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Optimal Element-Wise Distributions of Structural Theories from Neural Networks

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Abstract

This paper presents a novel approach to developing refined structural theories for finite element models. The proposed methodology stems from the synergistic use of various methods. First, refined structural theories are built using the Carrera Unified Formulation, and 2D finite elements are used. Each element can be assigned a different structural theory through the Node-Dependent Kinematics approach. The axiomatic/asymptotic method is used to evaluate the accuracy of each structural theory distribution over a numerical mesh. Finally, neural networks are employed to obtain surrogate models, find optimal distributions of theories, and minimize computational costs. The numerical results consider free vibrations of composite shells with various stacking sequences and thickness ratios. Such input parameters are included as features of the surrogate models to avoid lengthy finite element simulations. The use of the proposed methodology provides guidelines on the proper modelling by indicating the areas of the structure in which refined models are most needed. Furthermore, the adoption of neural networks leads to significant reductions in computational overheads.

Keywords: Node-Dependent Kinematics, Finite Elements, Structural Theories, CUF, Neural Networks.

1 Introduction

The development of refined structural theories to improve the accuracy and efficiency of finite elements (FE) has met growing interest over the last decades [1].

The aim is to use 1D or 2D models with advanced kinematics to avoid 3D FE. Among the others, the Carrera Unified Formulation (CUF) has emerged as a versatile approach to build any-order theory for various structural problems [2, 3]. In CUF, governing equations and FE matrices are obtained using index notations allowing expansion functions for the unknown variables. Within CUF, one of the latest developments is the Node-Dependent Kinematics (NDK) [4] to build models in which the structural theory can vary pointwise. In other words, considering an FE model, each node can have a different structural theory. The use of NDK leads to higher efficiency as refined theories are used only where necessary.
The choice of the most appropriate model to adopt for a given problem can be made through the Axiomatic/Asymptotic Method (AAM), a method to evaluate the influence of generalized variables and build best theories and models [5]. Moreover, the AAM provides sensitivity analyses concerning problem attributes: thickness ratio, material properties and stacking sequence, and boundary and load conditions. However, to obtain optimal distributions of structural theories, many trial models are required and, thus, high computational costs. This paper presents an alternative approach to reduce such overheads based on supervised learning techniques. Neural networks (NN) [6], lately, have seen their employment increase enormously in many fields, including structural analysis [7, 8], thanks to their accuracy and computational efficiency. NN are used as surrogate models to substitute FE models and obtain structural responses. The combined use of CUF, AAM, NDK, and Neural Networks (NN) is a promising approach to build surrogate models that can provide information on the structural theory and finite element discretization for a given problem [9, 10].

2 Methods

CUF introduces a formalism to derive the governing equations and FE matrices hierarchically and independently of the type and order of the theory. In a 2D case, the displacement field ($\boldsymbol{u}$) is

$$\boldsymbol{u}(x,y,z) = N_i(x,y)F_\tau(z)\boldsymbol{u}_{\tau i}(z) \tag{1}$$

where F_τ is the expansion along the thickness direction and $N_i(x,y)$ is the shape function. While "i" is the standard index for nodes, "τ" is an index for the generalized variables or expansion terms. A third-order model, for instance, has $\tau = 1,4$ and the following displacement field:

$$\begin{aligned} u_x &= u_{x1} + zu_{x2} + z^2u_{x3} + z^3u_{x4} \\ u_y &= u_{y1} + zu_{y2} + z^2u_{y3} + z^3u_{y4} \\ u_z &= u_{z1} + zu_{z2} + z^2u_{z3} + z^3u_{z4} \end{aligned} \tag{2}$$

Using the Principle of Virtual Displacements, the stiffness matrix components can be obtained in a unified manner by using a 3X3 nucleus. For the sake of brevity, the nucleus components are not reported here but can be found in [3]. An NDK model has structural models like Eq. 2 assigned to each node independently. In other

words, one node can have a third-order model and the following one a second-order, for instance. In this paper, NN are trained with sets of nodal distributions. Two structural theories were considered: the First-Order Shear Deformation Theory (FSDT) and a complete fourth-order, equivalent single-layer (N=4). After training, NN can provide the structure's natural frequencies for a given distribution of structural theories and thickness ratio. The machine learning technique employed consisted of a convolutional neural network (CNN). The input was built by encoding the mesh distribution into a 16-bit array, then reshaped as a 4-by-4 matrix to correctly identify the position of each 2D element above the reference plane. A multi-channel input was later developed to consider the thickness, the stacking sequence, and the boundary conditions. The network's architecture includes an input layer, several hidden convolutional layers paired with pooling ones, followed by a dense and an output layer.

3 Results

The results have been obtained considering a relatively thick plate (a/h = 10), with the top and bottom edges clamped and the lateral ones free and stacking sequence 90/0. Figure 2 shows the best spatial distributions for different amounts of FSDT and N=4 elements. The first figure, for instance, shows the optima distribution of 4 FSDT; in other words, the locations in which we can insert 4 FSDT and obtain the best accuracy in calculating the first ten natural frequencies. Such distributions indicate the areas of the structure in which the higher-order models are most needed. In this case, the clamped edges are the areas in which the fourth-order models are required. The NN obtained these results after training with a given set of samples.

4 FSDT, 12 N=4, E = 1.51% | 8 FSDT, 8 N=4, E = 3.24% | 12 FSDT, 4 N=4, E = 5.29%

Figure 1: Element-Wise distributions of structural theories and accuracies over a 4X4 mesh, 90/0, a/h = 10, clamped-free

A second set is considered a simply-supported structure with a/h = 50 and stacking sequence 0/90/0. The results are shown in Figure 2 and highlight the strong problem-dependency of the optima mesh distribution.

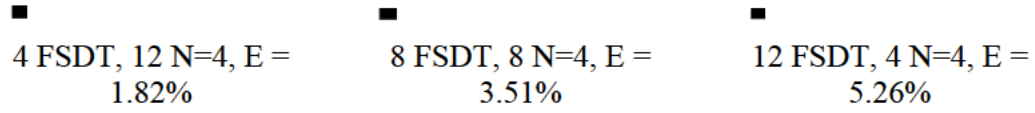

Figure 2: Element-Wise distributions of structural theories and accuracies over a 4X4 mesh, 0/90/0, a/h = 50, simply-supported

4 Conclusions

This paper presents numerical examples obtained using surrogate models based on neural networks in place of FE analyses to build optimized structural models for the analysis of composite structures. The Node-Dependent Kinematics capabilities of the Carrera Unified Formulation were exploited to build FE models with structural theories varying node-wise. Structural dynamics problems were considered, and natural frequencies were computed. Traditional FE models were used for verification purposes and to evaluate the precision of the surrogate models in detecting natural frequencies. The network employed used structural attributes as input parameters, successfully capturing the underlying relationship. The surrogate model correctly identified the most critical zones of the structure in which refined structural theories are mandatory. The network also considerably reduced computational times otherwise traditionally required for this kind of analysis. The proposed methodology can evaluate the role of refined terms and obtain the best theories with superior accuracy and computational efficiency.
Furthermore, using machine learning algorithms is promising to obtain indications on building refined models and FE discretizations. Future developments should include including failure parameters to use surrogate models for evaluating the damage onset. Then, nonlinear problems should be tackled to exploit the computational efficiency further, such as progressive failure and damage.

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