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# Multifidelity Domain-Aware Scheme for Cross-Regime Airfoil Shape Optimization

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## Abstract

This paper proposes a domain-aware multifidelity scheme to speed-up the airfoil shape optimization in a cross-regime scenario where the aerodynamic domain evolves with the Mach number. Our strategy relies on a multifidelity Bayesian framework based on a surrogate model iteratively updated through a multifidelity acquisition function that selects the next design configuration and level of fidelity to query. We implement the multifidelity Gaussian process as the aerodynamic surrogate model and formulate an original domain-aware multifidelity acquisition function informed by the evolution of the fluid domain. This property allows to wisely select the level of fidelity of the aerodynamic model considering the compressibility and non-linear effects at higher speed regimes, improving the accuracy of the surrogate model. We validate our approach for the benchmark test-case of the constrained shape optimization problem of a RAE 2822 airfoil. The results suggest that our strategy outperforms popular multifidelity and single-fidelity methods reducing the drag coefficient of the optimized airfoil with an improvement of the 24% respect to the baseline airfoil with a limited computational budget.

**Keywords:** multifidelity method, domain-awareness, Bayesian optimization, active learning, cross-regime, aerodynamic optimization.

## 1 Introduction

Aerodynamic design optimization is a complex and demanding problem characterized by expensive evaluations of the objective function and a large number

of design variables and constraints [1]. To produce superior aerodynamic configurations, modern engineering approaches require to accurately predict the fluid domain through high-fidelity models based on the numerical solution of Reynolds-Averaged Navier-Stokes (RANS) equations, which is a challenging and expensive task [2]. In particular, accurate models are essential to depict the aerodynamic field in the transonic regime, where the flow is characterized by local supersonic regions with shock waves that cause the separation of the viscous boundary layer.

However, the enormous computational cost required to repeatedly compute numerical simulations makes unfeasible to rely exclusively on high-fidelity models for trade-off analysis. To address these difficulties, aerodynamic optimization can take advantage of low-fidelity models which introduce approximations to reduce the computational cost, but also may not be reliable to predict non-linear physics and compressibility effects that occurs at higher flight speeds. This motivates the interest for computational strategies capable of wisely including high-fidelity simulations and capture promising design configurations that may be excluded by low-fidelity optimization. This is particularly attractive in cross-regime optimization scenario, where the selection of the aerodynamic models poses additional challenges to balance between the accuracy of the prediction of non-linear high-speed phenomena and the savings in computational cost.

This paper presents a multifidelity Bayesian optimization framework [3,4,5,6] based on a domain-aware active learning scheme for the dynamic principled selection over multiple aerodynamic models at different levels of fidelity, accounting for the evolution of the fluid regime and the associated physical phenomena. Our strategy combines the multifidelity observations of the objective function into a single surrogate model, and repeatedly maximizes a multifidelity acquisition function to select the next design configuration and the appropriate level of fidelity of the aerodynamic representation to query. The formulation of our multifidelity acquisition function comes with a domain-aware utility function that permits to grasp the evolution of the fluid domain, encouraging the use of high-fidelity predictions at higher speed regimes where transonic phenomena occur. This element of the computational strategy permits to include the knowledge of the physical domain in the active learning scheme, improving the selection of the level of fidelity to query and the accuracy of the aerodynamic surrogate model. We demonstrate our strategy using the benchmark constrained shape optimization problem of a RAE 2822 developed by Quagliarella and Diez [7].

## 2 Methods

Our strategy synthesizes the representations of the objective function  $\{f^{(l)}\}_{l=1}^{l=L}$  at different levels of fidelity  $l = 1, \dots, L$  into a surrogate model defined extending the Gaussian process (GP) [8] to a multifidelity setting using an autoregressive scheme and modeling the first prior  $f^{(1)}$  as a GP [9] :

$$f^{(l)} = \rho f^{(l-1)}(\mathbf{x}) + \delta^{(l)}(\mathbf{x}) \quad l = 2, \dots, L \quad (1)$$

where  $\rho$  is a scaling factor, and  $\delta^{(l)}(\mathbf{x})$  models the discrepancy. Based on the surrogate model, we define the multifidelity acquisition function  $U$  conceived for the cross-regime aerodynamic optimization as follows:

$$U(\mathbf{x}, l) = MFEI(\mathbf{x}, l) \alpha_{DA}(\mathbf{x}, l, M) \quad (2)$$

where  $MFEI(\mathbf{x}, l)$  is the multifidelity expected improvement [4] and  $\alpha_{DA}$  is the domain-aware utility function:

$$\alpha_{DA}(\mathbf{x}, l, M) = \begin{cases} 1 & \text{if } l \neq L \\ \frac{M^*}{M^* - M(\mathbf{x})} & \text{if } l = L \end{cases} \quad (3)$$

where  $M^* = 1$ . The utility function  $\alpha_{DA}$  is sensitive to the Mach number, accounting for the evolution of the aerodynamic regime. As the Mach number approaches the sonic condition, the flow is characterized by local shock waves, large-scale separation and unsteadiness. The accurate prediction of these phenomena requires a refined computational mesh that permits to numerically estimate the effects on the aerodynamic performances. Therefore,  $\alpha_{DA}$  is conceived to increase the values of the acquisition function when  $M > 0.8$ , encouraging the use of the highest-fidelity model at higher speed regimes.

The constrained airfoil optimization problem aims at computing the optimal shape and flight Mach number that jointly minimize the drag coefficient  $C_d$  of a RAE2822 airfoil subject to maintain a prescribed lift coefficient  $C_l$  and range of the pitching momentum coefficient  $C_m$ :

$$\begin{aligned} \min_{\mathbf{x} \in \mathcal{X}} \quad & C_d \\ \text{s. t.} \quad & C_l = 0.824 \\ & -0.1 \leq C_m \leq -0.01 \\ & t/c = 0.1211 \\ & r \geq 0.007c \\ & \tau \geq 5^\circ \\ & t_{85}/c \geq 0.02 \\ & \mathcal{X} = I_w \times I_M \end{aligned} \quad (4)$$

Additional geometry constraints include the thickness to chord ratio  $t/c$ , the trailing edge angle  $\tau$ , and the thickness at the 85% of the chord  $t_{85}$ . The design variable  $\mathbf{x}$  includes the weights of the shape modification functions  $\mathbf{w} = \{w_1, \dots, w_6\}$  and the Mach number  $M$ . The design space  $\mathcal{X}$  is bounded by the move limits on the weights

$I_w = [-0.5, 0.5]^6$  and the Mach number  $I_M = [0.6, 0.99]$ . As developed by [7], the computational grids are dynamically generated through GMSH v4 and the numerical solution of the RANS are computed using SU2 v6.2.0. We define three levels of fidelity for the aerodynamic modeling modifying the element scale factor  $ES$  of the mesh:  $ES^{(3)} = 5.5$  for the high-fidelity model (44200 cells),  $ES^{(2)} = 12$  for the mid-fidelity model (29500 cells), and  $ES^{(1)} = 20$  for the low-fidelity model (16000 cells). Figure 1 illustrates the mesh for the three models.

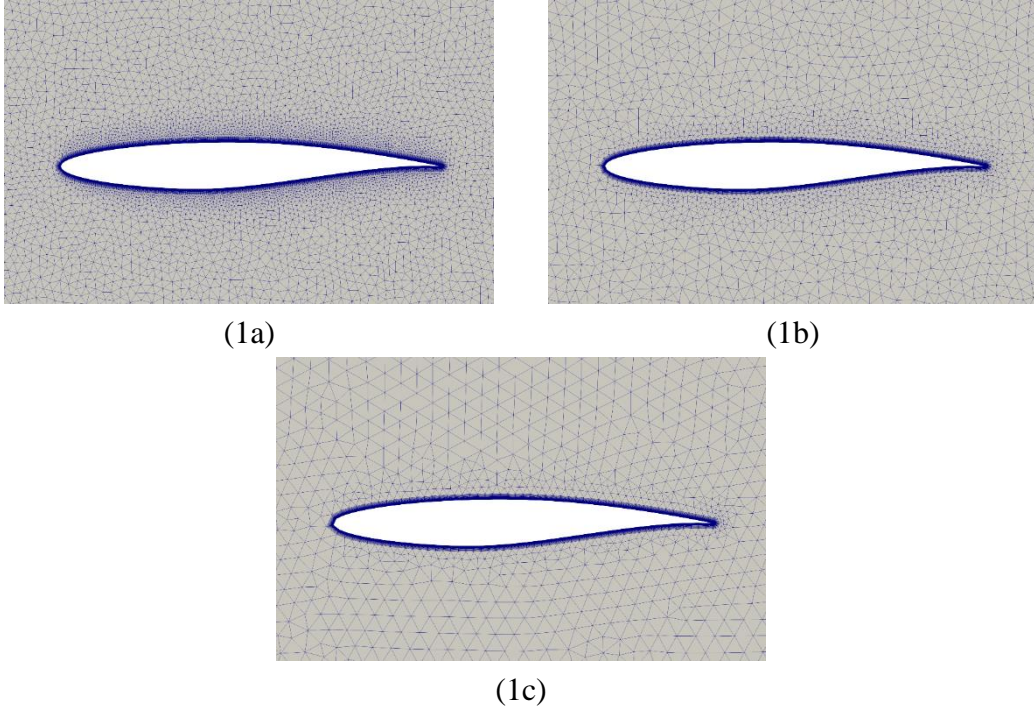


Figure 1: High-fidelity (1a), mid-fidelity (1b) and low-fidelity (1c) mesh.

### 3 Results

In this section, we illustrate the results of the RAE2822 shape optimization comparing the proposed Domain-Aware Multifidelity Bayesian Optimization (DA-MFBO) framework with the standard Multifidelity Bayesian Optimization framework (MFBO) proposed by Huang et. al [4] and the single high-fidelity efficient global optimization (EGO) algorithm developed by Jones et. al. [10].

Figure 2 illustrates the minimum drag coefficient as a function of the cumulative cost at each iteration of the optimization. We set the cost of each fidelity as the relative element scale  $\lambda^{(l)} = ES^{(l)}/ES^{(L)}$  and impose a maximum computational cost of 100. All the three methods are initialized with the same initial observations determined through a Latin hypercube sampling and are capable to progressively improve the baseline solution of the unmodified RAE2822. However, it can be observed that our DA-MFBO is more effective than MFBO and EGO, providing larger improvements

in the reduction of the drag coefficient with a limited computational cost. Table 1 summarizes the overall minimum of the  $C_d$  and the best design configurations determined by the three algorithms. In particular, the DA-MFBO algorithm outperforms the competing methods reaching a  $C_d$  equal to 0.01347, which corresponds to a design improvement of about the 24% with respect to the baseline. The outcome suggests that DA-MFBO computes more accurate aerodynamic surrogate model thanks to the domain-awareness property, which allows to use high-fidelity data at higher Mach numbers and capitalizes over lower-fidelity observations at lower speed regimes where the predictions of the aerodynamic models are closer. This aspect permits to effectively inform our multifidelity acquisition function guiding the selection of promising shape configurations and the appropriate level of fidelity to evaluate. Figure 3 compares the optimized airfoil shapes obtained with EGO, MFBO and DA-MFBO with the unmodified RAE2822 airfoil, while Figure 4 presents the corresponding pressure coefficient distribution. DA-MFBO generates a profile that determines a weaker shock wave reducing the loss of pressure downward and increasing the efficiency.

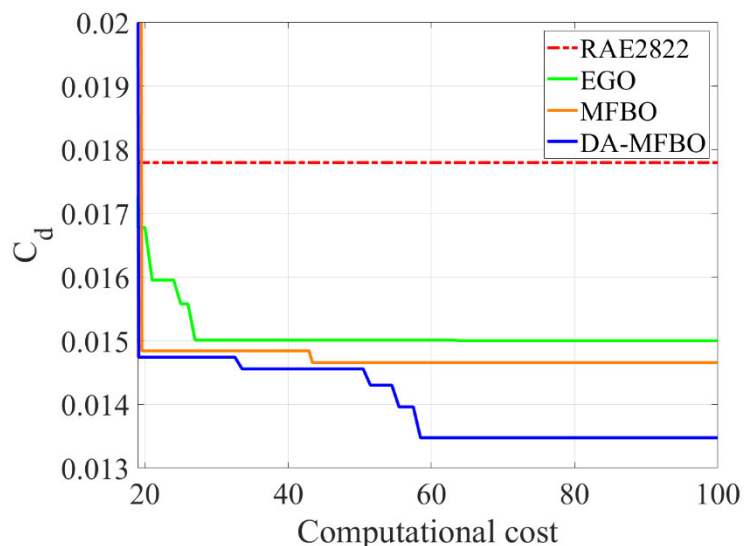


Figure 2: Performances of the DA-MFBO method compared to MFBO, EGO and the baseline RAE2822.

| Method  | $C_d$   | $w_1$  | $w_2$  | $w_3$  | $w_4$  | $w_5$  | $w_6$  | $M$   |
|---------|---------|--------|--------|--------|--------|--------|--------|-------|
| EGO     | 0.01500 | 0.272  | 0.061  | -0.107 | -0.424 | -0.117 | 0.162  | 0.664 |
| MFBO    | 0.01446 | -0.359 | -0.450 | 0.135  | 0.246  | 0.435  | -0.337 | 0.657 |
| DA-MFBO | 0.01347 | 0.498  | -0.454 | -0.098 | -0.266 | -0.356 | -0.124 | 0.656 |

Table 1: Optimum design configurations.

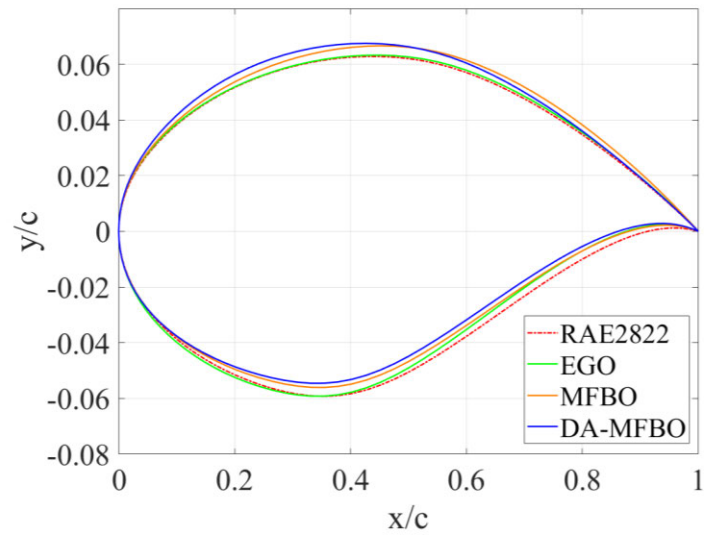
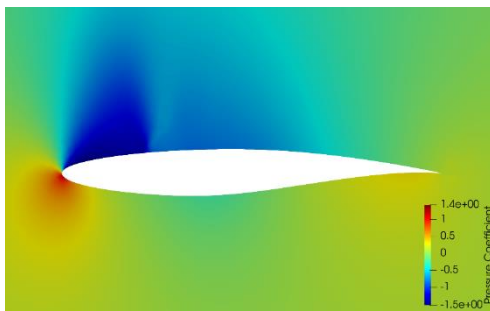
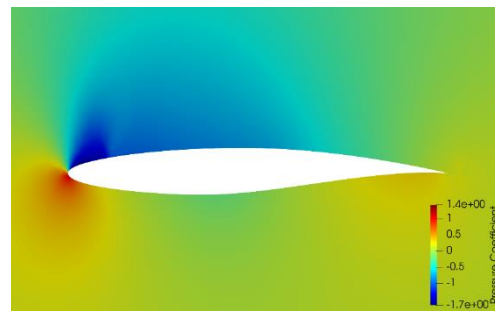


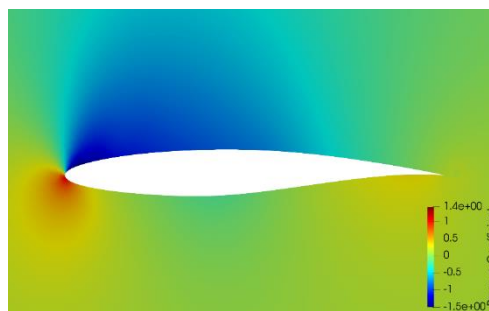
Figure 3: Airfoil shapes comparison.



(4a)



(4b)



(4c)

Figure 4: Pressure coefficient for the design configurations obtained with EGO (4a), MFBO (4b) and DA-MFBO (4c) algorithm.

## 4 Conclusions and Contributions

This paper introduces a domain-aware multifidelity Bayesian framework for the aerodynamic design and optimization in a cross-regime scenario that permits to: (i) leverage multiple models at different levels of fidelity to compute an efficient aerodynamic surrogate model, and (ii) iteratively learn the surrogate model through a domain-aware active learning scheme that includes the informative content about the fluid domain. We use the multifidelity Gaussian process as the surrogate model and propose a tailored formulation of the multifidelity acquisition function to effectively guide the active learning task. The objective is to define a computational framework to wisely include high-fidelity aerodynamic models in the optimization process without unfeasibly increasing the computational cost.

Our multifidelity acquisition function is computed starting from the information of the surrogate model, and includes a domain-aware utility function that is sensitive to the current fluid regime to select promising design configurations and the accuracy of the aerodynamic model to query. During the iterative process, the domain-aware utility function captures the evolution of the fluid domain with the flight Mach number prioritizing the expensive high-fidelity model at higher speed regimes, where cheap low-fidelity models do not accurately predict compressibility and non-linear effects. This aspect improves the accuracy of the aerodynamic model, enhancing the performances of the optimization process.

We demonstrate our domain-aware strategy in the optimization of the shape of a RAE2822 airfoil to minimize the drag coefficient maintaining a constant lift coefficient and subject to constraints on the pitching moment coefficient and the airfoil geometry. The aerodynamic domain is modelled computing the numerical solution of RANS through three aerodynamic models at different levels of fidelity, where the higher the fidelity the more refined is the discretization of the computational grid and time consuming. We compared our method against two well accepted Bayesian algorithms: the EGO single high-fidelity strategy and the multifidelity Bayesian framework based on the multifidelity expected improvement. The results demonstrate that the domain-awareness introduced in the active learning scheme allows to achieve better design solutions than competing methods leveraging the fluid domain information to compute an efficient aerodynamic surrogate model.

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