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Simulation of Dynamic Interaction and Long-Term Processes of Void Accumulation in Ballast Layer

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Abstract

This paper presents an approach for simulation of the dynamic interaction of the sleeper and ballast in void zones, and the long-term processes of the settlement accumulation. A DEM model of a ballast box and one sleeper with spring support was developed to reflect the influence of the rails in the void zone. The model allows to simulate the void development as well as the void control. The ballast model was prepared to produce the results useful for long-term prediction. Additionally, an approach is proposed to accelerate the simulation time by using parallel simulations with different controlled initial voids. The simulation results indicate quick settlement accumulation after appearing of a void. Analysis of the dynamic interaction explains the reasons for the settlement acceleration: the impact loading and vibration that appears while void closing. The dynamic processes correspond to the experimentally measured ones: impact interaction appears in time before the maximal cyclic loading. The impact loading increases while the void growth, but the cycling loading decreases due to the loading transfer in the spring support. It results in a void of 5-6mm with the maximal settlement intensity where the amplitudes of the dynamic and the cyclic loadings are about the same level. The further increase of the void decreases the intensity. The results indicate that the impact loading is absolutely dominating in the settlement accumulating. The study results could improve the present ballast settlements' phenomenological equations to take into account the impact and vibration effects.

Keywords: ballast layer, void zones, DEM simulation, hanging sleeper, impact interaction, settlement intensity.

1 Introduction

Track geometry deteriorates due to a number of failure mechanisms, one of which is sleeper voids or unsupported sleepers, which are one of the earliest and most frequent failures. Sleeper voids and resulting track irregularities usually arise in the transition zones to bridges, culverts, underground communications, rail welds, and support parts of railway turnouts [1–7]. However, due to the multitude of influencing factors and complex mechanisms of their influence, the prediction of void development is difficult.

The ballast layer under the sleepers in zones with unsupported sleepers is exposed to different loading patterns [8], which depend on the position of the sleepers in the zones. The appearance of the particular impact interaction is demonstrated by the experimental measurements [9, 10, 11] of the train-track dynamic interaction in a void. Numerous other investigations have indicated the impact of unsupported sleepers on the dynamics and stability of railway tracks [12, 13, 14, 15].

In studies [9], rail deflection data (Figure 1) clearly describe the dynamic behavior in the void zone. Rail accelerations that occur before time to the maximum rail deflections clearly demonstrate the impact interaction (Figure 1, right). In contrast to simple geometric imperfections, the primary characteristic of dynamic interaction in void zones is the appearance of impact oscillations prior to the wheel entering the middle part of the void zone. Consequently, when the wheel outside is moving, the highest dynamic interaction occurs in the first half of the void zone, while the maximal interaction in the geometrical irregularity appears usually in the second part of it.

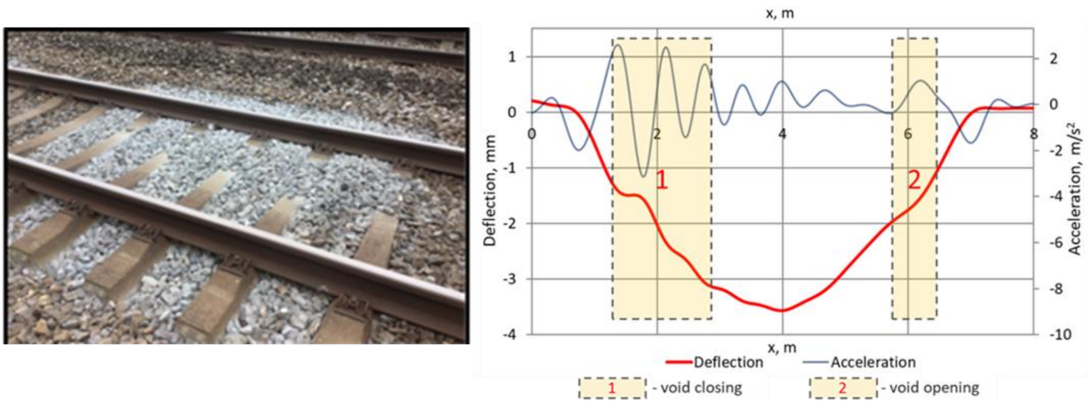

Figure 1: Track-side rail deflections and accelerations in the void zone for one axle (right), the ballast pulverization in the void zone (left) [9].

The prediction of long-track geometry deterioration in void zones, which is considered in many studies [16-22], is usually based on different phenomenological equations. However, most of the models take into account only the influence of the cyclic ballast loading and do not consider the influence of the vibration, impact, and sleeper unloading, which appear in the void zones. The impact loadings are usually considered to have the same influence as the quasistatic loading.

Discrete element modeling is one of the methods that could potentially reflect the long-term processes of ballast settlements. Recently, there has been an increase in the use of DEM for imitating ballast settlements. The discrete element technique allows for the examination of intrinsic mechanical processes, in contrast to empirical phenomenological equations that have little mechanical substantiation.

Discrete element analysis was used by Chen & McDowell [16] to look into transition zones from a micromechanical viewpoint. Nevertheless, the rail's elastic influence remained ambiguous, so the hanging sleepers were ignored. Due to the lack of free slopes in the ballast box, the model was unable to account for the sleeper settlements caused by the ballast flow along the sleeper.

For the modeling of 0–5 consecutively unsupported sleepers within 15 ones, Fang et al. [17] presented a model integrating the DEM and MBD. Nevertheless, the authors did not examine the long-term mechanisms of ballast settlement.

Using a DEM model, the researchers [18, 19] examined the settling behavior of a half-sleeper in a square ballast box. Throughout 1000 loading cycles, the variations in ballast porosity and pressure were examined. The models were nevertheless missing any free slopes that would have allowed for the lateral distribution of the ballast along the sleeper.

The major stress rotation impact was described by Bian et al. [20] using a DEM simulation of a ballast box with five sleepers that was 30 cm wide. However, the quasi-2D model was also unable to account for the ballast's lateral flow.

In examining ballast deformation under high-speed train loads, the study [21] highlighted the significance of load frequency while applying cyclic loads. The DEM model clearly demonstrates the increase in settlement intensity at loading frequencies greater than 15 Hz, even with a small-scale ballast box that has a 30 cm sleeper section. It implies that the vibration factor is an independent element that affects ballast resilience directly and cannot be considered through the maximum loading amplitude.

In order to predict the settlement behavior of full-scale test sections under repeated severe axle train loading, Tutumluer et al. [22] employed a DEM model in their investigation. There was a 2000 loading cycle limit on the number of simulation cycles. It is shown that the ballast DEM simulations with only 2000 car passes closely expected the settlement performance by correctly accounting for the initial compaction conditions. In the experimental observations, 580 thousand loading cycles equated to the initial compaction in the DEM model.

The analysis of the present studies on long-term settlement accumulation in void zones using DEM shows that many authors emphasize the importance of impact and vibration loading. However, many authors present 2D models that consider sleeper-ballast dynamic interaction in a void but do not consider the ballast flow along the sleeper. On the contrary, other authors consider the 3D models with ballast flow but without the impact loading. Due to computational costs, the track model can only be used for small ballast boxes with a single sleeper and no slopes, or it can only be a 2D model for large ballast boxes.

The present study aims an investigation the sleeper permanent settlements' behavior under the cyclic and impact loadings in the void zones using the DEM approach. A ballast box with a single sleeper that is suspended by springs is developed. The model enables to observe the sleeper settlements and void development. The void growth is accompanied by the impact increase and the decrease of the cyclic loading. Moreover, the initial void can be controlled, which allows to apply the parallelized simulation and to reach a time-effective settlement solution for the whole settlement lifecycle. Finally, the short-term and long-term processes are analyzed.

2 Methods

The method is based on numerical simulation of short- and long-term processes of void accumulation by using the discrete element method (DEM). Modeling of long-term processes of void accumulation is produced in the following steps:

- Preparation of the ballast box with a sleeper that is supported by springs. The springs enable the simulation of the hanging sleeper and void under the sleeper.
- Stabilisation procedure of the ballast layer for producing the maximal ballast compaction. The step enables avoiding the first nonlinear stabilization phase and further close to linear settlement accumulation. The linear settlement rate simplifies and makes more certain the long-term prediction based on DEM simulations.
- Parallelisation of the settlement accumulations during the lifecycle by controlling the initial void. The step enables many times reduction of the simulation times for the settlement prognosis over the lifecycles of the zones with voids.

The DEM models are generally not appropriate for the simulation of the long-term processes of the whole lifecycles over millions of loading cycles. The plausible simulation times are limited to thousands of cycles. Therefore, due to the high computational cost of the DEM model, it is maximally simplified. The model complexity is selected to reflect properly the process of ballast flow that is assumed to be a reason for the void accumulation. The additional assumptions are: particle breakage and attrition are not taken into account; the ball particles are employed with rolling resistance to reflect the angularity of the particles; the sleeper is an undeformable body.

The model of the ballast box with a sleeper is supported by springs (Figure 2). The springs correspond to the bending stiffness of the rails for the case of hanging sleepers in the void zone. The overall stiffness of the springs, 15 kN/mm, corresponds to the void length of 2.4 m and the rails UIC60.

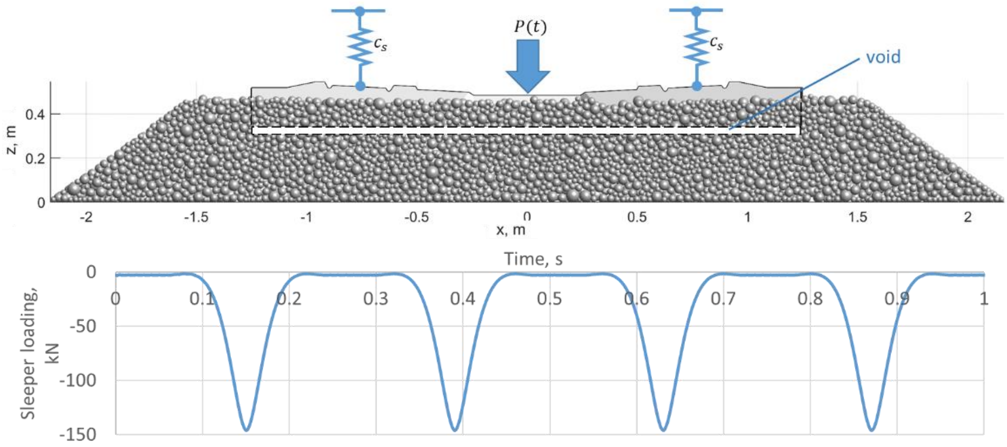

Figure 2: DEM model for simulation of the sleeper settlements with void development (top); sleeper loading pattern (bottom).

The geometry of the sleeper matches that of the standard concrete sleeper. The particle shape is presented by simple balls with a standard size distribution of 22.5-63 mm. The rolling radii, non-linear contact law, tangential stiffness, rolling resistance, and other properties are used to characterize the ball particles. The tangential force can be described by the Mindlin-Deresiewicz model.

As the normal force model, a Hertzian spring with viscous damping was selected. The impact of real particle shape is considered in the rolling resistance. The model's parameters are chosen based on theoretical and experimental research [23–30].

Boundaries' material characteristics are as follows: right and left trog walls: 20 Gpa Young's modulus, 0.3 Poisson's ratio, and 1700 kg/m^3 bulk density; 10 Gpa Young's modulus, 1600 kg/m^3 bulk density. The following are the interaction parameters between the boundaries and the ballast particles: static friction 0.6, dynamic friction 0.58, and restitution coefficient 0.72 between the trog bottom and the particles; friction near zero and restitution coefficient 0.65 between the right and left trog walls and the particles.

Boundaries have the following material characteristics: The right and left trog walls have a Young's modulus of 20 Gpa, a Poisson's ratio of 0.3, and a bulk density of 1700 kg/m^3. The trog bottom has a Young's modulus of 10 Gpa and a bulk density of 1600 kg/m^3. The following are the parameters of the interaction between the boundaries and the ballast particles: between the trog bottom and the particles, there is static friction of 0.6, dynamic friction of 0.58, and restitution coefficient of 0.72; between the right and left trog walls and the particles, there is friction that is almost zero and restitution coefficient of 0.65.

The static friction coefficient is 0.60, the dynamic friction coefficient is 0.58, and the restitution coefficient is 0.72 for the interaction between the sleeper and the ballast particles. The ballast layer is filled into a 60 cm wide ballast trog, and the ballast height beneath the sleeper is roughly 30 cm.

In the DEM simulations, the sleeper is represented as a rigid body, with its center bearing all external loadings. Particles are put into the sleeper sides to create a ballast bed that slopes and has 40 cm ballast shoulders. The quantity of particles varies according to the type of sleeper: 25788 for a conventional sleeper and 21287 particles for a wide sleeper.

The external loading on the sleeper describes the vertical loading on the sleeper from rails. It is assumed that the process of loading corresponds to the beam on an elastic foundation with a wavelength of about 3-4 m. Actually, the void wavelength is, on the one hand, longer than the elastic one (Figure 1), but on the other hand – the loading increase in the void zone could be higher for the good sleeper support. The loading cycle period is 0.15 s (Figure 2), which approximately corresponds to a train velocity of 100 km/h. The sleeper loading is cyclic with isolated four loadings/s and the short delays between them for the oscillation damping until the text loading.

The maximal loading is selected at 146 kN, which is about 1.5 times more than for the normal sleeper. Such increased loading allows the plausible simulation times for the void accumulation.

The stabilization procedure of the ballast layer is used to reach the maximal compaction and thus close to linear settlement accumulation. In this way is proposed to avoid the problem of the initial compaction uncertainty. The close-to-linear settlement accumulation allows prediction of the settlement accumulation over time much longer than the simulation time. The detailed procedure for the ballast stabilization in the model is described in [8].

The parallelization procedure of the settlement accumulations during the lifecycle is based on the idea of controlling the initial void by the movement of the ballast box downsides. It is supposed that the further void accumulation will correspond to a certain moment within the overall lifecycle process. The assumption enables a reduction of the simulation times for the overall lifecycle due to the distribution of the calculation to many separate simultaneous calculations with different initial voids. However, the procedure assumes that the ballast state is constant and the variation of its compaction and geometry variation or strengthening effects are neglected. An additional problem is combining the parallel simulation fragments into one process.

3 Results

The simulation is performed for two cases: the reference case as the normal sleeper without void and the sleeper with spring support and controlled initial void. Due to the specialty of the simulation data organization, the simulation results are presented in two ways: the long-term processes with low discretization step 0.02 s and the high frequency (0.001 s) short fragments of several cycles many times along the complete loading cycles. Such data organization allows for avoiding data explosion and saving dynamic details. Therefore, the analysis is performed correspondingly: the continuous long-term processes; the short-term dynamic interaction; and the pointwise analysis of the quick dynamic processes along with the void accumulation.

3.1 Long-term processes of the settlement accumulation and cyclic loadings

The analyzed parameters of the slow long-term processes are the maximal sleeper displacement, ballast void, and cyclic loading of the sleeper on the ballast. The results of the settlement accumulation for the sleepers with and without voids, as well as their comparison, are shown in Figure 3. The simulation of the reference case presents a close-to-linear trend of the settlement accumulation. It allows a certain estimation of the settlement intensity. Therefore, the simulation was stopped after 2500 loading cycles.

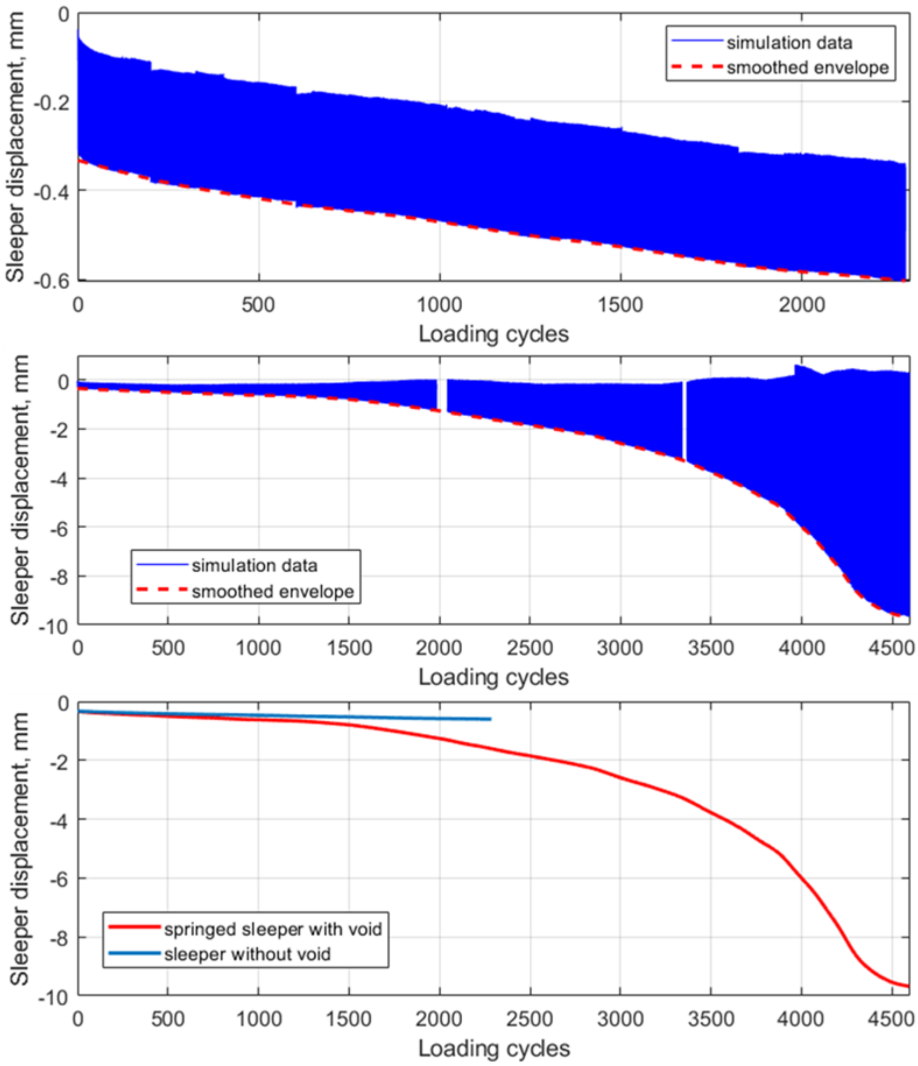

Figure 3: Sleeper displacements for the sleeper without void (top), sprung sleeper with voids (center), and the comparison of the maximal displacements of the both cases (bottom).

In contrast, the simulation of the sprung sleeper with voids presents nonlinear settlement accumulation due to the influence of increasing impacts, ballast vibration, and cyclic load reduction with increasing void. The parallelization of the simulations was applied with a controlled initial void at 0, 1, 3, 4, 5, 6, and 7 mm. After that, the simulation fragments were collected together to depict the whole void development process. Thereby, the length of the separate simulations was determined by the settlement intensity. After reaching the displacement that is at the start of the next simulation fragment, it was stopped. Nevertheless, many simulation displacements were overlapping, and some were not enough, which are visible as gaps in Figure 3 (center). The gaps were trimmed to fit to best the settlement trend.

The comparison of the maximal rail deflections for both cases (Figure 3, bottom) shows that the sleeper displacements and the settlement intensity are close in the first loading cycles. The general trend of the sleeper deflections for the sprung sleeper is the s-formed with maximal intensity in the middle part. The trend for the sleeper without voids is close to linear. After the 500 loading cycles, the maximal displacement indicates the evident deviations between the trends. The accelerated growth of the deviations is after 1500 cycles observable. At the last cycles (more than 4500 cycles), the settlement process is slowing down, and it is expected that it will not exceed 10 mm, which corresponds to the sleeper spring stiffness of 15 kN/mm and the maximal external loading on the sleeper 150 kN.

The comparison of the settlement intensities is presented in Figure 4. The sleeper settlement intensity for the case without void is about 0.11-0.15 mm/1000 cycles. The settlement intensity for the voided sleeper is close to that with normal support until about 1200 loading cycles. After that the settlement intensity accelerates until the 4100-4250th cycle with the maximal value of 9.5 mm/1000 cycles and afterwards decreases until the last cycles.

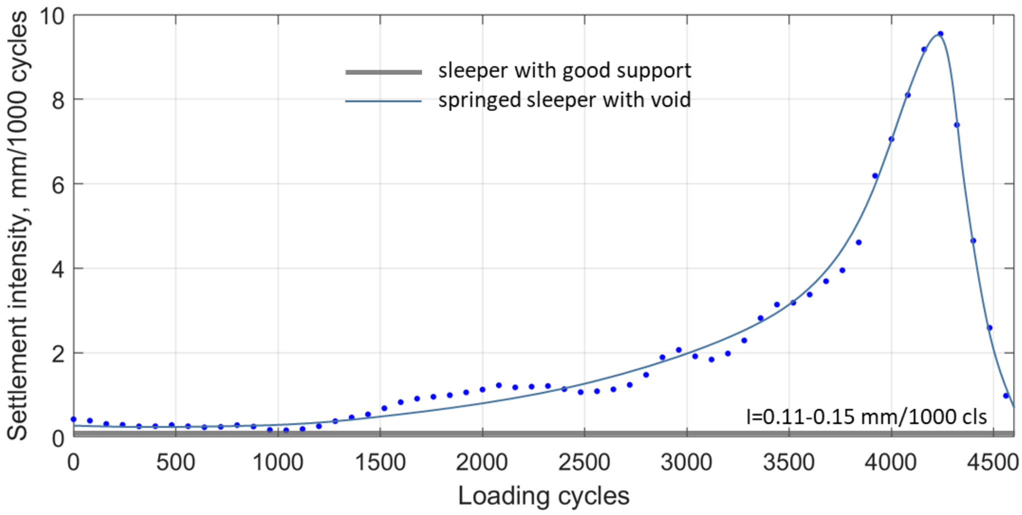

Figure 4: Sleeper settlement intensity for a good sleeper support and for a sleeper with spring support.

3.1 Short-term processes of the dynamic interaction

Accumulation of the void is accompanied by dynamic effects due to the interaction between the sleeper and the ballast bed. The ballast bed was partially unloaded at the beginning of the settlements of the sprung sleeper. After a full unloading appears, a void that causes impact and vibration while the void closes. Figure 5 presents a comparison of the interaction with good sleeper support and with void 2 mm in the form of the particle velocities distribution in the moment of the maximal interaction. The impact velocity is more than 20 times higher in the case of the void than for the normal sleeper.

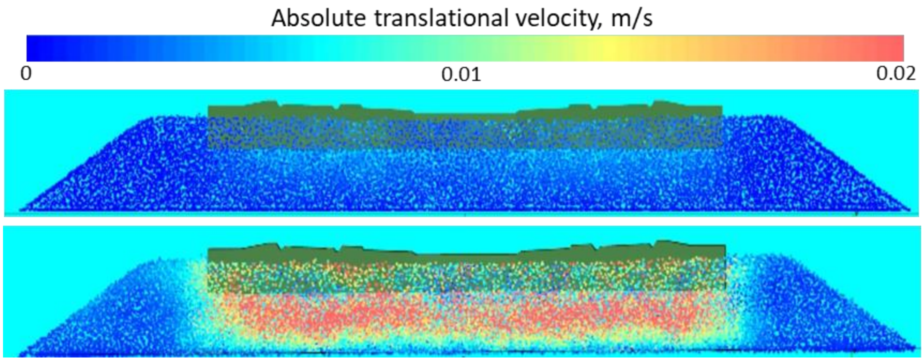

Figure 5 Particle velocity distribution across the ballast bed for the good support (top) and for the case with a void 2 mm (bottom).

The short-time dynamic interaction is studied in 10 time moments along the whole cycles of the settlement accumulation (Figure 6), which presents the sleeper velocity and the ballast loading for one loading cycle. The ballast bed loading (red line) corresponds to the external loading on the sleeper in the first loading cycles. However, with more passed cycles, the maximal cyclic loading decreases due to the sleeper settlements and redistribution of more loading at the springs. At the same time, about after 300 cycles, an additional dynamic loading occurs due to void closing. The impact loading appears before the maximal cyclic loading, which is similar to the experimental measurements. The dynamic loading is presented by several peaks with a frequency of 100-150 Hz that corresponds to the ballast and sleeper stiffness mass. The maximal impact loading becomes notable after 1800 cycles and increases up to 80 kN in the last loading cycles, where the void reaches about 6 mm. Thereby, the cyclic loading decreases to 70-100 kN at this loading point. The sleeper velocity process (blue line) corresponds in general to the loading process. However, during the unloading, an oscillation appears with a frequency of about 65 Hz corresponding to the spring support and sleeper mass.

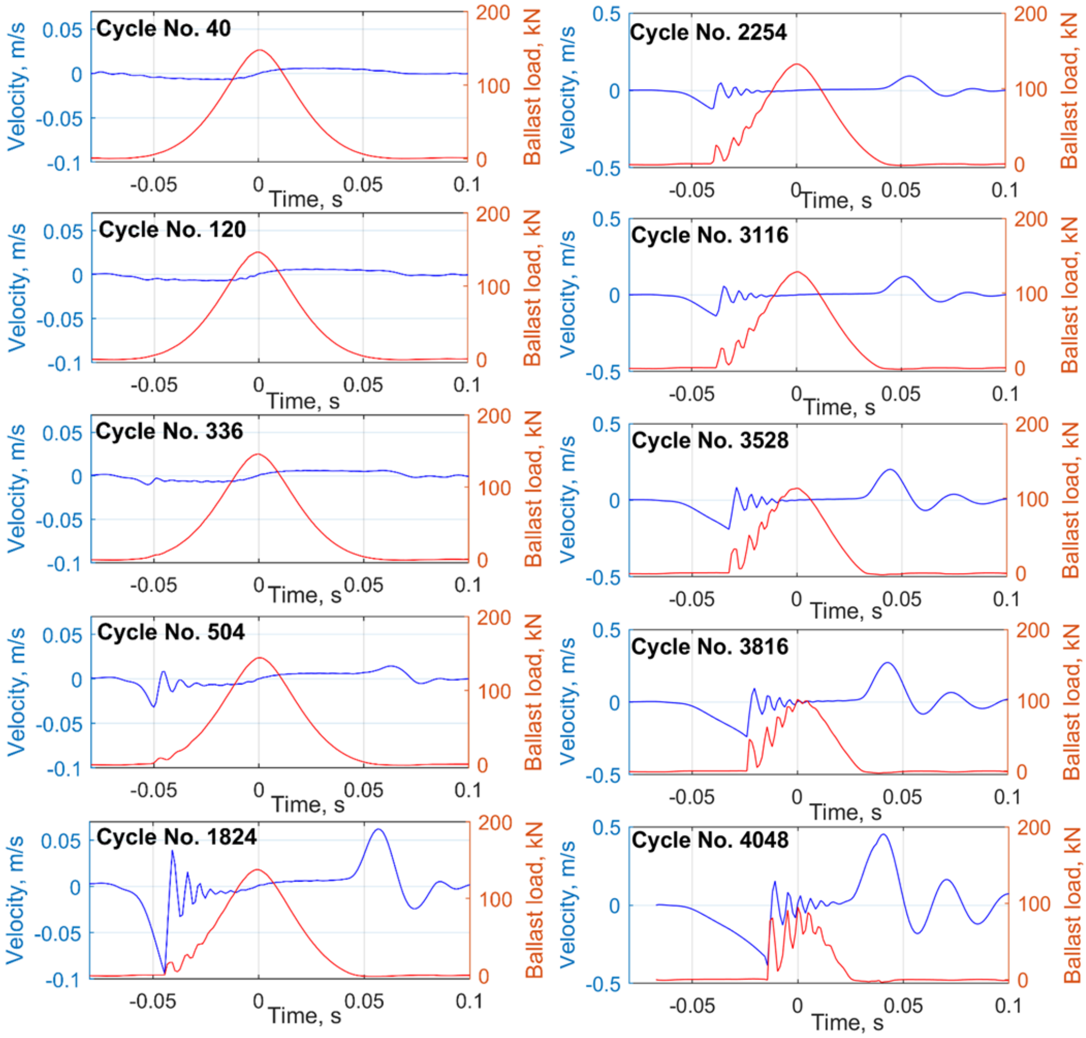

Figure 6: Dynamic interaction of the sleeper and ballast in the moments along the whole loading cycles.

The processes of the impact loading and the cyclic one are presented in Figure 7.

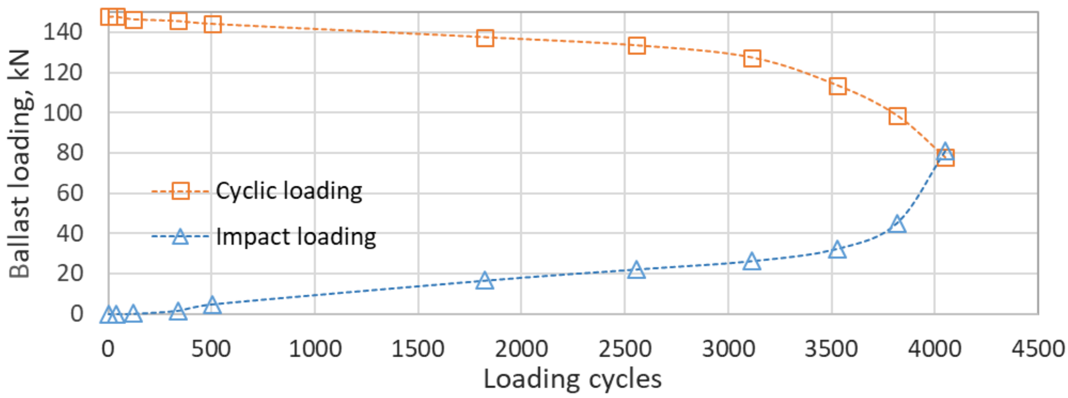

Figure 7: Quasistatic cyclic loading and impact loading within the whole lifecycle.

4 Conclusions and Contributions

The analysis of the present studies on long-term settlement accumulation in void zones shows the importance of considering the impact and vibration loading. The present approaches of using DEM are devoted to either the short-term dynamic interaction of the hanging sleeper or the long-term processes of the settlement accumulation with normal support. Moreover, the present approaches of long-term settlement simulation usually demonstrate very high settlement intensities and highly nonlinear processes that correspond to the stabilization phase. The settlements in the phase depend on the initial settlement, which is difficult to estimate. This does not allow to predict reliably the further settlement process.

The present approach aims to improve the disadvantage of the DEM models concerning long-term prediction. A novel model of the ballast box with a sleeper is presented that is supported by springs and enables simulation of the void development. The stabilization phase is excluded by the stabilization procedure of the ballast layer. It makes it possible avoiding of the first nonlinear stabilisation phase and further close to linear settlement accumulation. Parallelisation of the settlement accumulations allows many times reduction of the simulation times during the lifecycle by controlling the initial void.

The simulation results show similar experimental dynamic behavior in the void zone. Both the experimental measurements (Figure 1) and the simulations (Figure 6) indicate a similar loading pattern: the impact before the cyclic loading. The impact loading increases together with the growing void but the cycling loading decreases because the spring supports transfer the part of the external loading.

Both cyclic and dynamic loadings cause the accumulation of residual settlements. In the initial cycles, both sprung and void-less sleepers have almost the same settlement intensities. However, the settlement growth, caused by the ballast unloading, void impacts, and vibration, results in the acceleration of the settlements. The acceleration reaches the maximal value at the time point where the influence of the cyclic loading and the impact one come in the balance. The further increase of the void causes the reduction of the settlement intensity. The study results show that cyclic loading and impact loading have quite different contributions to the settlement intensity and cannot be considered as one factor. The impact and vibration are a dominating factor in the settlement intensity. It is supposed that the influence of both factors should be separately considered in the present phenomenological equations for the prediction of track geometry deterioration.

Although the simulation shows a similar to the experiment's dynamic effect and the plausible long-term process, only the relative results and the qualitative trends can be considered by comparing them with the results of other studies. Many critical aspects should be mentioned. The DEM model is oversimplified with ball particles and stiff sleepers without rotation and stiff subgrade. The sleeper loading is overdimensioned and it corresponds to the beam elastic line and not to the form of the void geometry. The void development is a complex process with varying void length and hanging stiffness.

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