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# Numerical Evaluation of Wave Energy Radiated from a Tunnel due to Bogie-Track Interaction

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## Abstract

This paper presents a semi-analytical method for evaluation of the wave energy radiated from a tunnel in bogie-track-tunnel-soil interaction problems. Due to the periodicity of the sleeper spacing, the interaction problem is reduced to that in a unit cell. The solution is derived with the aid of the Floquet transformation. The transformed solution is expressed by Fourier series in the track direction. Waves caused by both the railhead random roughness and the parametric excitation are considered. The former is evaluated within the framework of the mathematical expectation, while the latter is calculated deterministically. The wave energy passing through a virtual cylindrical boundary immersed in the soil region is evaluated for three types of track structure. The first one is the direct fixation track. The second one is the track with under-sleeper pads. The third one is the direct fixation track with an under-slab sheet. Although performance of the under-slab sheet is superior to that of the under-sleeper pad at around 40 Hz, these are equivalent above 60 Hz. The frequency dependence of the directivity of radiated waves is almost the same for both responses due to the railhead roughness and the parametric excitation.

**Keywords:** 3-D analysis, periodic track, random roughness, mathematical expectation, parametric excitation, directivity of radiated waves

# 1 Introduction

Ground-borne vibration induced by a train running in a tunnel may affect the surrounding residential environments. In taking measures to reduce the vibration, it is neccessary to assess the vibration level of the tunnel and soil in advance. For this purpose, different numerical evaluation methods have been developed.

In order to simulate the dynamic reaction due to moving loads, three-dimensional modelling of the tunnel-soil subsystem is desired. If the tunnel can be modelled as a cylindrical structure with uniform cross section, by virtue of the Fourier transformation with respect to the tunnel longitudinal direction, the coupling problem can be reduced to a two-dimensional problem [1, 2]. However, consideration of the track periodicity is essential to evaluation of the parametric excitation due to the interaction between a moving wheel and a discretely supported rail. Application of the Floquet transformation enables to reduce such periodic structure to a unit cell representing its periodicity [3, 4, 5, 6].

An analysis method based on the Floquet transformation has also been developed in reference [7]. In that paper, to reduce the computational effort, the interaction problem is divided into two sub-problems consisting of the wheel-track and the track-tunnel-soil subsystems. In the latter, the moving contact force between the wheel and the rail is replaced with a stationary harmonic load. This simplification contributes to saving the computation time, while the dynamic effect of the moving load cannot be considered precisely [8]. Furthermore, the vibration reaction resulting from a random roughness on the railhead is evaluated by smoothing the response over each 1/3 octave frequency band. Although this process makes it possible to approximate the average of random vibration, a reduction in frequency resolution might be unavoidable.

A semi-analytical method for bogie-track-tunnel-soil dynamic interaction problems taking into account the uncertainty in railhead roughness has been developed in reference [9]. In that method the mathematical expectation of the energy spectrum density of acceleration at observation points inside the tunnel is derived explicitly.

Although estimation of the vibration level at the tunnel is important, it will also be worth while to investigate the wave propagation characteristics in the ground which is radiated from the tunnel. This paper presents a semi-analytical method for evaluation of the transmission of wave energy passing through a virtual boundary immersed in the soil region far from the tunnel. The bogie-track-tunnel-soil interaction is calculated using the semi-analytic solution derived in reference [9]. The wave transmission originating from the parametric excitation is calculated deterministically, while that resulting from a random roughness is evaluated in the sense of the mathematical expectation. Through numerical analyses, directivity of the radiated waves and effect of the vibration reduction measures such as installation of the under-sleeper pad and the under-slab sheet on the wave radiation are investigated.

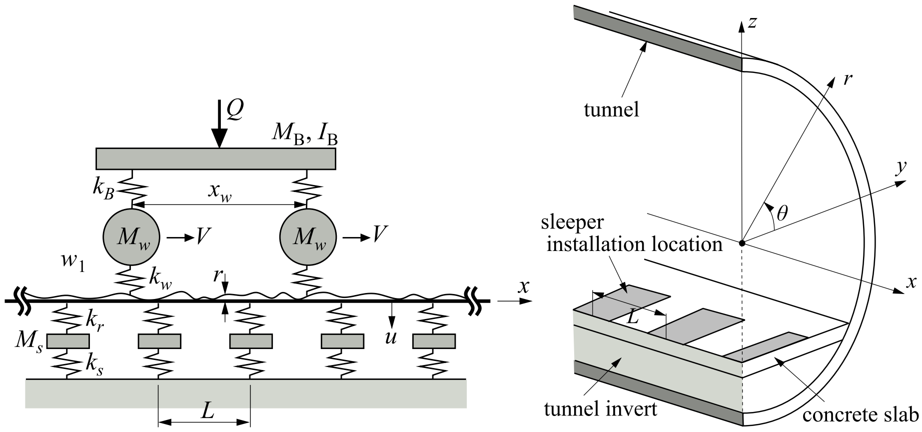


Figure 1: Bogie-track-tunnel-soil interaction problem considered in this study; (a) the bogie-track subsystem; (b) the tunnel-soil subsystem.

# 2 Outline of the mathematical model [9, 10]

A bogie-track-tunnel-soil interaction problem illustrated in Figure 1 is considered. A two-axle bogie is running with a speed of $V$ on an infinite track. A continuous welded rail is supported by sleepers with a regular spacing $L$. The rail is modelled as a Timoshenko beam. The railhead roughness $r$ is defined as a stationary random process. The rail pad and under-sleeper pad are given by springs $k_r$ and $k_s$, respectively. In the frequency-domain analysis these are described as complex stiffnesses. The sleepers and wheelsets are modelled by point masses of $M_s$ and $M_w$, while the bogie frame is represented by a rigid body of mass $M_B$ and moment of inertia $I_B$. Stiffness of the primary suspension is $k_B$. The wheelbase is $x_w$. The weight of car body is considered as a static load $Q$. Its consideration is essential to simulation of the parametric excitation. Contact stiffness between the wheel and the rail is modelled by a linear spring $k_w$.

A single-track shield tunnel is modelled as Figure 1(b). The concrete slab and the tunnel invert are given by viscoelastic bodies. The concrete lining is modelled as a cylindrical shell. The soil region is represented by an infinite homogeneous elastodynamic field.

# 3 Semi-analytic solution

In this section outline of the developed analysis method is described.

### 3.1 Derivation of solution for random railhead roughness

In order to solve the present problem, the Floquet transformation is applied to the solution. The Floquet transform [3,4] of a function $f(x)$ with a length $L$ is defined by

$$\tilde{f}(\tilde{x},\kappa) := \sum_{n=-\infty}^{\infty} f(\tilde{x}+nL)e^{in\kappa L}, \quad \left(-\frac{L}{2} \le \tilde{x} \le \frac{L}{2},\ 0 \le \kappa \le \frac{2\pi}{L}\right), \tag{1}$$

where $\tilde{f}$ is the Floquet transform of $f$ and $\kappa$ is a parameter called Floquet wavenumber. Inverse Floquet transform is given by

$$f(\tilde{x}+nL) = \frac{L}{2\pi}\int_0^{2\pi/L} \tilde{f}(\tilde{x},\kappa)e^{-in\kappa L}\,\mathrm{d}\kappa, \quad (n \in \mathbb{Z}). \tag{2}$$

In this study the Floquet transform of rail deflection $\tilde{\hat{u}}(\tilde{x},\omega,\kappa)$ is expressed in a Fourier series as [9,10],

$$\tilde{\hat{u}}(\tilde{x},\omega,\kappa) = \sum_n u_n(\omega,\kappa)e^{-i\kappa_n\tilde{x}}, \quad \kappa_n := \frac{2n\pi}{L}+\kappa, \tag{3}$$

where $\omega$ stands for the circular frequency and $u_n$ is the Fourier coefficient. Other terms such as the wheel/rail contact force and wheel vertical motion are also expressed in similar forms. $u_n$ is given by

$$\begin{aligned} u_n(\omega,\kappa) &= \sum_m u_{nm}\left[f_{1m}\left(\kappa-\frac{\omega}{V}\right)+f_{2m}\left(\kappa-\frac{\omega}{V}\right)e^{i\kappa_m x_w}\right], \\ u_{nm} &:= \frac{1}{VX_n}\left(\delta_{nm}-\frac{1}{X_m}\cdot\frac{1}{\frac{1}{k_e}+\sum_l\frac{1}{X_l}}\right), \\ X_n &:= GAK\kappa_n^2-\rho A\omega^2-\frac{(GAK\kappa_n)^2}{GAK-\rho I\omega^2+EI\kappa_n^2}, \end{aligned} \tag{4}$$

where $f_{1m}$ and $f_{2m}$ are Fourier coefficients of the wheel/rail contact forces at the rear and front wheels, respectively. $\delta_{nm}$ stands for the Kronecker delta. $G, K, E$ and $\rho$ are shear modulus, shear factor, modulus of elasticity and density of the rail. $A$ is the cross-sectional area and $I$ is the geometrical moment of inertia. $k_e$ is the dynamic equivalent stiffness representing the sleeper-tunnel-soil subsystem and expressed as

$$k_e(\omega,\kappa) = k_r\frac{k_s k_T-(k_s+k_T)M_s\omega^2}{k_s k_T+(k_s+k_T)(k_r-M_s\omega^2)}, \tag{5}$$

where $k_T(\omega,\kappa)$ is the dynamic equivalent stiffness corresponding to the slab-tunnel-soil substructure. The concrete slab and tunnel invert are discretized with finite elements in the tunnel section ($y-z$ plane), while in the tunnel direction the Fourier series is applied. Derivation of $k_T$ was described in reference [7].

In this analysis the interaction problem is reduced to the infinite linear equations of the Fourier coefficients of wheel/rail contact forces as [9,10]

$$
\begin{aligned}
&\left(\frac{1}{k_w}-\mu_{1n}\right) f_{1n}-\mu_{2n} f_{2n}+\sum_m A_{nm}(0) f_{1m}+\sum_m A_{nm}\left(\frac{x_w}{V}\right) f_{2m} e^{i\kappa_m x_w} \\
&\qquad\qquad\qquad = r_n-\gamma_n(\mu_{1n}+\mu_{2n}) Q_n, \\
&-\mu_{2n} f_{1n}+\left(\frac{1}{k_w}-\mu_{1n}\right) f_{2n}+\left[\sum_m A_{nm}\left(\frac{-x_w}{V}\right) f_{1n}\right. \\
&\quad \left.+\sum_m A_{nm}(0) f_{2m} e^{i\kappa_m x_w}\right] e^{-i\kappa_n x_w}=r_n e^{-i\kappa_n x_w}-\gamma_n(\mu_{1n}+\mu_{2n}) Q_n, \\
&A_{nm}(\alpha, \kappa):=\frac{1}{2\pi} \int_{-\infty}^{\infty} u_{nm}\left(\omega, \kappa+\frac{\omega}{V}\right) e^{i\alpha\omega}\, \mathrm{d}\omega,
\end{aligned} \tag{6}
$$

where $\mu_{jn}$ and $\gamma_n$ are coefficients relevant to the bogie motion and $r_n$ is the coefficient of railhead roughness.

Matrix notation of Equation (6) is given by

$$
[\mathbf{A}(\kappa)]\{\mathbf{f}(\kappa)\}=[\mathbf{B}(\kappa)]\{\mathbf{r}(\kappa)\}-[\mathbf{C}(\kappa)]\{\mathbf{Q}(\kappa)\}, \tag{7}
$$

where $\{\mathbf{f}\}$ is a vector composed of $f_{1n}$ and $f_{2n}$, $[\mathbf{A}]$ is a matrix corresponding to the left-hand side of Equation (6), $\{\mathbf{r}\}$ is a vector composed of $r_n$, $[\mathbf{B}]$ and $[\mathbf{C}]$ are matrices given by the first and the second terms on the right-hand side respectively, and $\{\mathbf{Q}\}$ is a vector corresponding to the static load $Q$; $Q_0=2\pi/LQ\tilde{\delta}(\kappa)$, $Q_n=0(n\neq 0)$, here $\tilde{\delta}$ is the periodic delta function with periodicity of $2\pi/L$.

From Equation (7), we can obtain the frequency response of rail deflection $\hat{u}$ as [9,10]

$$
\begin{aligned}
&\hat{u}(\tilde{x}, \omega)=\frac{L}{2\pi} \int_0^{2\pi/L}[\boldsymbol{\alpha}^T(\tilde{x}, \omega, \kappa)]\left\{\mathbf{r}\left(\kappa-\frac{\omega}{V}\right)\right\} \mathrm{d}\kappa-Q\beta(\tilde{x}, \omega), \\
&[\boldsymbol{\alpha}^T]:=\left[\mathbf{T}^T(\tilde{x}, \omega, \kappa) \mathbf{A}^{-1} \mathbf{B}\left(\kappa-\frac{\omega}{V}\right)\right], \\
&\beta:=\gamma_0(0)[\mu_{10}(0)+\mu_{20}(0)]\left[\mathbf{T}^T\left(\tilde{x}, \omega, \frac{\omega}{V}\right) \mathbf{A}^{-1}(0) \mathbf{I}_0\right],
\end{aligned} \tag{8}
$$

where the vector $\{\mathbf{T}\}$ is composed of $\sum_m u_{nm}e^{-i\kappa\tilde{x}}$ and $\sum_m u_{nm}e^{-i\kappa\tilde{x}-x_w}$, and components of the vector $\{\mathbf{I}_0\}$ are given by unity when these correspond to $f_{10}$ and $f_{20}$.

The Floquet transforms of displacement $\tilde{\hat{\mathbf{u}}}_G$ and traction $\tilde{\hat{\mathbf{p}}}_G$ due to the random roughness evaluated on the boundary immersed in the soil region are described as

$$
\begin{aligned}
&\tilde{\hat{\mathbf{u}}}_G(\omega, \kappa)=\tilde{\hat{\mathbf{u}}}_{0G}[\boldsymbol{\alpha}'^T(\tilde{x}, \omega, \kappa)]\left\{\mathbf{r}\left(\kappa-\frac{\omega}{V}\right)\right\}, \\
&\tilde{\hat{\mathbf{p}}}_G(\omega, \kappa)=\tilde{\hat{\mathbf{p}}}_{0G}[\boldsymbol{\alpha}'^T(\tilde{x}, \omega, \kappa)]\left\{\mathbf{r}\left(\kappa-\frac{\omega}{V}\right)\right\}, \\
&\{\boldsymbol{\alpha}'\}:=k_c\{\boldsymbol{\alpha}\}, \quad k_c:=\frac{k_T k_s k_r}{k_s k_T+(k_s+k_T)(k_r-M_s\omega^2)},
\end{aligned} \tag{9}
$$

where $\tilde{\hat{\mathbf{u}}}_{0G}$ and $\tilde{\hat{\mathbf{p}}}_{0G}$ are the Floquet transforms of the displacement and traction on the evaluation boundary due to a unit harmonic loading on the slab. The soil displacement $\tilde{\hat{\mathbf{u}}}_{0G}$ is expressed by [7, 11]

$$\tilde{\hat{\mathbf{u}}}_{0G} = \nabla\phi + \nabla \times \{\psi \mathbf{e}_x + \ell \nabla \times (\chi \mathbf{e}_x)\}, \tag{10}$$

where $\mathbf{e}_x$ is the unit vector in the direction of $x$, $\ell$ is of arbitrary length. $\phi, \psi$ and $\chi$ are expressed as follows in cylindrical coordinate system:

$$g = \sum_{n,m} a_{nm}(\kappa) \mathrm{H}_m^{(2)}(k_n r) e^{im\theta} e^{-i\kappa_n \tilde{x}}, \quad k_n^2 = \frac{\omega^2}{C^2} - \kappa_n^2, \tag{11}$$

where $g$ is either function of $\phi, \psi$ and $\chi$, $a_{nm}$ is an expansion coefficient, and $\mathrm{H}_m^{(2)}$ is the $m$-th order Hankel function of the second kind. $C$ is either speed of the longitudinal wave $C_L$ and the transversal wave $C_T$, and given by $C_L$ for $g = \phi$ and $C_T$ for $g = \psi$ or $\chi$.

Substituting Equation (11) into Equation (10), the soil displacement and traction can be given by

$$\tilde{\hat{\mathbf{u}}}_{0G} = \sum_{n,m} [\mathbf{U}_{nm}]\{\boldsymbol{\Phi}_{nm}\} e^{im\theta} e^{-i\kappa_n \tilde{x}}, \quad \tilde{\hat{\mathbf{p}}}_{0G} = \sum_{n,m} [\mathbf{S}_{nm}]\{\boldsymbol{\Phi}_{nm}\} e^{im\theta} e^{-i\kappa_n \tilde{x}}, \tag{12}$$

where the vector $\{\boldsymbol{\Phi}_{nm}\}$ is composed of $\phi_{nm}, \psi_{nm}$ and $\chi_{nm}$, $[\mathbf{U}_{nm}]$ and $[\mathbf{S}_{nm}]$ are $3 \times 3$ matrices.

Inner product of the displacement and traction can be obtained by way of the inverse Floquet transformation as

$$\hat{\mathbf{u}}_G^* \cdot \hat{\mathbf{p}}_G = \left(\frac{L}{2\pi}\right)^2 \iint_0^{2\pi/L} \tilde{\hat{\mathbf{u}}}_{0G}^* \cdot \tilde{\hat{\mathbf{p}}}_{0G} [\boldsymbol{\alpha}'^*] \left\{\bar{r}\left(\kappa - \frac{\omega}{V}\right)\right\} \left[r^T \left(\zeta - \frac{\omega}{V}\right)\right] \{\boldsymbol{\alpha}'\} \,\mathrm{d}\kappa \mathrm{d}\zeta, \tag{13}$$

where $(\cdot)^*$ and $(\bar{\ })$ stand for conjugate transpose and complex conjugate, respectively.

## 3.2 Mathematical expectation of transmitted wave energy

From Equation (13), the mathematical expectation of $\hat{\mathbf{u}}_G^* \cdot \hat{\mathbf{p}}_G$ is derived as

$$\mathrm{E}(\hat{\mathbf{u}}_G^* \cdot \hat{\mathbf{p}}_G) = \frac{1}{2\pi} \int_0^{2\pi/L} \tilde{\hat{\mathbf{u}}}_{0G}^* \cdot \tilde{\hat{\mathbf{p}}}_{0G} \sum_n S_r \left(\kappa_n - \frac{\omega}{V}\right) |\alpha_n'|^2 \,\mathrm{d}\kappa, \tag{14}$$

where $\mathrm{E}(\cdot)$ stands for the mathematical expectation, $S_r$ is the power spectrum density (PSD) of the railhead roughness. The expectation value of wave energy per unit angle transmitted through a cylindrical evaluation boundary immersed in the soil region with radius $R$ from the tunnel center and unit length in $x$ direction is approximately evaluated as follows:

$$\mathrm{E}(e_{T1}) \approx \frac{R\omega}{2} \mathrm{Im}(\mathrm{E}(\hat{\mathbf{u}}_G^* \cdot \hat{\mathbf{p}}_G)), \tag{15}$$

where $e_{T1}$ is the transmitted wave energy arisen from the random roughness and $\mathrm{Im}(c)$ stands for the imaginary part of a complex number $c$. The displacement and traction on the boundary are evaluated at $\tilde{x} = 0$.

The transmitted wave energy due to the parametric excitation $e_{T2}$ can be obtained deterministically by

$$e_{T2} \approx \frac{R\omega}{2} Q^2 |k_c \beta|^2 \mathrm{Im}(\tilde{\tilde{\mathbf{u}}}^*_{0G}(\omega, 0) \cdot \tilde{\tilde{\mathbf{p}}}_{0G}(\omega, 0)). \quad (16)$$

# 4 Results

In this study three types of track structure are considered. The first one is the direct fixation track, in which sleepers are fixed on the concrete slab and any vibration reduction measures are not taken. The second one is the track with under-sleeper pads. The third one is the direct fixation track with an under-slab sheet, in which a microcellular polyurethane elastomer sheet is placed between the concrete slab and the invert. UIC60 is assumed for the rail. $k_r = 83$ MN/m and $k_s = 7$ MN/m for the track with under-sleeper pad and $k_r = 30$ MN/m for other cases. The dynamic stiffness of the sheet is $7.5$ MN/m$^3$. The damping of these materials is considered by a loss factor of 0.2. The mass of sleeper $M_s$ is 100 kg per rail and the sleeper spacing $L$ is 0.6 m.

Inner radius and thickness of the tunnel lining are 3.25 m and 0.25 m. Its damping is represented by a loss factor 0.1. The soil density is 2200 kg/m$^3$, and $C_L = 412$ m/s and $C_T = 220$ m/s.

Parameters relevant to the bogie are $M_B = 1500$ kg, $I_B = 500$ kgm$^2$, $M_w = 1000$ kg, $x_w = 2.1$ m and $Q = 100$ kN. The contact stiffness $k_w$ is set to 1.5 GN/m. In the following, results for $V = 20$ m/s are shown.

## 4.1 Transmitted wave energy due to random roughness

The following PSD is considered for the railhead roughness:

$$S_r(k) = \frac{S_0}{k^4}, \quad (17)$$

where $k$ is the wavenumber and $S_0 = 4.5 \times 10^{-7}$ 1/m [10].

The expectation of the transmitted wave energy per unit angle $\mathrm{E}(e_{T1})$ is shown in Figure 2 for 40 and 60 Hz. Since the far-field solution given by taking the limit of $R \to \infty$ cannot be obtained explicitly, $\mathrm{E}(e_{T1})$ is evaluated on the cylindrical boundary of radius $R = 100$ m. Notice that this value can be a good approximation of the far-filed wave radiation.

Although the transmitted wave energy for the track with under-sleeper pad is comparable to that for the direct fixation track at 40 Hz, its remarkable reduction is achieved at 60 Hz. The lowest dominant frequency of the track with under-sleeper pad is about 40 Hz which is characterized by the resonance of wheel-track coupling

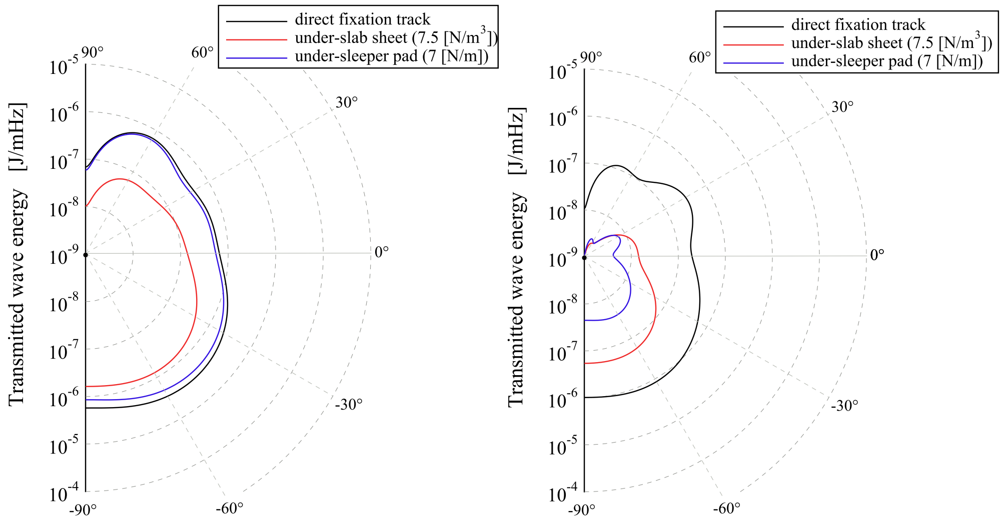


Figure 2: Expectation of transmitted wave energy per unit angle $\mathrm{E}(e_{T1})$ resulting from railhead roughness; (a) 40 Hz; (b) 60 Hz.

system. On the other hand, that of the track with under-slab sheet is about 20 Hz which corresponds to the natural frequency of a mass-spring system consisting of the slab and sheet. Therefore, the vibration reduction of the latter track is achieved above 20 Hz.

It is obvious that, regardless of track structure, most of the wave energy is radiated downward. The wave energy propagating to upward has rather clear directivity. The main radiation directions are $60°$ at 40 Hz and $30°$ at 60 Hz.

## 4.2 Transmitted wave energy due to parametric excitation

The transmitted wave energy due to the parametric excitation $e_{T2}$ is shown in Figure 3. The dominant frequencies are the sleeper passing frequency of $V/L = 33.\dot{3}$ Hz and its higher harmonic frequencies. However, due to the Doppler effect, the main frequency response splits into two peaks [9]. Because of this, in Figure 3 results at peaks of 34 Hz and 68 Hz, which are slightly higher than $V/L$ and $2V/L$, are shown. The radiated wave energy of the track with under-sleeper pad is the largest of the three tracks at 34 Hz, while its vibration reduction performance is comparable to that of the track with under-slab sheet at 68 Hz. The frequency dependence of the directivity of radiated waves is similar to that of waves caused by the railhead roughness. It can be found that, as far as these frequencies are concerned, the wave energy observed at far field is dominated by vibrations originating from the parametric excitation.

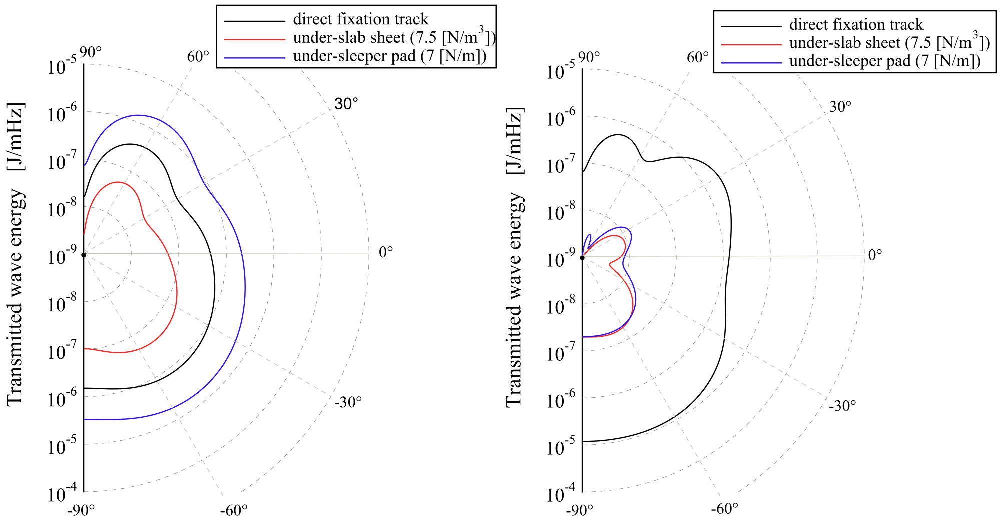


Figure 3: Transmitted wave energy per unit angle $e_{T2}$ resulting from parametric excitation; (a) 34 Hz; (b) 68 Hz.

# 5 Conclusions

Evaluation of the transmitted wave energy passing through a boundary immersed in the far-field soil region was attempted for the bogie-track-tunnel-soil interaction problems. To achieve this, a semi-analytical method has been developed with a combination of the Floquet transformation and the Fourier series in the track direction. Waves arisen from both the railhead random roughness and the parametric excitation were considered. The former was evaluated in the sense of the mathematical expectation. On the other hand, the latter was calculated deterministically.

Three track structures of the direct fixation track, the track with under-sleeper pad and that with under-slab sheet were compared in terms of vibration reduction. Although performance of the under-slab sheet is superior to that of the under-sleeper pad at around 40 Hz, these are equivalent above 60 Hz. The wave energy propagating to upward has rather clear directivity. The frequency dependence of the directivity of radiated waves is almost the same for both responses due to the railhead roughness and the parametric excitation. The radiated wave energy can be dominated by vibrations originating from the parametric excitation at around the sleeper passing frequency and its higher harmonic frequencies.

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