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Analysis of the Periodic Orbit in a Discretely Supported Railway Track Under a Moving Load

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Abstract

This paper studies the periodic orbit that appears in the deformation of a railway track on ballast with discrete supports due to a moving load. The track modeling methodology is based on the finite element method. A convergence analysis is performed to determine the number of elements per span needed to obtain accurate and reliable results without unnecessarily increasing computational costs. A sensitivity analysis of the main track parameters is also performed to study their influence on the periodic orbit. Different orbits are obtained depending on the velocity on the moving load and the value of the load itself when the velocity is constant, respectively. Additionally, the influence of the stiffness of the railpads and the ballast, respectivelly, on the maximum deflection of the orbit is analyzed. The result obtained is that the shape of the orbit depends little on the speeds considered, but the deflection increases with speed. As logic indicates, the deflection in the orbit increases as the load increases for a constant speed. Regarding the influence of the stiffness of the railpads and the ballast, the conclusion is that it is greater for low stiffness and decreases as the stiffness increases.

Keywords: finite elements, railway track, flexible track, sensitivity analysis, periodic orbit, discrete support

1 Introduction

The dynamic analysis of railway vehicles requires an accurate representation of the vehicle, the track geometry and structure, and their interaction [1].

For the modeling of track dynamics, a number of tools are available. One of them is the finite element method, that can be used to get the most realistic track models, normally requiring high computational costs [2]. Many finite elements programs are commercially available and applicable to the railway sector for simulating the vehicle, the complete track structure and the terrain [3]. A survey of railway track modeling can be found in [4, 5, 2] .

The ballasted track is the traditional structure of the track used worldwide and still the most used. For a ballasted track, the rails, sleepers and ballast bed contribute to the system vibration. Accurately simulating the dynamics behaviour of ballasted track, especially the discrete ballast bed, is still a challenging topic in the railway community [2].

In practical applications, although models with continuous support provide a good approximation of the overall trends in dynamic behavior, the discrete nature of track supports created by the sleepers has a significant impact. The most pronounced effect occurs at the pinned-pinned frequency, where the sleeper spacing corresponds to half a bending wavelength in the rail [6].

Generally, the Euler beam or Timoshenko beam model could be applied to model the continuous elastic rails. The wheel-rail interaction force obtained by using the two models do not show a significant difference [2]. When discrete support is considered, different support models and different track models can be used ([7, 8, 1, 9]).

In recent years, there has been a growing interest in understanding the behavior of discretely supported railway tracks, particularly focusing on periodic orbits – repetitive patterns of track behavior that could have significant implications for simplifying modeling [10, 11, 12].

In this paper, we will investigate the influence of different parameters, such as train speed, load, and support stiffness, on the characteristics of periodic orbits obtained with a 2D finite element railway track model. The study of the periodic orbits that appear in tracks with discrete supports will enable the development of methods that reduce computational costs compared to traditional methods in the simulation of tracks with discrete supports. For example, simplified models of tracks with variable parameters that generate the same periodic orbits can be developed, and it will also advance the development of the Moving Modes Method [13, 14] for the case of tracks with discrete supports.

2 Finite Element Model of the Railway Track

This section describes the model a railway track on ballast with discrete supports representing the sleepers using the finite element method. The track model has a finite length but is long enough such that the periodic orbit can be obtained without edge effects.

2.1 Equations of motion of the track

The equations of motion for a discrete support railway track can be written as follows:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q_g} + \mathbf{F}(t) \tag{1}$$

where $\mathbf{q}$ has dimension $n_q = n_p - n_{pf}$, being n_p the total number of nodal coordinates (2·number of nodes) and n_{pf} the fixed nodal coordinates; $\mathbf{Q_g}$ is the generalized force due to self weight, and $\mathbf{F}(t)$ is the generalized force of the applied load (moving load). Displacement is measured using the undeformed track as reference position. A description of the nodes and degrees of freedom can be found in the next subsection.

2.2 Generalized coordinates

If one element is considered per span, the track is discretized with nodes located at the rail supports on the sleeper, nodes on the sleepers, and nodes on the support. Each section between two sleepers would be defined by six nodes, two at rail level, two at sleeper level and two at support level. Each node has two degrees of freedom. Connection between nodes, degrees of freedom and the coordinate system considered can be seen in Fig.1. More than one element can be considered per span, increasing the number of nodes in the system. In Fig.1, two elements per span have been considered. Therefore, the vector of nodal coordinates, $\mathbf{p}$, includes the nodal coordinates of the rail, the sleepers, and the support, and is given by the following:

$$\mathbf{p} = [\mathbf{p}^{rail} \quad \mathbf{p}^{sleepers} \quad \mathbf{p}^{support}]^T \tag{2}$$

being

$$\mathbf{p}^{rail} = [u_z^1 \quad \theta_y^1 \quad u_z^2 \quad \theta_y^2 \quad \cdots \quad u_z^n \quad \theta_y^n]^T \tag{3}$$

$$\mathbf{p}^{sleepers} = [u_z^{s1} \quad \theta_y^{s1} \quad \cdots \quad u_z^{sn} \quad \theta_y^{sn}]^T \tag{4}$$

$$\mathbf{p}^{support} = [u_z^{su1} \quad \theta_y^{su1} \quad \cdots \quad u_z^{sun} \quad \theta_y^{sun}]^T \tag{5}$$

where the two degrees of freedom of the support are fixed, as well as the sleeper rotation (see Fig.2).

Vector $\mathbf{q}$ includes of the following coordinates:

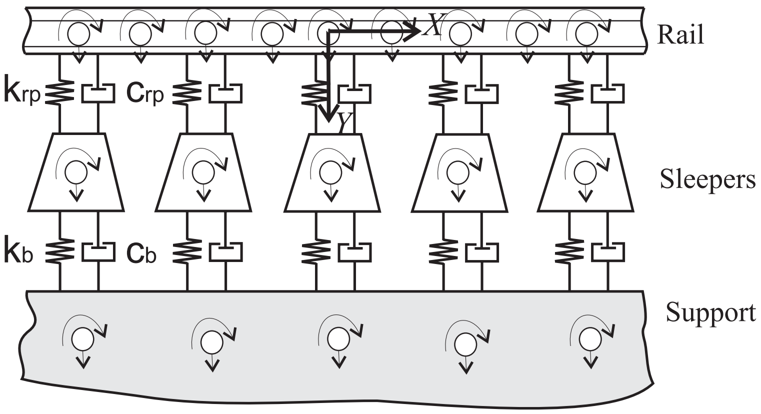

Figure 1: Railway track model. The circles represent the nodes with the degrees of freedom included in vector $\mathbf{p}$

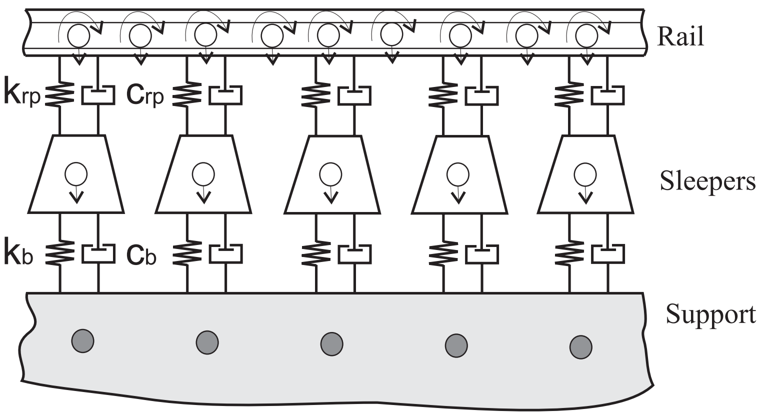

Figure 2: Nodes degrees of freedom included in vector $\mathbf{q}$

$$\mathbf{q} = [u_z^1 \quad \theta_y^1 \quad \cdots \quad u_z^n \quad \theta_y^n \quad u_z^{s1} \quad \cdots \quad u_z^{sn}]^T \tag{6}$$

Vectors $\mathbf{p}$ and $\mathbf{q}$ are related by the connectivity matrix $\mathbf{B}$ as follows:

$$\mathbf{p} = \mathbf{B}\mathbf{q} \tag{7}$$

The relationship is as follows:

$$\begin{bmatrix} u_z^1 \\ \theta_y^1 \\ \vdots \\ \theta_y^n \\ u_z^{s^1} \\ \vdots \\ \theta_y^{s^n} \\ u_z^{su^1} \\ \vdots \\ \theta_y^{su^n} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} u_z^1 \\ \theta_y^1 \\ \vdots \\ u_z^n \\ \theta_y^n \\ u_z^{s^1} \\ \vdots \\ u_z^{s^n} \end{bmatrix} \tag{8}$$

having the identity matrix $\mathbf{I}$ dimension n_qxn_q, and the matrix $\mathbf{0}$ dimension n_{pf}xn_q.

2.3 Mass, damping and stiffness matrices

The rail spans are modeled as cubic beam elements. The railpads and ballast are modeled as massless rods that deform axially in a linear fashion. With the FEM technology the mass, stiffness and damping matrices of each element, $\mathbf{M_e}$, $\mathbf{K_e}$ and $\mathbf{C_e}$, respectively, can be calculated independently and included in the system matrices as follows:

$$\mathbf{M} = \mathbf{M} + \mathbf{B}^T \mathbf{C_{ei}}^T \mathbf{M_e} \mathbf{C_{ei}} \mathbf{B} \tag{9}$$

$$\mathbf{K} = \mathbf{K} + \mathbf{B}^T \mathbf{C_{ei}}^T \mathbf{K_e} \mathbf{C_{ei}} \mathbf{B} \tag{10}$$

where $\mathbf{C_{ei}}$ is the Boolean connectivity matrix that selects the four coordinates of an element in vector $\mathbf{p}$. It has dimension [4xn_p] and by pre-multiplying this matrix by $\mathbf{Bq}$, i.e. $\mathbf{C_{ei}Bq}$, it selects the four coordinates of an element.

The damping matrix of the elements representing the spans is considered to be zero, and that of the elements representing the railpads and the ballast has the same form as the stiffness matrix of these elements, changing the value of the stiffness constants for the damping constants.

2.4 Generalized force

The generalized force associated with the weight of each rail element, $\mathbf{Q_g^e}$, and the weight of each sleeper, $m_s g$, are calculated with the FEM technology. This vector is included in the total generalized force vector, $\mathbf{Q_g}$, as follows:

$$\mathbf{Q_g} = \mathbf{Q_g} + \mathbf{B}^T \mathbf{C_{ei}}^T \mathbf{Q_g^e} \tag{11}$$

The generalized force associated to the moving load, $\mathbf{F}(t)$, is calculated as $\mathbf{F} = (\mathbf{N}(Vt)\mathbf{C_{ei}}\mathbf{B})^T \cdot P \cdot g$, where $\mathbf{N}(Vt)$ is the shape function evaluated at the point of application of the load P that is moving with velocity V.

3 Results

Table1 shows the parameter values used in the simulations performed for the convergence and sensitivity analyses. In the various sensitivity analyses, one of the parameters is varied while the rest maintain the values shown in the table.

Parameter	**Value**
Railpads	
Stiffness, k_{rp} (N/m)	1.5e9
Damping, c_{rp} (N/m/s)	67.5e3
Height, h_{rp} (m)	0.1
Ballast	
Stiffness, k_b (N/m)	0.18e9
Damping, c_b (N/m/s)	120e3
Height, h_b (m)	0.5
Sleepers	
Mass, m_s (kg)	150.5
Span, L_s (m)	0.6
Rail	
Mass per unit length, ρA (kg/m)	60
EI (Nm^2)	6400800
Load	
P (kg)	784+6000

Table 1: Parameters used

3.1 Convergence analysis

In a discretely supported track modeled with finite elements, the appropriate number of elements between supports should be analyzed by conducting a convergence study. The following figure (Fig.3) shows the periodic orbit obtained with 1, 2, 4, and 6 elements per span, respectively. The vertical displacement shown in the figure corresponds to the moving load application point. If only the displacement of the midpoint between supports is of interest, 2 elements would suffice to obtain accurate and reliable results without unnecessarily increasing computational costs. However, to obtain the complete orbit, 4 elements are more appropriate. The reason can be observed in Fig.3: the complete periodic orbit is not precisely obtained with 2 elements per span. Both with 4 and 6 elements, the same orbit is obtained (the difference is considered negligible), leading to the conclusion that 4 is the adequate and sufficient number of elements to represent the complete orbit. It is observed in the figure that the difference between using 4 and 6 elements is negligible.

The orbit that differs the most is the one obtained with 1 element per span. In this orbit, it is observed that the ends of the orbit, corresponding to the rail supports, displace more than the central area, corresponding to the rail section between supports. This behavior is due to the fact that the nodal coordinates at the rail level are located at the supports. These nodal coordinates are greater when the load passes directly over the support compared to when the load is in the central area of the span. If the nodal coordinates are smaller when the load passes through the central area, interpolating to obtain the deflection at the load application point results in a deflection greater than that at the supports at that moment, as logic dictates, but smaller than that at the supports when the load is applied directly on them.

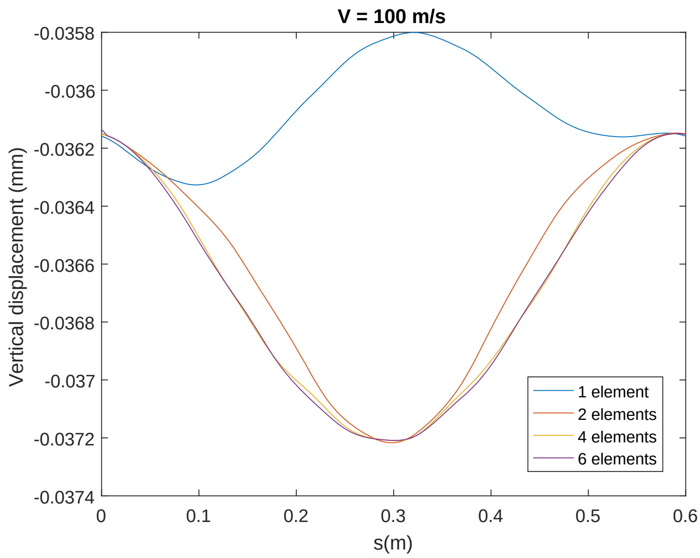

Figure 3: Periodic orbit with 1, 2, 4 and 6 elements, respectively, when the velocity is 100 m/s and the load 784 kg

3.2 Sensitivity analysis

A sensitivity analysis is conducted to examine the influence of certain track parameters on the periodic orbit that appears between supports.

The orbits for different speeds of the moving load, ranging from 20 to 50 m/s, are shown in Fig.4. The vertical displacement shown in the figure corresponds to the moving load application point. It can be seen that the track displacement increases with increasing speeds.

Figure 5 shows the resulting orbit when a moving load of different weights travels along the track at a speed of 50 m/s. The vertical displacement shown in the figure

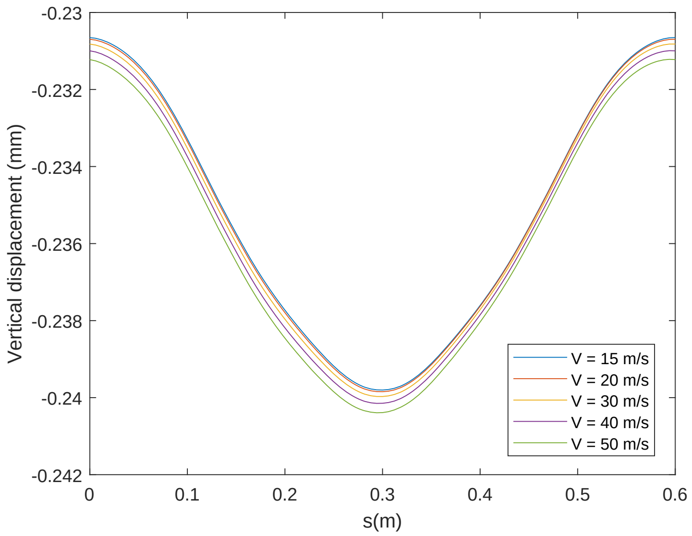

Figure 4: Periodic orbit for different velocities of the moving load.

corresponds to the moving load application point. The orbits for different loads are compared with the static deflection of the track when no load is applied.

Maximum vertical displacement of the orbit while changing the stiffness of the railpads and the ballast, respectively, are shown in Figs.6 and 7. The vertical displacement shown in the figures corresponds to the moving load application point. It can be observed in these figures that for lower stiffness values, the variation in maximum displacement of the orbit is more abrupt, while it smooths out as the stiffness increases.

4 Concluding remarks

A 2D finite element model for a ballasted railway track with sleepers has been described. The periodic orbit is analyzed by varying certain parameters such as load, speed, ballast stiffness, and rail pad stiffness.

This model will be expanded in future work by converting it to 3D, and the parts of the track currently modeled as Euler-Bernoulli beams may be modeled as Timoshenko beams. The study of periodic orbits will enable the development of more computationally efficient methods for modeling railway tracks on ballast with discrete supports in the future.

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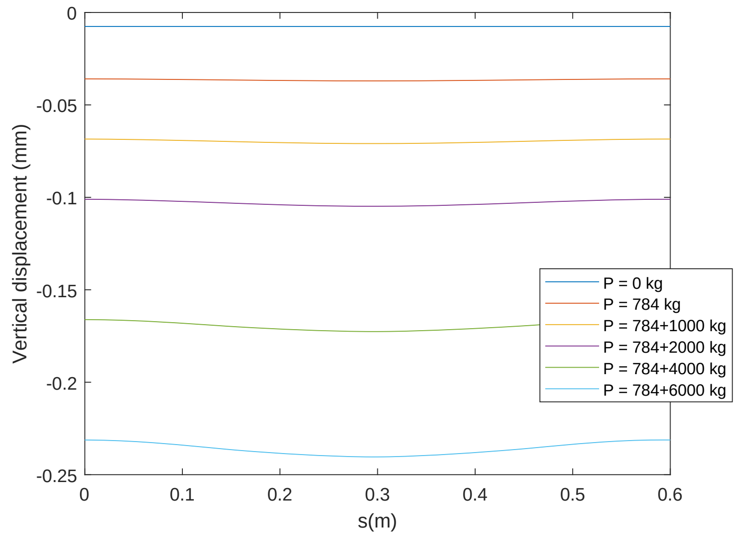

Figure 5: Periodic orbits for different loads moving at 50 m/s.

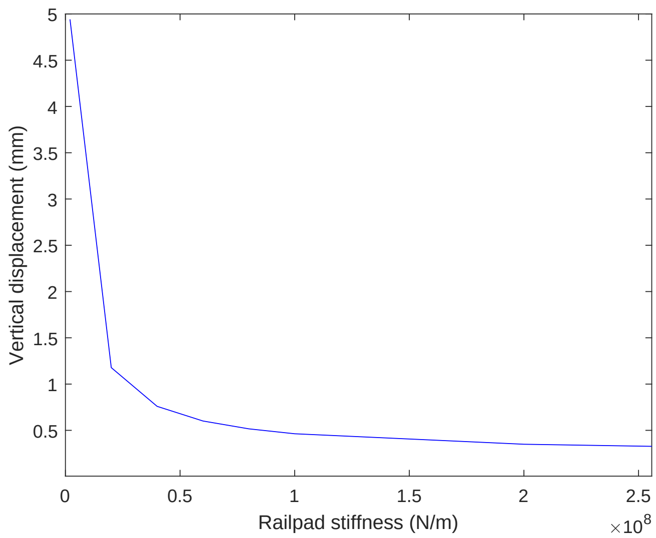

Figure 6: Maximum displacement of the orbit with varying railpad stiffness

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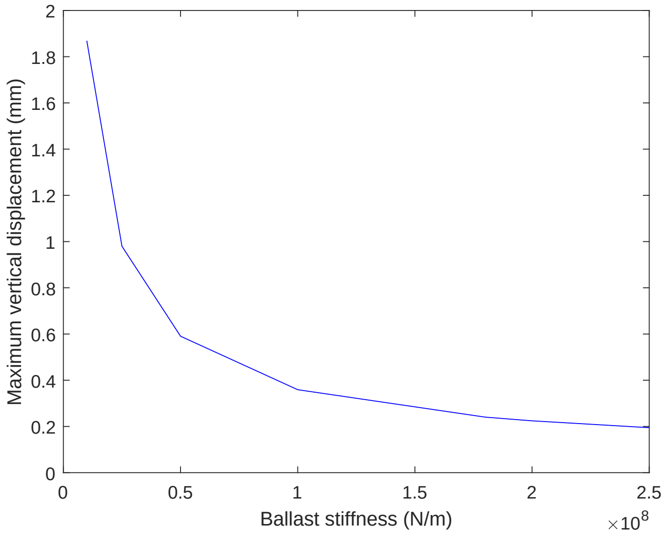

Figure 7: Maximum displacement of the orbit with varying ballast stiffness

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