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# Design of a Scaled Bridge for Validation of Vehicle-Bridge Interaction Models

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# Abstract

A gap exists between field experiments and laboratory scaled experimental set-ups, which hampers the validation of models and methods, and subsequently the application to practise. A similarity analysis is a useful tool in the design process of an experimental set-up to help bridging this gap for vehicle-bridge interaction systems. Expressing relevant quantities in dimensionless numbers allows for meaningful scaling, maintaining the desired dynamic behaviour of the scaled laboratory set-up while also maintaining the manufacturability. The relevant dynamic quantities are identified for the Boyne Viaduct in Ireland and the Diesel Multiple Unit trains passing this bridge, to design a scaled set-up of this bridge. The first experiments of the realised set-up show a deviation between the designed and actual values, which are mainly attributed to stiffness deviations of the bridge's lattice structure. However, it is also shown that the resonance frequency changes according to the theoretical expectation for a mass being positioned at different locations of the bridge. After mitigation of the stiffness issues with the lattice structure, dynamic measurements will be started to validate the numerical models and to further close the gap between theory and practise.

Keywords: similarity analysis, vehicle bridge interaction, dynamics, bridge, laboratory experimental tests, instantaneous frequency.

# 1 Introduction

Bridges are essential elements in rail infrastructure and hence avoidance of disrupted operation is of crucial importance for both a safety and availability perspective. A fair amount of research is directed towards bridge monitoring as it is an efficient method to assess the current condition of the bridge.

An important distinction between road bridges and railway bridges is that, typically, the mass ratio between vehicle and bridge is significantly higher for railway bridges. This implies in terms of modelling of the bridge dynamics that a moving force model suffices for road bridges, while a moving mass model is required for railway bridges [\[1\]](#page-12-0).

Moreover, safe operation of bridges is on the one hand related to the dynamic response itself and the avoidance of exciting resonance frequencies as these harm the structural integrity of the bridge or even result in immediate dangerous situations. On the other hand, slowly developing damage, such as corrosion or fatigue related damage, can be identified using the dynamic response of the bridge.

Most of the research presented in literature focusses on the extraction of dynamic properties of the bridge, reasoned by the fact that changes in the dynamic response are indicative for the presence of damage or degradation of the structural elements in the bridge. In many cases the dynamic characteristics, specifically modal properties such as natural frequencies, are extracted from the free-vibration response of the bridge. The free-vibration is either cause by wind loading, or a remnant of the vibrations caused by a passing train.

Although it has been shown to be possible to extract natural frequencies from the free-vibration of a bridge, a number of limitations is associated with this approach. Firstly, the signal is relatively weak compared to the strength of the dynamic signal during passage of a train, possibly leading to inaccurate estimations of the modal properties. Secondly, the response is sensitive to environmental conditions, which makes it hard to distinguish between a change of modal properties due to damage and due to environmental conditions.

Using the traverse phase, during which the train passes the bridge, offers opportunities at the cost of a more complex feature extraction process. Accurate extraction of the Instantaneous Frequency (IF) is an essential step: the relatively high mass ratio between train and bridge causes the resonance frequencies to vary with the location of the train on the bridge. Earlier research by Mostafa et al. [\[2\]](#page-12-1) showed that the Wavelet Synchro-Squeezed Transformation (WSST) can be used to extract the IF in case of trains passing a bridge with a velocity well below the critical velocity. Subsequently, a method was proposed to identify damage, independent of environmental and operational conditions [\[3\]](#page-12-2), and which train type (in terms of their dynamics) results in the

strongest interaction between bridge and vehicle, facilitating the damage identification [\[4\]](#page-12-3).

The above cited research is predominantly numerical. Bridge data is acquired during a two-year measurement campaign on the Boyne Viaduct in Ireland in the framework of the EU DESTinationRail project [\[5\]](#page-12-4), but, similar to many other cases in literature, the gap between numerical models and field data is substantial and not suitable for a proper validation of the models and methods. An experimental programme, centered around a scaled bridge, is defined to close the gap between the models and field experiments. This paper discusses the development of the set-up, the scaling procedure that is applied and the preliminary results obtained from the set-up.

# 2 Methods

The subject of scaling is the Boyne Viaduct in Ireland (fig. [1\)](#page-2-0), a single track steel bridge with three spans, of which the center one is 80 meters long. This center section is the section that is monitored and hence only this section is scaled instead of the complete bridge. Two different types of trains pass this bridge: a *Diesel Multiple Unit* (DMU) train with 4 to 8 carriages, with each their own Diesel engine and equal dynamic properties, and a *Enterprise* Diesel locomotive combined with up to 9 passenger wagons. Mass and suspension characteristics of the locomotive and wagons, and with that the train dynamics, are significantly different.

<span id="page-2-0"></span>

Figure 1: Boyne Viaduct near Drogheda, Ireland. *Photo by R. Loendersloot.*

#### 2.1 Scaling of the Bridge & Train

Scaling of an actual bridge to a size that is suitable for laboratory experiments is more than reducing the dimensions by a fixed factor. Scaling by a fixed factor leads to possibly infeasible dimensions, such as very thin beams. It also leads to dissimilar dynamic behaviour of the scaled bridge with respect to the original bridge, since dimension can be scaled, but material properties such as elastic modulus and density cannot be scaled in a similar manner.

The scaling procedure is based on the definition of dimensionless numbers [\[6\]](#page-12-5), which are quite common in Fluid Mechanics, but less common in Solid Mechanics. In general, it can be stated that a problem can be defined by  $N$  independent quantities. The dimensionless numbers are derived for these quantities based on the primary or physical quantities. Typically, three primary quantities are used in mechanics: length L, mass M and time T. A dimensionless product (the Buckingham-Pi product  $\Pi_i$ ) is constructed for the quantities  $P_i$  that need to be made dimensionsless:

<span id="page-3-1"></span>
$$
\Pi_i = \left(\prod_{k=1}^{N_p} Q_k^{\alpha_{k,i}}\right) P_i = 1 \tag{1}
$$

where *i* refers to the *i*<sup>th</sup> Buckingham-Pi product,  $Q_k$  to the  $k^{\text{th}}$  primary quantity and  $\alpha_{k,i}$ to the power of the  $k^\text{th}$  primary quantity related to the  $i^\text{th}$  quantity and  $N_P$  to the number of primary quantities (here: 3). The number of independent dimensionless products  $N_{\text{II}}$  equals, according to the Buckingham-Pi theorem [\[6\]](#page-12-5), the difference between the number of independent physical quantities  $N$  and the number of primary quantities  $N_P$ :

<span id="page-3-0"></span>
$$
N_{\Pi} = N - N_P \tag{2}
$$

Equation [\(2\)](#page-3-0) shows that there are  $N_{\Pi}$  equations and N independent quantities, leaving  $N_P$  quantities, in this case three, to be chosen freely. These three quantities are chosen conveniently: in this case the length of the bridge is chosen, as well as the modulus of elasticity of the material and its density. The first allows to choose a convenient length of the scale bridge, in relation to the available laboratory space and other practical considerations. A length of 2 meters is chosen. Choosing the modulus and density effectively implies the material of which the scaled bridge is made can be chose a priori. Here, steel was chosen. All other quantities in the model follow from solving the coefficients  $\alpha_{k,i}$  in Buckingham-Pi products defined in eq. [\(1\)](#page-3-1).

The bridge is modelled as a lattice frame structure, comparable to the real bridge. To reduce the complexity of the model, a constant height rather than an arch shape of the mid-section is of the Boyne viaduct is designed. The train is modelled as a single unit train, with on either side a three-axle bogy.

A schematic representation of the bridge and train is presented in fig. [2.](#page-4-0) All relevant independent quantities are indicated in the figure as well. The input parameters for the train are taken from [\[7\]](#page-12-6), initially focussing on the DMU trains. The dynamic

<span id="page-4-0"></span>

Figure 2: Schematic representation of the bridge and vehicle to be scaled, indicating all relevant independent physical quantities.

behaviour of the train is dominated by the dynamic behaviour of the sub-structures: the two bodies and the train body. Masses, mass moments of inertia, stiffness and damping values of the wheel, bogie and train body, including the axle spacing, bogie spacing and train length are explicitly taken into account in the scaling analysis. The bridge is defined by its material properties (modulus of elasticity and density), cross sectional area, area moment of inertia and length. The lattice structure itself is not considered explicitly. The bridge is therefore represented by a single line in fig. [2.](#page-4-0)

A total of  $N = 19$  relevant quantities are identified, of which  $N_P = 3$  are free and chosen beforehand, leaving  $N_{\Pi} = 16$  derived quantities. More details on the choice of quantities and the analysis to find the values of the scaled quantities can be found in [\[8\]](#page-12-7). The results are listed in table [1.](#page-5-0)

#### 2.2 Design and Manufacturing of the Scaled Train Bridge System

The motion of the vehicle is controlled by an electro-motor driven belt running along the bridge and below. The motor is chosen such that a constant velocity of 2 m/s can be reached. The maximum velocity is well beyond the representative velocity for the case of the Boyne viaduct, since the trains pass the bridge with a relatively low velocity o roughly 5 m/s due to the nearby Drogeda railway station.

The scaled bridge is manufactured from galvanised steel plate, cut and folded to the right dimensions and shape. The available tools at the workshop of the University of Twente imposed some limitations on the dimensions of the different parts, resulting in the bridge being constructed from several segments. All elements of the bridge are bolted, in order to mimic damage by unfastening one or multiple bolts.

The train is manufactured in a similar manner. The bogies and train body are modular. A single bogie, with either a sprung or unsprung mass can be use instead of the full train. Moreover different train masses can be tested by changing the mass of the train body.

	Quantity	Unit	<b>Dimension</b>	<b>Full scale</b>	<b>Scaled</b>	<b>Realised</b>	$\Delta$ [%]
Free	$E_{\rm bridge}$	$Nm^{-2}$	$ML^{-1}T^{-2}$	$2.050 \cdot 10^{11}$	$2.050 \cdot 10^{11}$	$2.050 \cdot 10^{11}$	$\overline{0}$
	$\rho_{\rm bridge}$	$\text{kg m}^{-3}$	$\cal{M}L^{-3}$	$7.850 \cdot 10^3$	$7.850 \cdot 10^3$	$7.850 \cdot 10^3$	$\theta$
	$L_{\rm bridge}$	m	L	80.77	$\overline{2}$	2	$\overline{0}$
Derived	$I_{\text{bridge}}$	m <sup>4</sup>	$L^4$	1.220	$4.586 \cdot 10^{-7}$	$4.877 \cdot 10^{-7}$	6.35
	$A_{bridge}$	m <sup>2</sup>	$L^2$	0.150	$9.197 \cdot 10^{-5}$	$9.727 \cdot 10^{-5}$	5.76
	$m_{\text{bridge}}$	kg	M	$1.018 \cdot 10^4$	$1.546 \cdot 10^{-1}$	$1.546 \cdot 10^{-1}$	$\overline{0}$
	$J_{\text{bridge}}$	$\text{kg m}^2$	$ML^2$	$1.155 \cdot 10^4$	$1.075 \cdot 10^{-4}$	$4.685 \cdot 10^{-4}$	335.81
	$m_{\rm tb}$	kg	M	$6.448 \cdot 10^{4}$	$9.780 \cdot 10^{-1}$	$9.872 \cdot 10^{-1}$	0.84
	$J_{\text{tb}}$	$\text{kg m}^2$	$ML^2$	$2.002 \cdot 10^6$	$1.864 \cdot 10^{-2}$	$1.084 \cdot 10^{-2}$	41.85
	$m_{\rm w}$	kg	M	$4.520 \cdot 10^3$	$6.862 \cdot 10^{-2}$	$6.887 \cdot 10^{-2}$	0.36
	$J_{\rm w}$	$\text{kg m}^2$	$ML^2$	$4.520 \cdot 10^3$	$4.208 \cdot 10^{-5}$	$1.648 \cdot 10^{-6}$	96.08
	$k_p$	$N m^{-1}$	$MT^{-2}$	$1.470 \cdot 10^6$	$3.640 \cdot 10^{4}$	$3.630 \cdot 10^4$	0.27
	$c_p$	$N s m^{-1}$	$MT^{-1}$	$4.000 \cdot 10^3$	2.453		
	$k_{s}$	$N m^{-1}$	$MT^{-2}$	$6.300 \cdot 10^5$	$1.560 \cdot 10^4$	$1.610 \cdot 10^{4}$	3.21
	$\mathcal{C}_{\mathcal{S}}$	$N$ s $m^{-1}$	$MT^{-1}$	$2.000 \cdot 10^{4}$	12.26		
	$R_w$	m	L	0.508	$1.258 \cdot 10^{-2}$	$1.250 \cdot 10^{-2}$	0.64
	$L_{wb1}$	m	L	1.689	$4.182 \cdot 10^{-2}$	$4.000 \cdot 10^{-2}$	4.35
	$L_{wb2}$	m	L	2.019	$5.000 \cdot 10^{-2}$	$5.000 \cdot 10^{-2}$	$\overline{0}$
	$L_b$	m	L	13.41	0.332	0.332	$\overline{0}$

<span id="page-5-0"></span>Table 1: Relevant quantities of train and bridge included in the Buckingham-Pi scaling analysis. The realised quantities in the actual test object are indicated, as well as the relative difference with the target values.

A SolidWorks rendering of the bridge and the train is presented in fig. [3.](#page-6-0) A schematic side view is also added. Note that the span of the bridge is 2 meters, while the complete set-up has a length of 3.82 meters, leaving sufficient room to park the vehicle on either side of the bridge. A sliced view of the bogie subsystem is presented in fig. [4.](#page-6-1) The three axles are suspended using a lever system and a spring (red springs in the figure). A similar structure is used for the secondary spring system (blue spring). The system is configured such that the levers, for both the primary and secondary suspension are in approximate horizontal position in case they carry the normal mass of the bogie and train. No dashpots are added, in contracts to the real train systems. Practical considerations, such as availability of affordable dashpots with the correct characteristics and construction complexities, were compared with the relevance of the presence of dashpots in terms of representing the correct dynamic behaviour. Dashpots were deemed unnecessary.

<span id="page-6-0"></span>

Figure 3: Scaled experimental set-up. (a) SolidWorks rendering; and (b) schematic side view of the system.

<span id="page-6-1"></span>

Figure 4: SolidWorks sliced view of the bogie. Part of the train body is also added in the figure.

#### 2.3 Dynamic Characterisation

Dynamic measurements were done to validate the design. The first objective is to determine the resonance frequencies of the (sub-)system(s). Two different measurement techniques were applied:

- 1. Roving hammer test
- 2. Shaker driven Experimental Modal Analysis (EMA)

The roving hammer test is applied to the bodgie and the train (two bogies and train body). One single accelerometer is used, while the structure is hit at 10 locations on the bogie and 15 on the train body. A Meggit 2302-10 modal hammer with embedded force transducer is used, together with an Instron 256-100 accelerometer. Both are fed to a National Instruments 4431 data acquisition unit, which is connected to a normal personal computer. An in-house developed LabVIEW application is used to control the measurements. The mode shapes are reconstructed based on the impact hammer force input signal and accelerometer output using FEMtools, a commercial software for modal analysis.

The resonance frequencies of the bridge are determined using a Brüel and Kjær shaker type 4810 and a series of Instron 256-100 accelerometers attached to the bridge. A sampling rate of 48kS/s and measurement time of 2.1s were used. The shaker was activated with a sine-sweep signal from 20-200Hz covering the full duration of the measurements. Again FEMtools is used to reconstruct the mode shapes of the bridge.

Finally, the effect of a mass on the bridge is tested. A mass is placed on different locations of the bridge, after which a resonance frequency analysis is done, using the shaker and the accelerometers, with the same settings as used to determine the bridge resonance frequencies.

# 3 Results & Discussion

#### 3.1 Realised Design

The scaled values resulting from the similarity analysis have not been realised exactly, due to practical limitations. The realised values and the deviation from the target values are indicated in table [1.](#page-5-0) The mass moments of inertia of the bridge, train body and wheel deviate most. However, it turned out that these have a marginal effect on the dynamics of the system and hence are further neglected. Furthermore, no dampers are included. Damping has a limited effect on the resonance frequencies, provided the damping is under-critical. The practical implications of including a damper system parallel does not outweigh the small error made, in particular since an unknown amount of damping will anyway be present in the springs.

#### 3.2 Dynamics Characterisation

The resonance frequencies of the individual sub-systems resulting from the different dynamic measurements are listed in table [2.](#page-8-0) The frequency and mass ratios are evaluated to validate the similarity modelling as well as the realised experimental set-up. The frequency f and mass m ratios  $\eta$  are all relative to the bridge, according to:

$$
\eta_{q_s - q_{\text{bridge}}} = \frac{q_s}{q_{\text{bridge}}}, \quad q \in [f, m], \quad s \in [\text{body}, \text{logic}] \tag{3}
$$

A large deviation is found for the resonance frequency of the designed and realised bridge. The mass ratios are consistent, hence the stiffness of the bridge is deviating from the designed value.

<span id="page-8-0"></span>



The bridge is modelled as a simply supported beam in the numerical models that support this work [\[2–](#page-12-1)[4,](#page-12-3) [8\]](#page-12-7) and in line with common practise [\[1\]](#page-12-0). Cross-sectional area and area moment of inertia are set such that they align with the values of the real bridge. However, the frame structure behaves far from a slender beam structure. This is also highlighted by an analysis of the theoretically expected resonance frequency. The i<sup>th</sup> resonance frequency  $f_{n,i}$  for a slender beam with density  $\rho$ , modulus of elasticity  $E$ , length  $L$ , cross-section  $A$  and area moment of inertia  $I$  equals:

$$
f_{n,i} = i^2 \frac{\pi}{2} \sqrt{\frac{EI}{\rho A L^4}}
$$
 (4)

This relation shows that a factor of 4 is expected between the first and the second resonance frequency, which is not the case when comparing the first measured resonance frequencies of the scaled bridge as listed in table [3.](#page-9-0) The table also includes the frequency ratio between the  $i<sup>th</sup>$  and  $1<sup>st</sup>$  resonance frequencies.

<b>Frequency</b>						
		<b>Mode</b> Measured [Hz] $\bar{f} = \frac{f_i}{f_i}$ Theoretical [Hz] $\bar{f} = \frac{f_i}{f_i}$				
80.3		142.1				
124.3	2.5	568.4				
161.3	32	1.278.9				

<span id="page-9-0"></span>Table 3: Measured and theoretical resonance frequencies of the scaled bridge.

The area moment of inertia is of the bridge is approximated by calculating the area moment of inertia with the Section Property tool in SolidWorks at 2.5mm intervals along the bridge, after which the average is taken. No dynamic model, for example a finite element model, was made to verify resonance frequencies and mode shapes. The measured mode shapes, shown in fig. [5](#page-10-0) reveal two interesting observations:

- 1. The stiffness in vertical direction is insufficient to ensure top and bottom move similarly, as would be comparable to a beam structure;
- 2. The motion of the top side of the bridge in the third mode, resembles the motion of a first mode of a slender beam.

The first can be mitigating by adding vertical bars connecting the bottom and top sections of the bridge. This will add some weight to the structure, which is not considered to be problematic as long as the frequency ratio between bogie and bridge is tuned to be close to unity.

The second point explains why an initial roving hammer test, in which the structure was excited at the top of the bridge, indicated a first resonance frequency of 161.3Hz. The shaker based measurement, in which the shaker was exciting the bottom part of the bridge, showed that this was actually the third resonance frequencies.

Typically, boundary conditions also cause deviations between theoretical and experimental resonance frequencies. In practise, supports are not ideal hinges, resulting in higher measured resonance frequencies compared to the theoretical values in case a simply supported beam is assumed as is the case here. The substantially lower measured resonance frequency is most likely explained, next to the missing vertical elements, by the flexibility introduced by the assembly of the bridge elements. The laser cutter, CNC press brake and sheet folding machines used to manufacture the different parts could not handle elements of the size of the full length of the bridge. The

<span id="page-10-0"></span>

Figure 5: First three mode shapes of the scaled bridge. (a) Mode 1: 80.348 Hz, (b) Mode 2: 124.31Hz, (c) Mode 3: 161.34 Hz.

bridge bottom and top sections are therefore assembled from multiple parts, bolted together. Furthermore, all elements in the lattice structure are bolted as well. The choice for bolting was motivated by the ease of realising specific bolt connections to mimic damage in a later stage of the project. However, bolted connections introduce a small amount of flexibility, with a strong effect on the overall stiffness properties.

#### 3.3 Instantaneous Frequency

Ideally seen, a single bodgie, or complete train is pulled over the bridge to determine the effect of the suspended, moving vehicle on instantaneous resonance frequencies of the bridge. As shown by Mostafa et al. [\[3\]](#page-12-2), the frequency ratio between bridge and bogie should be close to unity to get a strong interaction and relatively large drop of the first resonance frequency. Either the bridge or the vehicle need to be adjusted, such that the frequency ratio is close to unity, which is not the case for the current set-up as elaborated earlier.

Instead of a dynamic measurement, the influence of an added mass on the first resonance frequency of the bridge-vehicle system is evaluated by positioning a mass of 0.500kg at eight different locations. The resulting resonance frequency is shown

in fig. [6.](#page-11-0) The dashed and dash-dotted line are spline fits with points 1, 4, 5 and 8 as reference points. In addition, the slopes at start and end are set to zero for the dashed line. The results show that the resonance frequency changes according to the theoretically expected behaviour.

<span id="page-11-0"></span>

Figure 6: Instantaneous first resonance frequency of the bridge with a 0.500kg mass placed a different locations along the bridge.

### 4 Conclusions & Future Work

Validation of model is a challenging task. The gap between field experiments and models as well as laboratory set-ups is large. One issue is the scaling of an experimental set-up. This paper showed that a similarity analysis is a helpful tool in the design of a laboratory, scaled set-up based on an actual asset, in particular a bridge and train combination. However, it also showed that the translation of the theoretical output of the similarity analysis to a realised experimental set-up is not in all cases trivial. The assumption that the dynamic behaviour of the lattice structure based bridge structure behaves sufficiently similar to a beam is only valid for specific lattice structures. In this particular case, vertical bars connecting the top and bottom will greatly improve the match between designed and actual resonance frequencies. Furthermore, the bolted connections form a potential issue, as it introduces flexibilities in the system that are difficult to capture in a dynamic model.

The experimental set-up is currently being further developed, such that a frequency ratio between bogie and bridge frequency approaches unity. Once that is achieved, dynamic measurements, which actual moving vehicles (bogie and full train) will be done and validation of models can start to finally better link models and laboratory experiments to field experiments. This is considered as an important step to validate theoretical models to a more realistic case and hence close the gap between theory and practise.

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