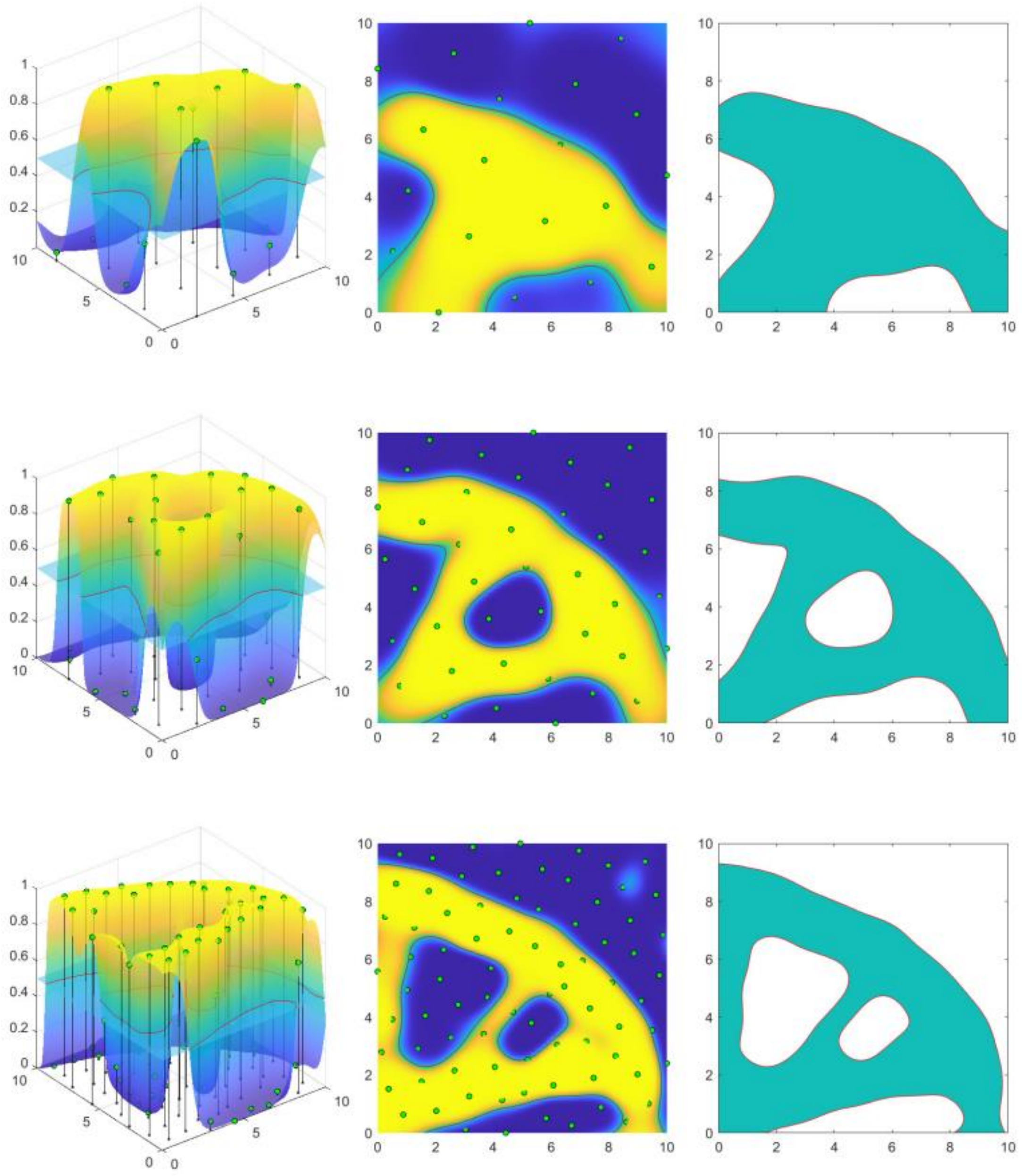

Figure 3 - Design of Experiments and Level Set Function representation for the optimisation results for 20 (top), 40 (middle) and 80 (bottom) design variables

The largest design variable study (80) shows the most resemblance to the results presented for this benchmark in the literature. The progression of the topology through the optimisation process is presented in Figure 2. It can be seen that the topology forms a quarter circle shape early on in the optimisation -- from here material is removed to create spoke-like members. Additionally, this methodology does not require holes to be initialised -- holes are created naturally by the optimiser if required. The process converged in 50 iterations, with a final compliance of 0.259.

4 Conclusions and Contributions

This paper presents a methodology based on the use of Explicit Level Set representation for topology optimisation that is based on the use of MAM, a metamodel- and trust region strategy-based optimisation process. The methodology is developed with the intended purpose of producing an effective framework for performing topology optimisation without the use of sensitivity data. This is to enable the topology optimisation with computationally expensive disciplines where the outputs may be polluted with numerical noise, and the design sensitivity information is unavailable. Two stages of DoEs and metamodels are used to solve a sequence of optimisation problems, one to build the LSF, and another within the MAM trust regions. The MAM utilises the Non-collapsible Latin Hypercube (NLH) DoE and the MLSM to build the metamodels within the sub-spaces of the design variable space (i.e. in trust regions). The methodology uses a Permutation Genetic Algorithm to effectively distribute DoE points, that will then act as the basis to build the LSF using a Kriging metamodel. The computational model is built using a MATLAB code, where the contours is extracted as explicit coordinate data and used to build a conforming discretisation mesh. The approximated problem is solved in a current trust region using a multi-start SQP process, referred to as candidate points. The information on the DoE and candidate points is used to guide the trust region strategy and increase the likelihood of converging to a global optimum. Additionally, parameters of the trust region strategy control satisfaction of the convergence criteria of the MAM.

The results present a solution to a single load Michell benchmark case for linear static topology optimisation using 20, 40 and 80 design variables, that all converged in less than 50 iterations. The results were achieved without the requirement for design sensitivities. The technique has been developed with the intention of being utilised for computationally expensive and difficult disciplines. In these applications, where design sensitivities are not available, methods that utilise fewer design variables are advantageous to keep the computational cost reasonable.

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