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Identification of the Lateral Offset of Railway Tracks from Train Accelerations

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Abstract

The aim of the work presented here is the inverse identification of the railway tracks irregularities and more precisely the lateral offset using a railway dynamics software with measured irregularities as inputs. The broader goal is to use on-board sensors to identify track irregularities for the purpose of facilitating the maintenance of the railways. First of all, this identification involves the search for statistical correlations with the train's accelerations and other irregularities. Then, an estimator is constructed in order to create an a-priori on the lateral offset irregularity.

Keywords: track irregularities, lateral offset, railway maintenance, inverse problem, linear estimation, train accelerations.

1 Introduction

Nowadays, the challenges facing the railway industry are numerous: the opening up to competition, the arrival of European companies on the French rail network, as well as the increasing economic and environmental constraints are driving the french railway company SNCF to innovate and strengthen its competitiveness. In this context, understanding and controlling the mechanical behavior of trains on railroad tracks is essential. In particular, estimating track irregularities (which are shown in figure 1) is crucial in order to organize maintenance operations as effectively as possible and thus guarantee the smooth running of the system.

The present work proposes a method for estimating the lateral offset track irregularities from wheelsets accelerations. The developments are illustrated by a numerical experiment: the lateral offset is estimated from simulated train accelerations. This work represents one step towards the reconstruction of railway track irregularities from on-board accelerations using on-board sensors on commercial trains which is our final goal. The track design is supposed to be perfectly known as well as the mechanical model of the train and the speed of the circulation. It will also be assumed that vertical offset and crosslevel can be accurately identified [1].

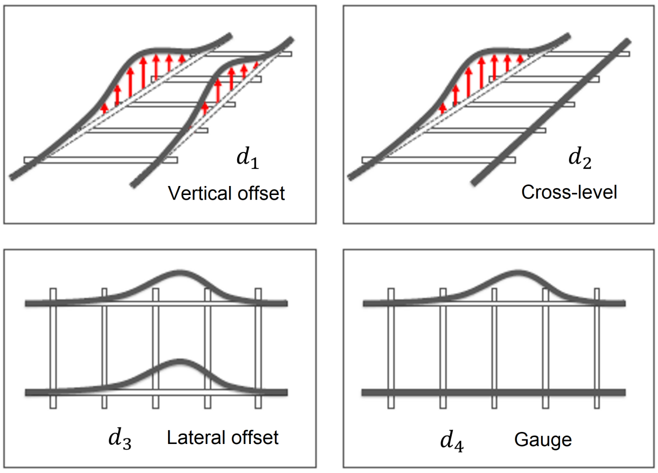

Figure 1: Classification of track geometric irregularities

Lateral offset will be estimated in three steps. First of all, statistical dependencies with the vertical offset and cross-level irregularities and the dynamic reactions of the studied train are estimated on a base of measured and /or simulated data. These dependencies are then exploited to build a first estimate of the lateral offset by conditioning the distribution previously obtained by the identified vertical offset, cross-level and train response on studied track portions. Finally, this first estimate will be refined by solving the inverse problem described in figure 2. This consists in an optimization problem where we must find the lateral offset which gives the closer dynamical

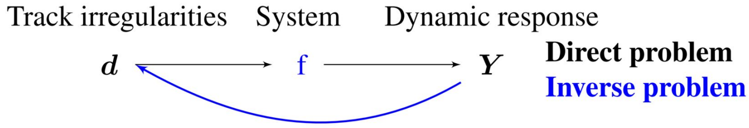

Figure 2: Direct & inverse problems diagram

response to the reference response [2–8].

Solving this problem presents several difficulties. The first is induced by the high degree of *non-linearity* of the system, and in particular of the contact between the wheel and the rail. Second, the quantities sought are *functional*. Therefore, dimension reduction and meta-modeling will be implemented [9, 10].

2 The Method

2.1 Problem definition and notations

The goal of this work is to identify $\boldsymbol{d_3}$, the lateral offset of the studied track from the simulated reactions of the train $\boldsymbol{Y}$ to real measured track irregularities $\boldsymbol{d}$. More precisely, we are going to identify the section $\boldsymbol{d_3}[\boldsymbol{S}, \boldsymbol{S}+\boldsymbol{l}[$ of length l from the accelerations of the different bodies of the train computed by a railway dynamic code. However, the reactions $\boldsymbol{Y}[\boldsymbol{S}, \boldsymbol{S}+\boldsymbol{l}[$ observed on the track section depend on the previous dynamics and in particular on the track geometry of the previous section $\boldsymbol{d}[\boldsymbol{0}, \boldsymbol{S}[$. In this initial work, we will therefore consider the geometry $\boldsymbol{d}[\boldsymbol{0}, \boldsymbol{S}[$ as known. A recursive approach will be adopted later to address the industrial issue.

Regarding the vertical irregularities of the track, a straightforward method to identify them is to utilize the displacements given by the software which are included in $\boldsymbol{Y}$. The vertical displacements of the wheels can give a very good estimate of the vertical offset irregularity $\boldsymbol{d_1}$ and the cross-level irregularity $\boldsymbol{d_2}$ [11]. Thus, these vertical irregularities of the track are going to be considered perfectly known. Other parameters and data regarding the initial design of the tracks like vertical offset and cross-level, the train model used and its speed will be considered to be perfectly known throughout our experiments and are going to be written as $\boldsymbol{U}$.

Having specified what we consider known and what we are trying to identify, we're interested in the following railway dynamics model which is the non-linear transient analysis of our software.

$$(\boldsymbol{d}[\boldsymbol{0}, \boldsymbol{S}+\boldsymbol{l}[, \boldsymbol{U}) \to \boldsymbol{Y}(\boldsymbol{d}[\boldsymbol{0}, \boldsymbol{S}+\boldsymbol{l}[, \boldsymbol{U}) \tag{1}$$

Consider the following notations:

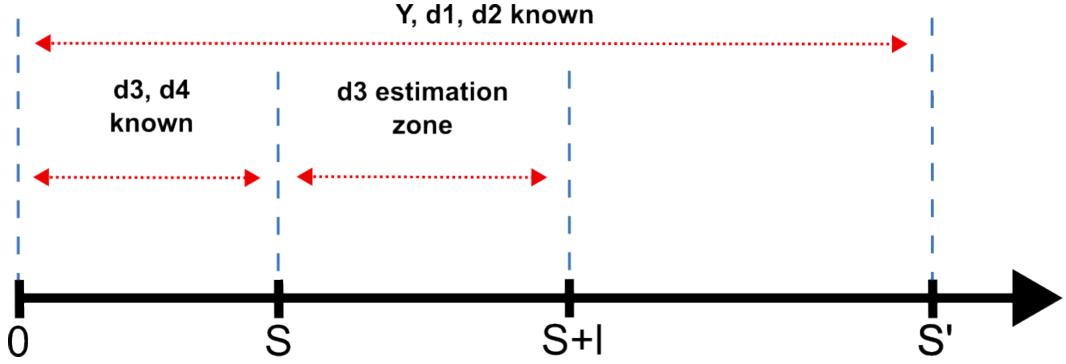

Figure 3: Known and unknown variables on the curvilinear abscissa

- s is the curvilinear abscissa over the considered zone of length S'. The vectors mentioned are discretizations in space on M points such that:
  $\underline{\boldsymbol{d_1}} := \{\boldsymbol{d_1(s_i)} / \ s_1, ...s_M \in [0, S'[\}, \quad \underline{\boldsymbol{d_1}}^{\text{prev}} := \{\boldsymbol{d_1(s_i)} / \ s_1, ...s_N \in [0, S[\}$
  $\underline{\boldsymbol{d_1}}^{\text{ref}} := \{\boldsymbol{d_1(s_i)} / \ s_{N+1}, ...s_P \in [S, S+l[\}$

- $\underline{\boldsymbol{U}}$: the vector containing the parameters of the track design, the train's model and the speed of the vehicle. It is considered known.

- $\underline{\boldsymbol{d}} := \left\{(\underline{\boldsymbol{d_1}}, \underline{\boldsymbol{d_2}}, \underline{\boldsymbol{d_3}}, \underline{\boldsymbol{d_4}})\right\}$ the rail irregularities in the following order: vertical offset, cross level, lateral offset and gauge.
  $\underline{\boldsymbol{d}}^{\text{prev}} := \left\{(\underline{\boldsymbol{d_1}}^{\text{prev}}, \underline{\boldsymbol{d_2}}^{\text{prev}}, \underline{\boldsymbol{d_3}}^{\text{prev}}, \underline{\boldsymbol{d_4}}^{\text{prev}})\right\}$ the track irregularities on the section of the track preceding $\underline{\boldsymbol{d}}$. It is considered known.

- $\underline{\boldsymbol{Y}}$: the positions and accelerations of the train bodies. n the number of signals characterizing the train's kinematics.
  $\underline{\boldsymbol{Y}} := \{(\boldsymbol{Y_j(s_i)}) \ / \ j \in [1, n], \ s_1, ...s_M \in [0, S'[\}$

- $\underline{\boldsymbol{Z}} := (\underline{\boldsymbol{Y}}, \underline{\boldsymbol{d_1}}, \underline{\boldsymbol{d_2}}, \underline{\boldsymbol{d_3}}^{\text{prev}}, \underline{\boldsymbol{d_3}}^{\text{ref}})$ the vector resulting from the concatenation of the known irregularities vectors and train response data. To simplify notation, we'll also define the vector $\underline{\boldsymbol{Z_{-3}}} = (\underline{\boldsymbol{Y}}, \underline{\boldsymbol{d_1}}, \underline{\boldsymbol{d_2}}, \underline{\boldsymbol{d_3}}^{\text{prev}})$.

For a given railway journey and a given train $\underline{\boldsymbol{U}}$, the goal of this work is to identify the lateral offset on the studied portion of the railway track $\underline{\boldsymbol{d_3}}^{\text{ref}}$. Having the train's kinematics $\underline{\boldsymbol{Y}}$, we will first estimate the vertical offset and the cross level irregularities $\underline{\boldsymbol{d_1}}$ and $\underline{\boldsymbol{d_2}}$ and then use all the known data we have to estimate the lateral offset. This first estimate is then going to be used as a starting point or an a-priori in an optimization. This optimization consists in the search of the lateral offset $\underline{\boldsymbol{d_3^\star}}$ that gives the closest train reaction to the reference reaction.

2.2 Estimation of the vertical offset and of the cross-level

The first and most intuitive approach to the identification of the vertical track irregularities is by assuming that the axles follow perfectly the rail. The position of the wheels represented here by the vertical position of the center of mass of the axle z_{axle} and its absolute roll angle $\boldsymbol{\theta_{axle}}$ is therefore assimilated to the real vertical positions of the rail. The vertical irregularities therefore result from comparing these displacements to the rail track design, which is represented by its vertical offset z_{track} and cross-level angle $\boldsymbol{\theta_{track}}$.

$$\boldsymbol{d_1^*} = z_{axle} - z_{track} \tag{2}$$

$$\boldsymbol{d_2^*} = Dsin(\boldsymbol{\theta_{axle}}) - Dsin(\boldsymbol{\theta_{voie}}) \tag{3}$$

with $\boldsymbol{d_1^*}$ the identified vertical offset irregularity, $\boldsymbol{d_2^*}$ the identified cross-level offset irregularity and D the distance between the left and right contact points between the axe and the rail which is assumed to be constant.

2.3 Constitution of the a priori

In order to expose the cross-correlations between $\underline{\boldsymbol{Z_3}}$ and $\underline{\boldsymbol{Z_{-3}}}$, we will start by computing the empirical autocovariance matrix of $\underline{\boldsymbol{Z}}$. The autocovariance matrix is estimated using a database of $\underline{\boldsymbol{d}}$ irregularities, consisting of numerous measurements on french high speed lines. The database was divided into training and test sets so that the same section of track measured at different times was not in both sets. Using numerical simulation, the $\underline{\boldsymbol{Y}}$ associated with the measured irregularities needed to complete this database were calculated. All variables are centered and reduced prior to these calculations. We construct here the empirical mean $\underline{\boldsymbol{\mu_0}}$ and variance $\underline{\underline{\boldsymbol{C_0}}}$ of $\underline{\boldsymbol{Z}}$:

$$\underline{\boldsymbol{\mu_0}} = \left[\underline{\boldsymbol{\mu_{-3}}}, \underline{\boldsymbol{\mu_3}}\right], \tag{4}$$

$$\underline{\underline{\boldsymbol{C_0}}} = \frac{1}{n}\sum_{i=1}^{n} \underline{\boldsymbol{Z}}_i \underline{\boldsymbol{Z}}_i^T - \underline{\boldsymbol{\mu_0}}\underline{\boldsymbol{\mu_0}}^T = \begin{bmatrix} Var(\underline{\boldsymbol{Z_{-3}}}) & Cov(\underline{\boldsymbol{Z_{-3}}}, \underline{\boldsymbol{d_3}}) \\ Cov(\underline{\boldsymbol{d_3}}, \underline{\boldsymbol{Z_{-3}}}) & Var(\underline{\boldsymbol{d_3}}) \end{bmatrix} = \begin{bmatrix} \underline{\underline{\boldsymbol{C_1}}} & \underline{\underline{\boldsymbol{C_{12}}}} \\ \underline{\underline{\boldsymbol{C_{12}^T}}} & \underline{\underline{\boldsymbol{C_2}}} \end{bmatrix}, \tag{5}$$

To create our a priori on $\underline{\boldsymbol{d_3}}^{\text{ref}}$, we'll use a linear MMSE estimation [12]. The created operator uses the identified vertical irregularities, the known previous lateral offset and the displacements and accelerations of bodies of the train $\underline{\boldsymbol{Z_{-3}}}^{\star}$ to give us the first moment $\underline{\boldsymbol{\mu_c}}$ of the law of $\underline{\boldsymbol{d_3}}^{\text{ref}}|\underline{\boldsymbol{Z_{-3}}}^{\star}$. The second moment is deduced from the computed covariance matrices and is noted $\underline{\underline{\boldsymbol{C_c}}}$. These are expressed as follows :

$$\underline{\boldsymbol{\mu_c}} = \underline{\boldsymbol{\mu_3}} + \underline{\underline{\boldsymbol{C_{12}}}}^T \underline{\underline{\boldsymbol{C_1}}}^{-1}(\underline{\boldsymbol{Z_{-3}}}^{\star} - \underline{\boldsymbol{\mu_{-3}}}), \tag{6}$$

$$\underline{\underline{\boldsymbol{C_c}}} = \underline{\underline{\boldsymbol{C_2}}} - \underline{\underline{\boldsymbol{C_{12}}}}^T \underline{\underline{\boldsymbol{C_1}}}^{-1} \underline{\underline{\boldsymbol{C_{12}}}}, \tag{7}$$

Due to problems with the conditioning of the matrix to be inverted $\underline{\underline{\boldsymbol{C_1}}}$, we had to use a reduced basis. Using SVD, we created a reduced basis for $\underline{\boldsymbol{Z_{-3}}}$ and another for $\underline{\boldsymbol{d_3}}$, so as to keep 99.99% of the variance and thus eliminate redundancies in the signal that reduce the rank of the covariance matrix. The transition matrices to the reduced bases are respectively $\underline{\underline{\boldsymbol{P_1}}}$ and $\underline{\underline{\boldsymbol{P_2}}}$ so that $\underline{\underline{\boldsymbol{P}}} = \left[\underline{\underline{\boldsymbol{P_1}}}, \underline{\underline{\boldsymbol{P_2}}}\right]$.

$$\underline{Z_{-3,rb}} = \underline{\underline{P_1}}\underline{Z_{-3}} \quad ; \quad \underline{d_{3,rb}} = \underline{\underline{P_2}}\underline{d_3} \quad ; \quad \underline{Z_{rb}} = \underline{\underline{P}}\underline{Z} \tag{8}$$

Equations 5 and 8 become :

$$\underline{\underline{C_{0,rb}}} = \underline{\underline{P}}\,\underline{\underline{C_0}}\,\underline{\underline{P}}^T = \begin{bmatrix} \underline{\underline{C_{1,rb}}} & \underline{\underline{C_{12,rb}}} \\ \underline{\underline{C_{12,rb}^T}} & \underline{\underline{C_{2,rb}}} \end{bmatrix}, \tag{9}$$

Thus the estimated lateral offset and its variance can be written as follows :

$$\underline{\mu_c} = \underline{\mu_3} + \underline{\underline{P_2}}^T \underline{\underline{C_{12,rb}}}^T \underline{\underline{C_{1,rb}}}^{-1} \underline{\underline{P_1}} (\underline{Z_{-3}}^\star - \underline{\mu_{-3}}), \tag{10}$$

$$\underline{\underline{C_c}} = \underline{\underline{P_2}}^T (\underline{\underline{C_{2,rb}}} - \underline{\underline{C_{12,rb}}}^T \underline{\underline{C_{1,rb}}}^{-1} \underline{\underline{C_{12,rb}}}) \underline{\underline{P_2}}, \tag{11}$$

We have therefore created a linear MMSE estimator for the lateral offset.

$$\underline{\mu_c} = \underline{\hat{d}_3}_{\text{MMSE}}(\underline{Z_{-3}}^\star) \tag{12}$$

3 Results

3.1 Identification of vertical offset and cross-level irregularities

A numerical experiment is carried out where real measured track irregularities and track designs measured on the French high speed railway network are used. The acceleration signals of the axles given by our software are then integrated twice to obtain its displacements. Then, these displacements are compared to the track design data as explained in 2.2.

The signals are pre-filtered by a rectangular high-pass filter with a cut-off frequency of $\lambda_c = 500m$ to eliminate signal drift. The operation results in reasonable identification errors relative to the amplitude of the irregularities (see figures 4 and 5). with RMSE errors of $\epsilon_{d1} = 0.155mm$ and $\epsilon_{d2} = 0.204mm$. Compared to the real irregularities, these errors have orders of magnitude several tens of times smaller.

3.2 Constitution of the a priori on the lateral offset

As in 3.1, we use as inputs to the software real track irregularities and design. The resulting simulated dynamical response of the elements of the train $\underline{Y}$ as well as the considered know irregularities will be used to try and identify the lateral offset $\underline{d_3}^{\text{ref}}$ used as input. Hereunder in figures 6 and 7 are shown examples of the reference lateral offset and the result given by the linear estimator $\underline{\mu_c}$. On figure 6, we show two examples on different portions of the railway track in curve and on figure 7 two examples in alignment.

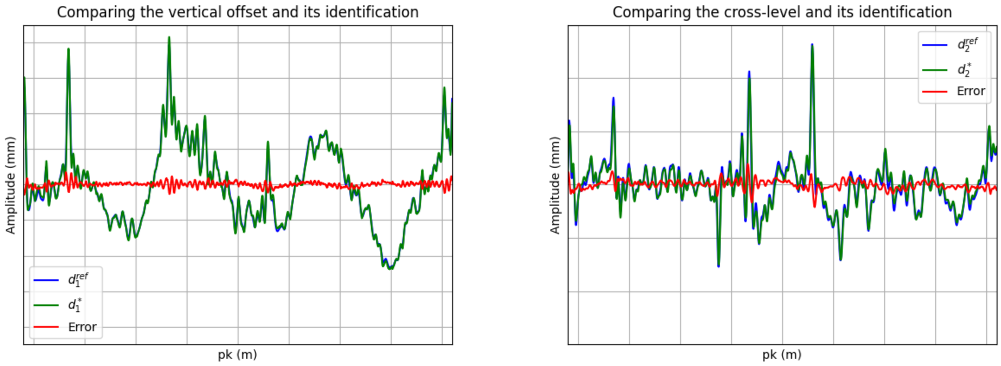

Figure 4: Vertical offset and cross-level irregularities identification from axle accelerations in curve

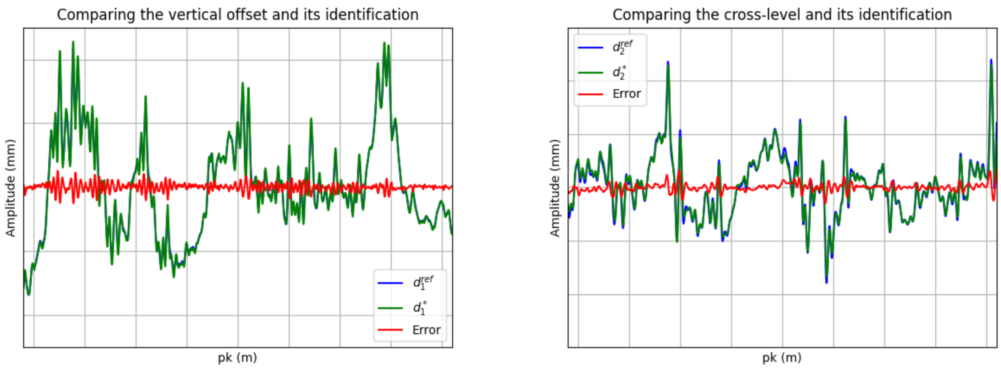

Figure 5: Vertical offset and cross-level irregularities identification from axle accelerations in alignment

We have tested our linear estimator on the test dataset and computed the average and the standard deviation of the root mean square error separately on both curved and aligned portions of the track. The results are in table 1. On a curved track, we get an average error of $\epsilon_{d3} = 1.16mm$ with a standard deviation of $\sigma = 0.35mm$ and on an aligned track, an error of $\epsilon_{d3} = 0.76mm$ and a standard deviation of $\sigma = 0.18mm$.

Track curvature	Curved	Aligned
RMSE (mm)	1.16	0.76
σ(RMSE) (mm)	0.35	0.18

Table 1: RMSE errors and the standard deviation on the identification of d_3

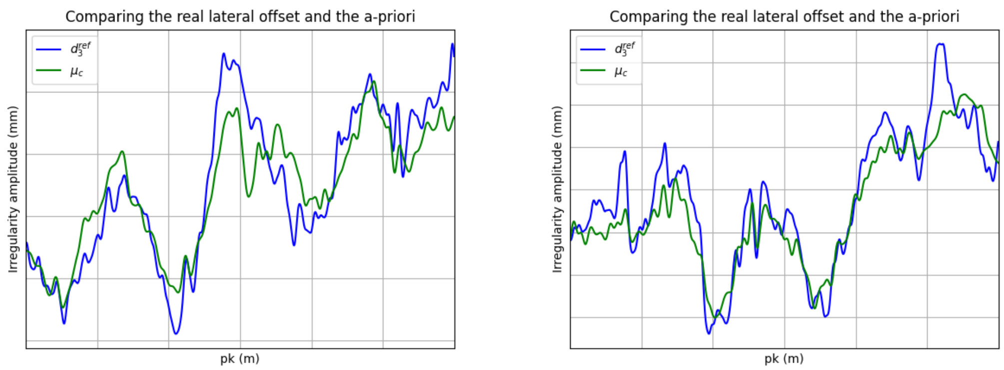

Figure 6: Examples of linear prediction of lateral offset from axle accelerations and known irregularities in curve.

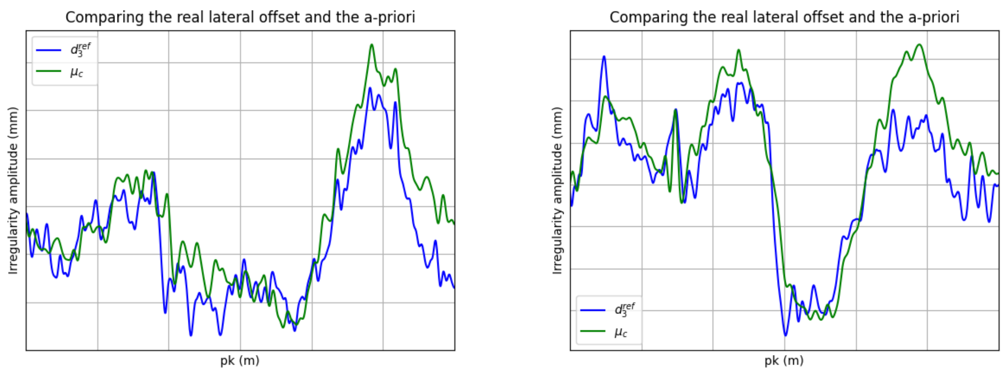

Figure 7: Examples of linear prediction of lateral offset from axle accelerations and known irregularities in alignment

4 Concluding remarks

In conclusion, we have used the statistical dependencies between measured track irregularities and simulated train dynamical responses to construct a linear operator intended to estimate the lateral offset track irregularities. The performance of the constructed operator was evaluated on a test database and has given promising results.

This first estimation is already a valuable tool for optimizing maintenance strategies and ensuring the operational integrity of railway systems. However, to ideally run the maintenance, a more refined prediction is needed. The current model, while promising, may benefit from further fine-tuning. For this reason, the variance associated with the estimator is going to be used next as the search space to refine the accuracy of the estimation of the lateral offset via the setting up of an optimization problem. The optimization problem is a complex one as it involves the creation of a cost function which compares the response of the train with the reference response.

Furthermore, in parallel to this work and in order to implement the desired solution on commercial trains, we are working on the processing of axle-box acceleration as the measured accelerations need to be expressed in the same reference frame as the simulated ones.

Acknowledgements

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