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Integrating Longitudinal Train Dynamics Simulations Within Multibody Models

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Abstract

This paper shows a novel approach for the integration of longitudinal train dynamics within the multibody formalism. The strategy relies on the definition of a dummy user force element, demanded to solve the dynamic states of several hidden vehicles in the train consist and to evaluate the in-train forces. The approach is validated against the LTDPoliTo code in simulation scenarios defined by an international benchmarking activity. Furthermore, the novel method is compared against multibody models with one body per each vehicle in the train, proving to have superior computational performances. Therefore, the novel approach is suitable for the development of detailed multibody models of railway vehicles while accounting for the effect of the in-train forces on the running safety, with no need for co-simulation architectures that can slow down the simulation times.

Keywords: longitudinal train dynamics, multibody dynamics, railway dynamics, parallel computing, coupler/bar, hook/buffer.

1 Introduction

In-train forces are known to affect the running safety of railway vehicles, especially during braking operations, whereby lateral and vertical components of longitudinal compressive forces (LCFs) can lead to a higher derailment risk [1-4], with an increase of all safety indexes derived from the wheel-rail contact forces as defined by the international standards [5]. Multibody (MB) models of isolated vehicles/small groups of wagons are developed to effectively calculate the wheel-rail contact forces and the

related running safety indexes, but they commonly neglect the interaction with the rest of the train composition. Conversely, longitudinal train dynamics (LTD) codes [6, 7] are adopted to estimate the in-train forces on the coupling elements, but they cannot calculate the wheel-rail contact forces, as they only consider a single degree of freedom (DOF) in longitudinal direction for each vehicle in the train consist.

Several strategies have been developed to combine LTD and MB simulations, aiming to account for the effect of the in-train forces on the wheel-rail contact forces. These range from simple MB models with the application of the in-train forces evaluated from preliminary LTD simulations [8] to co-simulation techniques [9, 10] and even to enhanced LTD codes [11-13] which can model the full-dynamics of some vehicles in the train composition. Furthermore, in recent years different researchers have explored on-the-edge techniques like machine learning and digital twins [14, 15], aiming to achieve excellent computational times thanks to closed-form surrogate models, which however miss the physical background of railway dynamics. Concerning the physical-based methods, the application of the in-train forces obtained from a preliminary LTD simulation does not allow to consider the mutual dependency between LTD and the full-dynamics of the single wagon, while co-simulation techniques are characterized by certain requirements for the synchronization of the simulation environments, which can drastically worsen the computational performances. Moreover, enhanced LTD codes can have strict requirements in terms of the integration step-size, as the introduction of detailed vehicles in the train consist involves the solution of the wheel-rail contact problem, which is usually not dealt with by LTD solvers. In past works, Bosso et al. [16, 17] built MB models of long trains featuring a mix of simplified and detailed vehicles, to exploit the numerical solvers of MB codes, which are suitable for the solution of the wheel-rail contact problem. However, the developed models proved to suffer from numerical instabilities and poor computational performances, especially when dealing with long trains with many vehicles in the composition.

The present paper concerns the development of a novel method to include the in-house LTDPoliTo code developed in past activities [18-21] within the MB formalism, thanks to the introduction of dummy force elements with additional dynamic states, which are solved by the same numerical integrator used for the MB model. The novel strategy is validated in the simulations of the international benchmark of LTD simulators [22, 23] and its computational performances are compared with typical literature approaches described above.

2 Methods

The novel approach for the integration of LTD within MB models is based on the development of a dummy user force element with related dynamic states and outputs. The dummy force element is applied between markers belonging to two different bodies, which are separated by a group of “hidden” vehicles, whose longitudinal position and speed are calculated as the dynamic states of the user force element. Furthermore, the dummy user force element provides the in-train forces as outputs. At the current stage of the activity, the authors have implemented the novel approach

in the SIMPACK commercial MB code, writing the force element as a FORTRAN90 routine, but the method can be adapted for other MB codes. For the SIMPACK implementation, the FORTRAN90 code is compiled as a dynamic library, so that the user force element is available among the other elements of the code library with the standard graphical user interface window.

The FORTRAN90 routine implements the equations of the LTDPoliTo code developed in previous activities in MATLAB, so that the LTD dynamic states are calculated solving the following system of ordinary differential equations:

$$\mathbf{M}_{LTD} \dot{\mathbf{u}}_{LTD}(t) = \mathbf{q}_{LTD}(t, \mathbf{u}_{LTD}, \dot{\mathbf{u}}_{LTD}) \quad (1)$$

$$\mathbf{M}_{LTD} = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & & m_1 & & \\ & & & & \ddots & \\ & & & & & m_N \end{bmatrix} \quad (2)$$

where t is time, \mathbf{M}_{LTD} is the mass matrix of the system, m is the mass of each of the N vehicles in the train composition, \mathbf{q}_{LTD} is the right-hand side vector, \mathbf{u}_{LTD} is the vector of dynamic states and $\dot{\mathbf{u}}_{LTD}$ is the vector of dynamic states derivatives. The latter vectors are expressed as:

$$\mathbf{u}_{LTD} = \{s_1 \cdots s_N \dot{s}_1 \cdots \dot{s}_N\}^t \quad (3)$$

$$\dot{\mathbf{u}}_{LTD} = \{\dot{s}_1 \cdots \dot{s}_N \ddot{s}_1 \cdots \ddot{s}_N\}^t \quad (4)$$

where s represents the position along the track curvilinear abscissa, \dot{s} is speed and \ddot{s} is acceleration. The right-hand side vector is written as:

$$\mathbf{q}_{LTD} = \left\{ \begin{array}{c} \dot{s}_1 \\ \vdots \\ \dot{s}_N \\ F_{T\backslash DB,1} - F_{res,1} + F_{c,front,1} - F_{c,rear,1} - F_{air,1} \\ \vdots \\ F_{T\backslash DB,N} - F_{res,N} + F_{c,front,N} - F_{c,rear,N} - F_{air,N} \end{array} \right\} \quad (5)$$

where $F_{T\backslash DB}$ is the traction/dynamic braking force, which is non-zero on locomotives only, $F_{c,front}$ and $F_{c,rear}$ are the in-train forces on the front and rear coupling systems, F_{air} is the force due to air braking, j is the vehicle counter and finally F_{res} is the contribution of resistant forces. The latter include ordinary, curving and slope resistances, as stated by:

$$F_{res} = \frac{m}{1000} \cdot \left[Q_{ad} \left(2.943 + \frac{89.2}{P_{ax}} + 0.0306V + \frac{0.122V^2}{P_{ax}n_{ax}} \right) + \frac{6116}{|R_c|} + gi_s \right] \quad (6)$$

where V is vehicle speed in km/h, n_{ax} is the number of axles, P_{ax} is the axle-load in ton, R_c is the curve radius, g is gravity and i_s is the slope in ‰.

Traction/dynamic braking forces are evaluated as a function of the running speed and notch level from the mechanical characteristics of the locomotives, while the air brake forces can be estimated with different approaches and different levels of modelling accuracy and complexity as reviewed by Wu et al. [24]. One of the biggest challenges in LTD simulations is represented by modelling of the strong nonlinearities of the mechanical impedance characteristics of in-train coupling systems. The FORTRAN90 routine relies on the same approach adopted in the LTDPoliTo code, which calculates the in-train forces from look-up-tables (LUTs) storing the loading and unloading curves of the coupling system, with the application of a smoothing transition [25] between the two curves when the deflection speed is below a threshold value, as stated by:

$$F_c(\Delta x, \Delta v) = \begin{cases} F_L(\Delta x), & |\Delta v| \geq v_\varepsilon \wedge \Delta x \cdot \Delta v \geq 0 \\ F_U(\Delta x), & |\Delta v| \geq v_\varepsilon \wedge \Delta x \cdot \Delta v < 0 \\ F_M(\Delta x) + |F_A(\Delta x)| \frac{\Delta v}{v_\varepsilon}, & |\Delta v| < v_\varepsilon \end{cases}$$

$$F_M(\Delta x) = \frac{F_L(\Delta x) + F_U(\Delta x)}{2} \quad (7)$$

$$F_A(\Delta x) = \frac{|F_L(\Delta x) - F_U(\Delta x)|}{2}$$

where F_c is the force on the coupling system, Δx is deflection, Δv is the relative deflection speed, F_L and F_U are the loading and unloading curves and finally v_ε is the threshold speed for the transition between the two characteristics.

The user force element written as a FORTRAN90 code cannot directly apply a force on the markers between which it is defined. In fact, the markers are separated by the hidden vehicles for which the LTD dynamic states are calculated, hence the forces on the markers are different and this would violate the action-reaction principle. To solve this issue, two additional trailing forces must be defined in the MB environment on the bodies in front and at the rear of the group of hidden vehicles, see Figure 1. The values of the trailing forces correspond to the in-train forces connecting the hidden vehicles to the bodies in the MB formalism. Each of the two trailing forces is applied between a marker defined on one body and a moving marker belonging to ground, which follows the body joint position along the track.

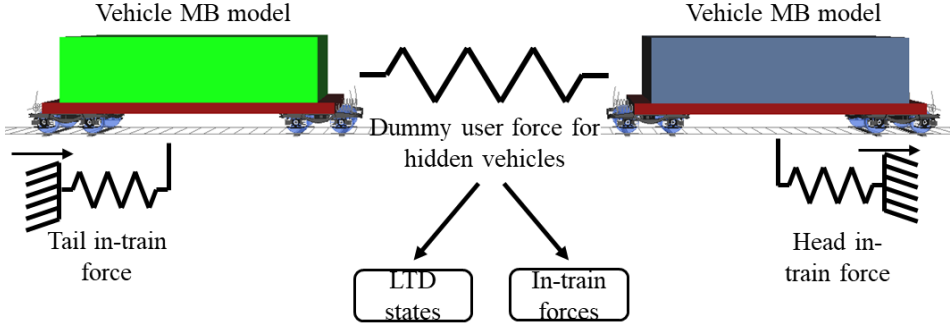


Figure 1: Sketch of the novel approach for LTD simulations with MB codes.

The novel dummy force element can be introduced within any MB model for the simulation of the full dynamics of isolated vehicles or small groups of vehicles. In this condition, the numerical integrator of the MB code is demanded to solve the following system of equations:

$$\begin{cases} \dot{\mathbf{p}} = \mathbf{R}\mathbf{v} \\ \mathbf{M}_{MB}\dot{\mathbf{v}} = \mathbf{Q}_M(\mathbf{p}, \mathbf{v}, \mathbf{u}_{LTD}, t, \boldsymbol{\lambda}) - \mathbf{J}^t(\mathbf{p}, t)\boldsymbol{\lambda} \\ \dot{\mathbf{u}}_{LTD} = \mathbf{Q}_{LTD}(\mathbf{p}, \mathbf{v}, \mathbf{u}_{LTD}, t, \boldsymbol{\lambda}) \\ \mathbf{g}(\mathbf{p}, t) = 0 \end{cases} \quad (8)$$

$$\mathbf{p} = \{\mathbf{y}_1 \quad \dots \quad \mathbf{y}_j \quad \dots \quad \mathbf{y}_M\}^t \quad (9)$$

$$\mathbf{y} = \{x \quad y \quad z \quad \alpha \quad \beta \quad \gamma\}^t \quad (10)$$

where \mathbf{p} is the vector of joint position states for the bodies in the MB model, \mathbf{R} is the matrix of angle transformations, \mathbf{v} collects the velocity of the joint states, x , y , z and α , β , γ refer to the 3 translations and 3 rotations with respect to the local reference frame of each body, \mathbf{M}_{MB} is the mass matrix for the bodies in the MB model only, \mathbf{Q}_M is the vector of external forces on the bodies of the MB model, $\boldsymbol{\lambda}$ is the Lagrange multiplier vector, \mathbf{J} is the Jacobian matrix for the constraint equations, \mathbf{g} collects in a vector the algebraic constraint equations and lastly \mathbf{Q}_{LTD} is the vector of generalized forces acting on the dynamic states. From comparison with Equation (1) the third equation in the DAE system in Equation (8) can be expressed as:

$$\mathbf{Q}_{LTD}(\mathbf{p}, \mathbf{v}, \mathbf{u}_{LTD}, t, \boldsymbol{\lambda}) = \mathbf{M}_{LTD}^{-1}\mathbf{q}_{LTD}(t, \mathbf{u}_{LTD}, \dot{\mathbf{u}}_{LTD}) \quad (11)$$

hence highlighting that the dynamic states of the LTD problem do not affect the mass matrix of the MB system.

The present paper compares the novel approach, referred to as MB user force hereafter, with the basic development of MB models including one body with a single DOF along the track per each vehicle in the train consist. The latter strategy is referred to as MB bodies in the rest of the paper. Both MB user force and bodies methods are implemented in the SIMPACK commercial code, and they are benchmarked against

the pure LTD code developed by the authors in past years in MATLAB (LTDPoliTo). The comparison considers both the main simulation outputs and the computational efficiency of all approaches. For the LTDPoliTo code, the ODE system in Equation (1) is solved using the ode15s solver available in MATLAB, which is a variable step-size numerical integrator based on numerical differentiation formulas. On the other hand, for the models built in SIMPACK, the solution is obtained with the SODASRT2 solver, which implements backwards differentiation formulas.

The comparison is performed for simulation scenarios 2 and 4 of the international benchmark [22, 23], which correspond to the simulation of the operation of long trains with remote locomotives, running on the same reference track. More in detail, train 2 includes two sets of 2 locomotives trailing 50 wagons, for a total of 104 vehicles (2 head locomotives, 2 remote locomotives and 100 wagons). On the other hand, train 4 is composed by 2 head locomotives trailing 120 wagons, and then followed by a remote locomotive trailing another group of 120 wagons. For both trains, rigid bars are used to build wagon pairs, which are then connected to adjacent pairs with couplers featuring a 10 mm clearance. The main data for the simulations, which include i) the properties of the locomotives and wagons of both trains, ii) the coupling system characteristics, iii) the track layout, iv) the driving command and v) the mechanical characteristics of the locomotives are all extracted from the benchmark. The interested reader can refer to references [22, 23].

For the novel MB user force method, train 2 is modelled by defining 6 bodies with a single DOF along the track, corresponding to all 4 locomotives, as well as to the tail wagon of each wagon set, see Figure 2a). The tail wagon of each set is connected to the second locomotive using a dedicated dummy user force element storing the LTD dynamic states of 49 wagons. A similar strategy is used for train 4, see Figure 2b), through the definition of single DOF bodies for the 3 locomotives, as well as for wagons 1, 61, 122 and 182 (with numbering from the train tail). All other wagons are modelled using 4 separate user force elements solving the states of 59 wagons each. The definition of separate dummy LTD user force elements is selected to enable the implementation of multithreading computation, which speeds up the wall clock time for the simulation.

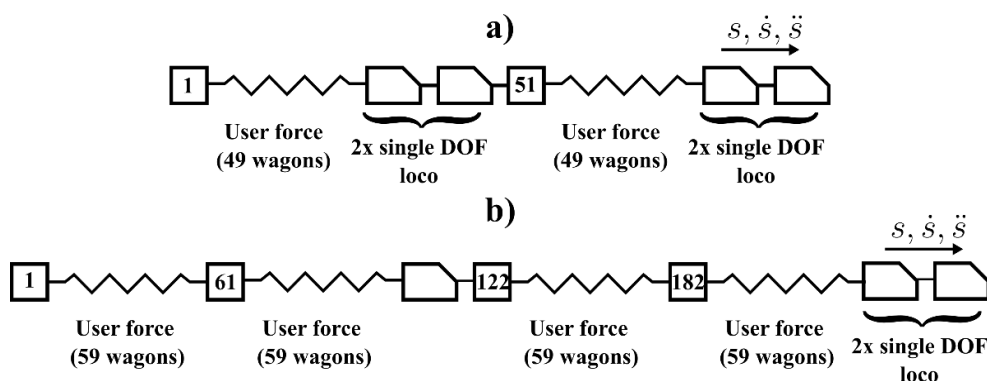


Figure 1: MB models with dummy user force element: a) Train 2 and b) Train 4 of the international benchmark of LTD simulators.

3 Results

This section shows the validation of the novel MB user force approach on the simulation scenarios of the international benchmark of LTD simulators, as well as the comparison of the computational times of the different tested strategies described in the previous section. The validation is performed in terms of the major simulation outputs defined in the benchmark [23], which include:

- The maximum and average speed along the simulation, extracted from all time steps and vehicles.
- The largest in-train force among all coupling systems and time steps, for draft and buff conditions.
- The mean of the maximum values of in-train forces on all coupling positions, for draft and buff conditions.
- The maximum deflection in draft and buff states on a reference coupling position.

Table 1 and Table 2 provide the major simulation outputs calculated by the LTDPoliTo code, the MB user force method, and the MB bodies approach for trains 2 and 4, respectively. It can be observed that both MB approaches implemented in SIMPACK provide results that are well comparable to those calculated by LTDPoliTo, which is considered here as a reference since it was validated in past activities against the other codes joining the benchmark. Minor differences in the outputs are mainly related to the different integrators adopted in the MATLAB and SIMPACK environment. Nonetheless, the MB user force and bodies approaches ensure a good numerical stability, as the maximum in-train forces and corresponding positions are close to the ones obtained with the LTDPoliTo code.

Model	Speed (km/h)		Largest in-train force* (kN)		Mean in-train force (kN)		Max deflection on coup. 61* (mm)	
	Max.	Avg.	Draft	Buff	Draft	Buff	Draft	Buff
LTDPoliTo	89.44	64.56	772@54	407@54	459	264	89.3	48.0
MB BODIES	89.42	64.55	775@54	407@54	457	265	89.3	48.0
MB user force	89.42	64.55	775@54	407@54	460	265	89.3	48.0

Table 1: Main results for train 2 (*couplers numbered in ascending order from head to tail according to benchmark rule).

Model	Speed (km/h)		Largest in-train force* (kN)		Mean in-train force (kN)		Max deflection on coup. 146* (mm)	
	Max.	Avg.	Draft	Buff	Draft	Buff	Draft	Buff
LTD PoliTo	81.81	45.00	1309@123	866@122	946	501	161.7	96.2
MB BODIES	81.77	45.00	1309@123	867@122	948	503	161.7	96.4
MB user force	81.79	44.99	1309@123	866@122	947	501	161.7	96.2

Table 2: Main results for train 4 (*couplers numbered in ascending order from head to tail according to benchmark rule).

The greatest benefit of the novel MB user force approach developed in this paper relies in its excellent computational efficiency when compared to the basic MB bodies strategy. In fact, treating the LTD states as additional dynamic states with no need for the definition of additional bodies in the MB model avoids the management of markers and geometry from the MB code, thus leading to higher computational speeds. Furthermore, splitting the LTD trains into separate user force elements can enable parallel computing, hence contributing to an improved wall clock time for the simulations. Table 3 and Table 4 provide information on the computational time for the LTD PoliTo code as well as for the methods implemented in the MB formalism. Precisely, in Table 3 and Table 4, the simulated time corresponds to the train trip time, and the CPU time and wall clock time refer to the time needed for the solution of states only.

For the MB models, simulations are run on 4 threads, while the simulations run with the MATLAB LTD PoliTo code do not benefit from multithreading, as the code does not rely on parallel computing at the current state. For the LTD PoliTo code, two conditions are considered, which are referred to as JAC ON and JAC OFF. For the JAC ON case, the numerical integration of the ODE system in Equation (1) is performed relying on an a priori specification of the Jacobian sparsity pattern and mass matrix properties, which greatly improves the computational times. Conversely, for the JAC OFF case, such information is not provided to the solver. From Table 3 and Table 4, it can be inferred that the CPU and wall clock time are reduced by a factor around 7 when shifting from the MB BODIES to the novel MB user force approach. Furthermore, the MB user force approach is even faster than the LTD PoliTo code when the latter does not apply the specification of the Jacobian and mass matrix properties.

MODEL*	Simulated time (s)	CPU time (s)	Wall clock time (s)
LTDPoliTo (JAC OFF)	2711	342.30	266.52
LTDPoliTo (JAC ON)	2711	59.36	42.43
MB BODIES*	2711	1169.73	296.12
MB user force*	2711	177.89	45.03

Table 3: Computational times for train 2 (*SIMPACK models are run on 4 threads).

MODEL*	Simulated time (s)	CPU TIME (s)	Wall clock time (s)
LTDPoliTo (JAC OFF)	3863	2433.31	2098.77
LTDPoliTo (JAC ON)	3863	194.08	141.11
SIMPACK BODIES*	3863	6212.14	1567.20
MB user force*	3863	883.05	223.22

Table 4: Computational times for train 4 (*SIMPACK models are run on 4 threads).

4 Conclusions and Contributions

The novel MB user force and the MB bodies approach are validated with success against the MATLAB LTDPoliTo code, providing outputs in excellent agreement and with good numerical stability for trains 2 and 4 of the international benchmark. Both MB strategies feature a good numerical stability, thus overcoming the inefficiencies of previous MB models for LTD simulations. Nonetheless, the user force approach has superior computational performances, which make it comparable with pure LTD codes. Therefore, the novel MB user force approach can lead the path towards the introduction of LTD within MB codes for the detailed analysis of the full dynamics of railway vehicles, with a minor impact on computational times.

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