



Proceedings of the Sixth International Conference on  
Railway Technology: Research, Development and Maintenance  
Edited by: J. Pombo  
Civil-Comp Conferences, Volume 7, Paper 16.3  
Civil-Comp Press, Edinburgh, United Kingdom, 2024  
ISSN: 2753-3239, doi: 10.4203/ccc.7.16.3  
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# **Identifying the Separate Contributions of Electromagnetic and Wave-Induced Instability Mechanisms to the Overall Dynamic Instability in Hyperloop and Maglev Systems**

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## **Abstract**

The Hyperloop, a developing transportation system, reduces air resistance by housing the vehicle within a depressurized tube and eliminates contact friction by using an electro-magnetic suspension/levitation system. Maintaining system stability poses a challenge due to the exceptionally high target velocities. Consequently, it is important to know a priori the velocity regimes in which the system can be unstable. The authors have recently investigated this aspect by, unlike previous studies, properly accounting for the frequency and velocity dependent reaction force provided by the infinite guideway. Furthermore, that study focused on the interplay between two fundamentally different instability sources, namely (i) the electro-magnetic suspension, and (ii) wave-induced instability, showing that stability domains drastically change above a certain vehicle velocity. The current study presents a methodology to distinguish the contribution of each instability mechanism to the overall system stability, and demon-

strates that the wave-induced instability mechanism is causing the drastic stability change at large vehicle velocities. This investigation offers physical insight into the mechanisms that can cause instability in the Maglev/Hyperloop systems, and can help engineers that develop this novel transportation system to avoid excessive vibrations and, in extreme cases, derailment.

**Keywords:** Hyperloop, magnetically-levitated vehicles, Maglev, dynamic stability, anomalous Doppler waves, moving-load dynamics.

## 1 Introduction

Hyperloop is an innovative transportation system that is currently under development. It minimizes air resistance by enclosing the vehicle in a de-pressurized tube and eliminates wheel-rail contact friction through the use of an electromagnetic suspension/levitation, similar to Maglev trains. This design can potentially achieve much higher velocities compared to traditional railways, positioning the Hyperloop as an environmentally friendly alternative to air transportation.

Some challenges encountered by the Hyperloop system have already been explored within the context of high-speed railways (e.g., vehicle-structure-soil interaction at large velocities [1–3], fatigue/degradation critical locations in the structure such as transition zones [4–8], influence of guideway periodicity on the system response [9] and system stability [10–12], etc.) and of magnetic levitation (Maglev) transportation systems [13] (e.g., levitation and propulsion design [14, 15], electro-dynamic instability [14], electro-magnetic instability [16, 17], aerodynamic instability [17, 18], parametric instability [19], etc.).

However, due to the substantially higher target velocities, it is anticipated that new hurdles will arise [20,21]. One such challenge involves ensuring the dynamic stability of the system at these elevated velocities, aiming to prevent excessive amplifications of the response. It is widely recognized that when a vehicle travels on an elastic guideway, its vibrations can become unstable once it surpasses a certain critical velocity [22]. Metrikin [23] demonstrated that instability arises when the energy associated with the radiation of anomalous Doppler waves, which feed back energy into the vehicle's vibration, exceeds that of normal Doppler waves. Identifying the velocity ranges in which the Hyperloop system may become unstable (i.e., determining critical velocities [2, 22, 24–30]) is crucial for its design and practical implementation.

The authors have previously investigated the interplay between the electro-magnetic and wave-induced instability mechanisms [31], and showed that the stability space changes significantly above a certain velocity. In other words, the control strategy can ensure the overall system stability only for a very limited range of its gains. The cause for this drastic change was attributed to the wave-induced instability mechanism [31]. The current study presents a methodology to distinguish the contribution of each instability mechanism to the overall system stability. More specifically, the

energy variation of the vehicle is divided in two components corresponding to (i) energy input by the electro-magnetic suspension and (ii) energy input by the guideway. While identifying unstable velocity regimes is practical for Hyperloop design, the ability to discern which mechanism causes system instability for a certain parameter combination is valuable can be crucial for efficient mitigation.

## 2 Model formulation

In this study, the system under consideration comprises an infinite Euler-Bernoulli beam characterized by mass per unit length  $\rho$  and bending stiffness  $EI$ . The beam is continuously supported by distributed springs ( $k_d$ ) and dashpots ( $c_d$ ). A vehicle with mass  $M$  and velocity  $v$  acts upon the guideway. The connection between the vehicle and the guideway involves a nonlinear electromagnetic force  $F$ , akin to the electromagnetic suspension utilized in Maglev trains, operating solely in attraction [18]. To facilitate the chosen solution approach, it is advantageous to express the governing equations in the reference frame moving with the vehicle, denoted by  $\xi = x - v\bar{t}$  and  $t = \bar{t}$ , where  $(x, \bar{t})$  and  $(\xi, t)$  represent the spatial and temporal coordinates in the stationary and moving reference frames, respectively. The system configuration is illustrated in Fig. 1, with its governing equations detailed in [18] and presented in the following:

$$EIw'''' + \rho(\ddot{w} - 2v\dot{w}' + v^2w'') + c_d(\dot{w} - vw') + k_d w = -F(t)\delta(\xi), \quad (1)$$

$$M\ddot{u} = F(t) - Mg, \quad (2)$$

$$F(t) = C \frac{I^2}{(w_0 - u)^2}, \quad (3)$$

$$\dot{I} = \frac{w_0 - u}{2C} \left( U - IR + 2C \frac{I}{(w_0 - u)^2} (\dot{w}_0 - \dot{u}) \right), \quad (4)$$

$$U = K_p(w_0 - u - \Delta^{ss}) + K_d(\dot{w}_0 - \dot{u}) + U^{ss}, \quad (5)$$

where primes and overdots denote partial derivatives in space  $\xi$  and time  $t$ , respectively,  $g$  is the gravitational acceleration,  $\delta$  represents the Dirac delta function,  $u$  is the mass displacement, and  $w_0 = w(\xi = 0)$  is the beam displacement under the moving mass. The electro-magnetic force  $F$  depends on the current intensity  $I$  and on the air-gap  $\Delta = w_0 - u$ , while  $C$  is a constant that depends on the electro-magnet properties [18]. Eq. (4) is a nonlinear differential equation governing the current intensity where  $U$  is the voltage and  $R$  is the circuit resistance.

In the absence of a control strategy, the formulated system exhibits inherent instability, even in the absence of vehicle horizontal motion. Consequently, a control strategy is implemented on the voltage  $U$  (Eq. (5)). A conventional proportional and derivative control approach is adopted, wherein  $K_p$  and  $K_d$  denote the position and velocity feedback gains, respectively. The error is defined as the deviation from the desired air-gap  $\Delta^{ss}$  (with the superscript  $ss$  indicating steady state);  $U^{ss}$  represents the voltage required to achieve the desired air-gap in the equilibrium state.

Since the system stability is dictated by the vehicle-guideway interaction, it suffices to investigate the response under the moving vehicle (i.e.,  $\xi = 0$ ). To this end, the guideway response  $w_0$  under the moving vehicle is written as follows:

$$w_0(t) = - \int_0^t G_0(t - \tau) F(\tau) d\tau + w_0^{\text{ic}}(t), \quad t \geq 0, \quad (6)$$

where  $G_0$  represents the Green's function of the guideway evaluated at  $\xi = 0$  due to a moving impulse load and  $w_0^{\text{ic}}$  represents free vibrations of the guideway at  $\xi = 0$  due to initial conditions corresponding to the system's equilibrium position. This term is necessary because the convolution integral captures just the response of the system with trivial initial conditions. The Green's function  $G_0$  can be obtained from Eq. (1) by replacing  $F$  with  $-\delta(t)$ . By applying the Laplace transform over time, the resulting equation can be solved, expressing the Laplace-domain solution as a superposition of wave modes [4, 5]. The analytical solution is then evaluated at  $\xi = 0$  to determine  $\hat{G}_0$  (where the hat denotes the quantity in the Laplace domain). Subsequently, its time-domain counterpart  $G_0$  can be obtained through numerical evaluation of the inverse Laplace transform.

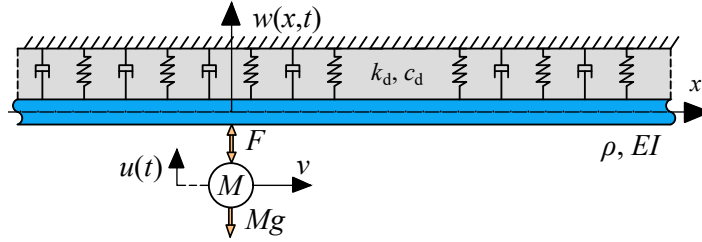


Figure 1: System representation: an infinite Euler–Bernoulli beam continuously supported by a visco-elastic foundation and subject to a moving mass. The vehicle-structure interaction is governed by the nonlinear electro-magnetic suspension.

In this study, we adopt a typical Hyperloop design previously outlined in Refs. [31, 32]. Since the current model does not consider the discrete nature of the supports (see Refs. [6, 9, 32] for a discussion on when system periodicity should and should not be accounted for), parameters from [32] are adjusted by dividing them by the support spacing to create an equivalent distributed foundation. The vehicle is suspended from above, and the displacement  $w$  is measured at the rail level, situated at the top of the tube. Consequently, the spring stiffness in our phenomenological model not only encompasses the support stiffness but also incorporates the flexibility of the tube-rail connection and, notably, the flexibility introduced by the ovalization of the tube. Parameter values are as follows:  $EI = 25 \times 10^6$  kNm<sup>2</sup>,  $\rho = 1400$  kg/m,  $k_d = 28 \times 10^3$  kN/m<sup>2</sup>,  $c_d = 20$  kNs/m<sup>2</sup>,  $M = 7650$  kg,  $C = 0.05$  Nm<sup>2</sup>/A<sup>2</sup>, and  $\Delta^{\text{ss}} = 15$  mm.

### 3 Linearisation of the nonlinear model

To explore the stability of the system, it is valuable to examine the linearized system around its equilibrium state. While the system described by Eqs. (2)–(6) may exhibit multiple equilibrium positions, our focus lies on the behavior around the operational equilibrium position, akin to the standard steady state of an equivalent mechanical system. To achieve this, we introduce a perturbation around the steady state by substituting  $w_0(t) = w_0^{ss} + w_0^{tr}(t)$ ,  $u(t) = u^{ss} + u^{tr}(t)$ ,  $F(t) = F^{ss} + F^{tr}(t)$ , and  $I(t) = I^{ss} + I^{tr}(t)$ , where the superscript *tr* denotes transient. Following mathematical manipulations, we employ the Taylor expansion to the governing equations corresponding to Eqs. (3) and (4). As a result, the linearized system is derived and expressed as follows:

$$w_0^{tr} = - \int_0^t G_0(t - \tau) F^{tr}(\tau) d\tau, \quad (7)$$

$$M\ddot{u}^{tr} = F^{tr}, \quad (8)$$

$$F^{tr} = \frac{2CI^{ss2}}{\Delta^{ss3}} \left( \frac{\Delta^{ss}}{I^{ss}} I^{tr} + u^{tr} - w_0^{tr} \right), \quad (9)$$

$$\dot{I}^{tr} = \frac{\Delta^{ss}}{2C} \left[ -I^{tr}R + K_p(w_0^{tr} - u^{tr}) + \left( K_d + \frac{2CI^{ss}}{\Delta^{ss2}} \right) (\dot{w}_0^{tr} - \dot{u}^{tr}) \right], \quad (10)$$

The stability of the equilibrium state can be investigated through the eigenvalues of the linearised system. To obtain the characteristic equation, the Laplace transform is applied to Eqs. (7)–(10) with respect to time. The expression of the electro-magnetic force is substituted in the Laplace-domain counterparts of Eqs. (7) and (8), and the resulting system of equations, in matrix form, reads

$$\hat{K}_{\text{dyn}}(s) \begin{pmatrix} \hat{w}^{tr} \\ \hat{u}^{tr} \\ \hat{I}^{tr} \end{pmatrix} = \begin{pmatrix} 0 \\ su^{tr}(t=0) + v^{tr}(t=0) \\ I^{tr}(t=0) \end{pmatrix}, \quad (11)$$

$$\hat{K}_{\text{dyn}}(s) = \begin{pmatrix} 1 - \hat{G}_0(s) \frac{2CI^{ss2}}{\Delta^{ss3}} & \hat{G}_0(s) \frac{2CI^{ss2}}{\Delta^{ss3}} & \hat{G}_0(s) \frac{2CI^{ss}}{\Delta^{ss2}} \\ \frac{2CI^{ss2}}{\Delta^{ss3}Ms^2} & 1 - \frac{2CI^{ss2}}{\Delta^{ss3}Ms^2} & -\frac{2CI^{ss}}{\Delta^{ss2}Ms^2} \\ \frac{(\Delta^{ss2}K_d + 2CI^{ss})s + \Delta^{ss2}K_p}{-2C\Delta^{ss}} & \frac{(\Delta^{ss2}K_d + 2CI^{ss})s + \Delta^{ss2}K_p}{2C\Delta^{ss}} & \frac{\Delta^{ss}R}{2C} + s \end{pmatrix},$$

where  $u^{tr}(t=0)$ ,  $v^{tr}(t=0)$ , and  $I^{tr}(t=0)$  are the initial conditions of the perturbation for the mass and current. The characteristic equation is obtained by equating the determinant of the coefficient matrix to zero:

$$\left| \hat{K}_{\text{dyn}}(s) \right| = 0. \quad (12)$$

It must be emphasized that the characteristic equation is neither a polynomial (since  $s$  appears also in  $\hat{G}_0$  under square roots) nor a transcendental equation, meaning that it has a finite amount of roots. The eigenvalues are determined numerically by using a root finding routine (i.e., *fsolve* in Matlab) with a multitude of initial guesses for  $s$  to cover the part of the complex plane relevant for this problem.

The stability results based on the eigenvalue analysis are presented in [31] and are not repeated here since the goal of the present work is to distinguish between the two different instability mechanisms that is presented in the following.

## 4 Energy analysis

The eigenvalue analysis presented in the previous section is straightforward and fully describes the system's stability, but it fails to differentiate between various instability sources. Consequently, discerning the primary contributing mechanism for effective mitigation remains impossible. To judge the contribution of different components to the stability of the system, we present hereafter an investigation based the energy variation of the moving mass as done by Metrikin [23]. Because the system considered in this study is slightly different to the one in [23], the expression for the energy variation is re-derived in the following. It must be emphasized that the linearized system is used for this investigation, as was used for the eigenvalue analysis. Also, the superscript tr is omitted in the following expressions for brevity.

The variation of the moving mass energy  $E_M$  can be obtained by multiplying both sides of its equation of motion (Eq. (8) without superscript tr) by its velocity  $\dot{u}$ . Re-writing the left-hand side as  $M\ddot{u}\dot{u} = \frac{\partial}{\partial t} \frac{1}{2} M \dot{u}^2$ , the following expression is obtained

$$\frac{\partial}{\partial t} E_M = F(t)\dot{u}, \quad E_M = \frac{1}{2} M \dot{u}^2. \quad (13)$$

The guideway contribution to the vehicle energy variation is implicitly incorporated in  $F(t)$ . To express it explicitly, we add and subtract the term  $F(t)\dot{w}_0$  to the right-hand side of Eq. (13), thus obtaining

$$\frac{\partial}{\partial t} E_M = \underbrace{-F(t)\dot{\Delta}}_{\text{Energy input } E_{\text{em}} \text{ by electro-magnetic force}} + \underbrace{F(t)\dot{w}_0}_{\text{Energy input } E_{\text{gw}} \text{ by the guideway}} \quad (14)$$

where the first term of the right-hand side represents the electro-magnetic force energy input  $E_{\text{em}}$  to the vehicle energy variation, while the second term is the guideway contribution  $E_{\text{gw}}$ . Positive input represents energy added to the mass vibration while negative input represents energy dissipation. The guideway contribution can be expressed from its equation of motion as follows [6, 33]:

$$F(t)\dot{w}_0 = - \left[ S(x, t) - v h_{\text{el}}(x, t) \right]_{x=vt-0}^{x=vt+0} + E_{\text{F,h}}(t), \quad (15)$$

$$S(x, t) = EI (w''' \dot{w} - w'' \dot{w}'), \quad h_{\text{el}}(x, t) = \frac{1}{2} \left[ EI (w'')^2 + \rho \dot{w}^2 + k_d w^2 \right],$$

$$E_{\text{F,h}}(t) = v F(t) w'_0,$$

where  $S(x, t)$  and  $h_{\text{el}}(x, t)$  represent energy flux and elastic density, respectively, and  $E_{\text{F,h}}(t)$  is the energy input by the horizontal force maintaining the constant vehicle velocity (for more details, see [6, 33]).

Eq. (14) shows that there are two main contributions to the energy variation of the vehicle: (i) the power input/dissipation by the electro-magnetic force, and (ii) the power input/dissipation by the guideway. Since both these contributors can cause instability, as discussed in the Introduction, they can either dissipate or input energy into the vehicle vibration. It must be emphasized that Eq. (14) (together with Eq. (15)) is the *exact* energy balance expression for the linearized system (Eqs. (7)–(10)) without any assumptions or simplifications made.

The system is unstable only if the energy variation averaged over one oscillation period is positive. Since the vibration frequency is not known a priori, the results from the eigenvalue analysis (Sect. 3) are used as input for the energy analysis performed in this section. More specifically, a harmonic motion is imposed to the mass with the frequency corresponding to the imaginary part of the eigenvalue(s) obtained in Sect. 3 and with arbitrary amplitude. The response to this system is derived in detail in [34] and is not repeated here for brevity. The averaged energy variation over one oscillation period reads

$$\begin{aligned} \Delta E_M &= \bar{E}_{\text{em}} + \bar{E}_{\text{gw}}, \tag{16} \\ \Delta E_M &= E_M(t + T_\Omega) - E_M(t), \quad \bar{E}_{\text{em}} = -\frac{1}{T_\Omega} \int_0^{T_\Omega} F(t) \dot{\Delta}(t) dt, \\ \bar{E}_{\text{gw}} &= -\left[ \bar{S}(x) - v \bar{h}_{\text{el}}(x) \right]_{-}^{+} + \bar{E}_{\text{F,h}}, \end{aligned}$$

where the overbar indicates that the quantities have been averaged over one period  $T_\Omega = \frac{2\pi}{\Omega}$  of oscillations, where  $\Omega$  is the imaginary part of the eigenvalue  $s$  determined in Sect. 3.

## 5 Results

A negative/positive  $\Delta E_M$  means that the free-vibration response due to a perturbation will exhibit a decay/increase with time, and this should be corroborated by the corresponding eigenvalue (from which the oscillation frequency  $\Omega$  was taken) having a negative/positive real part, while  $\Delta E_M = 0$  represents the boundary between stability and instability. It may seem that the energy analysis does not bring additional information to the stability investigation since it requires the eigenvalue analysis as an input, and its output merely confirms the already known stability boundaries from the eigenvalue analysis. This is right when it comes to the stability investigation, but it does allow to determine which instability mechanism (which contributor from Eq. (14)) is governing.

Fig. 2 presents  $\Delta E_M$  in the  $K_p$ – $K_d$  parameter space for a sub-critical and a super-critical velocity. The resulting stability boundaries (i.e., curves defined by  $\Delta E_M = 0$ ) match perfectly the ones obtained from the eigenvalues analysis (i.e., curves defined by  $\text{Re}(s_n) = 0$ ) when instability is caused by the complex-valued eigenvalue pair (i.e., stability is lost through a Hopf bifurcation). The left instability zone, caused

by the real-valued eigenvalue being positive, is not predicted by the energy analysis because this analysis assumes a harmonic motion at the stability boundary while the left boundary is characterized by a constant rigid-body motion, not a harmonic one. Nonetheless, the stability to the right of the left boundary is correctly predicted by the energy analysis, thus demonstrating that here the system stability is indeed governed by the energy variation in Eq. (16). Furthermore, the domains where the eigenvalue analysis predicted instability (grey background) coincide with a positive  $\Delta E_M$ , which further confirms the validity of the energy analysis.

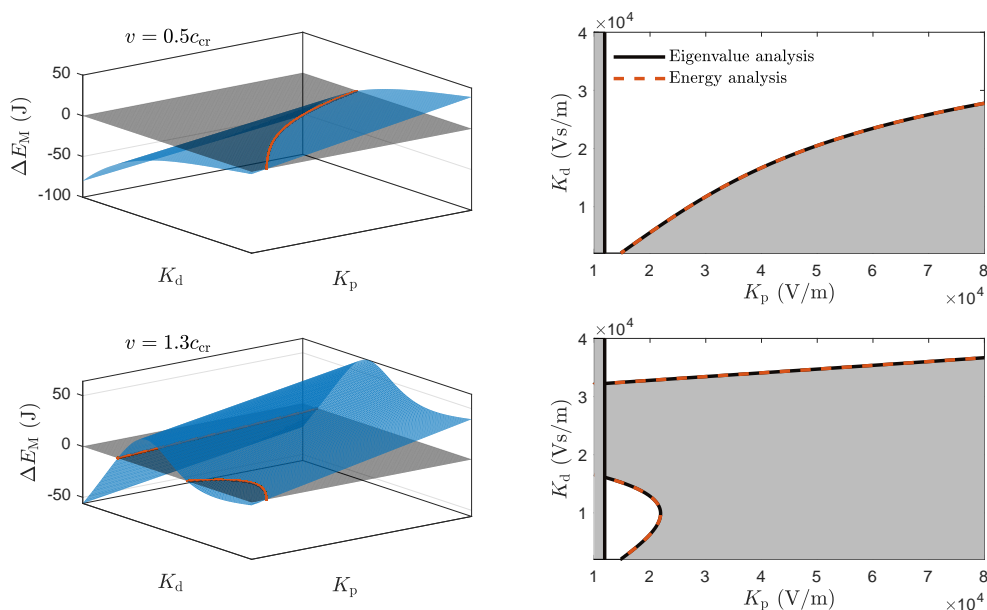


Figure 2:  $\Delta E_M$  (left panels) and  $\Delta E_M = 0$  (right panels) in the  $K_p$ – $K_d$  parameters space for two vehicle velocities:  $v = 0.5c_{cr}$  (top panels) and  $v = 1.3c_{cr}$  (bottom panels). The curves defined by  $\text{Re}(s_n) = 0$  from the eigenvalue analysis are superimposed in the right panels.

Taking advantage of the energy analysis, Fig. 3 presents the separate contribution of the electro-magnetic and wave-induced instability mechanisms. The contributions are presented along the  $K_d$  axis for one selected value of  $K_p$  (their 3-D representation in the  $K_p$ – $K_d$  parameter space was considered unclear by the authors). To emphasize the influence of the guideway, the curve describing zero energy input by the electro-magnetic force (i.e.,  $\overline{E}_{em} = 0$ ) is also presented; this curve represents the stability boundary when the guideway is considered as rigid [17, 18, 35].

On the one hand, the results for  $v = 0.5c_{cr}$  (Fig. 3) demonstrate that the instability at sub-critical velocities is caused by the electro-magnetic force having a positive energy input ( $\overline{E}_{em} > 0$ ). The guideway, through its negative energy input (i.e., energy dissipation), proves to be a stabilizing mechanism and, consequently, enlarges the stability domain ( $\overline{E}_{em} = 0$  curve is the reference in which the guideway is not accounted for). On the other hand, the results for  $v = 1.3c_{cr}$  show that, for a range of  $K_d$ ,  $\overline{E}_{gw}$  is positive and is the main cause of instability at super-critical velocities, while



$\overline{E}_{em}$  is mainly negative, thus being a stabilizing mechanism. Fig. 3 demonstrates the complex interaction between the electro-magnetic and wave-induced instability, where each one can be a stabilizing or destabilizing mechanism depending on the vehicle velocity.

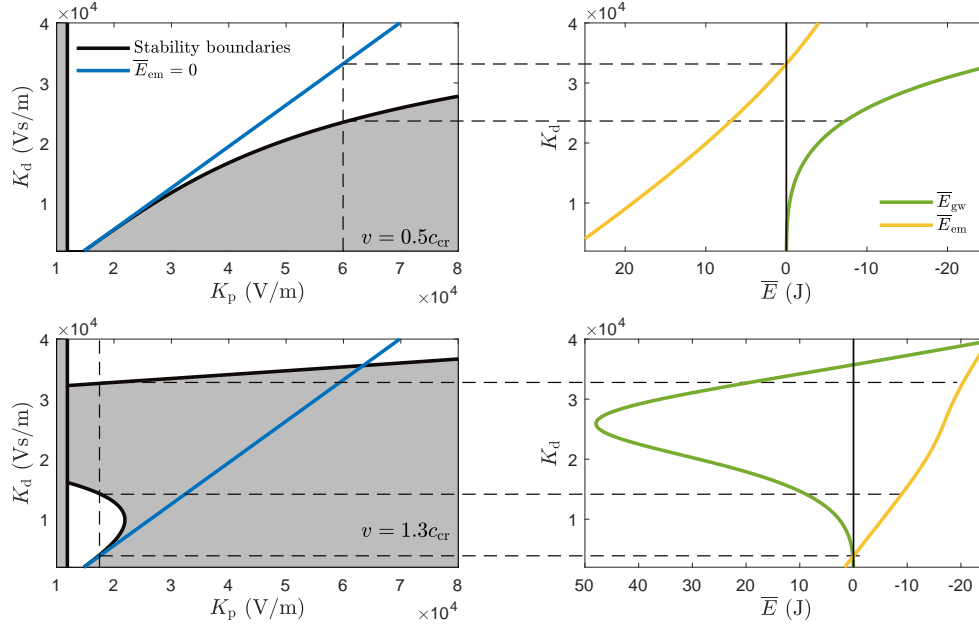


Figure 3: Left panels: Stable/unstable domains in the  $K_p$ - $K_d$  parameter space and the curve corresponding to no energy input by the electro-magnetic force (i.e.,  $\overline{E}_{em} = 0$ ). Right panels: energy input by the electro-magnetic force  $\overline{E}_{em}$  and by the guideway  $\overline{E}_{gw}$  vs  $K_d$  for one value of  $K_p$  indicated by the vertical dashed black line in the left panels. These results are presented for two vehicle velocities:  $v = 0.5c_{cr}$  (top panels) and  $v = 1.3c_{cr}$  (bottom panels).

## 6 Concluding remarks

The authors have previously investigated [31] the unstable velocity regimes of the Hyperloop vehicle-structure interaction and their dependency on the control gains by properly accounting for the frequency and velocity dependent reaction force provided by the infinite guideway. Furthermore, that study focused on the interplay between two fundamentally different instability sources, namely (i) the electro-magnetic suspension, and (ii) wave-induced instability, showing that stability domains drastically change above a certain vehicle velocity. The current study presented a methodology to distinguish the contribution of each instability mechanism to the overall system stability.

To discern the individual contributions, the energy variation of the vehicle is divided in two components corresponding to (i) energy input by the electro-magnetic

suspension and (ii) energy input by the guideway. The results obtained through the energy analysis are validated against the ones previously obtained using an eigenvalue analysis. The results of the energy analysis show that at low relative velocities, the electro-magnetic suspension can be destabilizing for certain control gains while the guideway is stabilizing for any control gains. At large relative velocities, the main destabilizing mechanism is the guideway while the electro-magnetic suspension can be either stabilizing or destabilizing depending on the control gains. The energy analysis demonstrates the complex interaction between the electro-magnetic and wave-induced instability, where each one can be a stabilizing or destabilizing mechanism depending on the vehicle velocity and control gains.

While identifying unstable velocity regimes is practical for Hyperloop design, the ability to discern which mechanism causes system instability for a certain parameter combination is valuable and can be crucial for efficient mitigation. This investigation offers physical insight into the mechanisms that can cause instability in the Maglev/Hyperloop systems, and can help engineers that develop this novel transportation system to avoid excessive vibrations and, in extreme cases, derailment.

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