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A New Methodology for Hybrid Simulation of Pantograph-Catenary Interaction Based on Hybrid System Response Convergence: Proposal and First Numerical and Experimental Tests

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Abstract

A new methodology is proposed for the hybrid simulation of pantograph-catenary interaction, which is based on the Hybrid System Response Convergence. Thanks to its open-loop nature, the methodology allows overcoming the simplification that are usually needed in the schematisation of virtual catenaries in HIL pantograph tests. The main characteristics of the methodology are introduced together with some numerical experiments aimed at its development and first verification. Finally, an example of laboratory application is addressed, to show the feasibility and effectiveness of the proposed methodology.

Keywords: pantograph-catenary interaction, hybrid simulation, HIL test, hybrid system response convergence, pantograph testing, pantograph dynamics.

1 Introduction

The aim of this work is to present a new methodology for the investigation of the pantograph-catenary interaction based on Hybrid system response convergence. The testing methodology proposed is a kind of hybrid testing, where a part of the system is simulated in a virtual domain, in this case the catenary, and the rest of the system is a physical sample and undergoes, using proper actuation systems, the perturbation generated by the interaction with the virtual domain. This concept is also called “Hardware in the loop” and has been already successfully applied to the pantograph-catenary application [2–5]. Differently from hybrid systems already developed in past works, the system proposed here presents the possibility to expand the virtual domain complexity. This forward step is allowed by the fact that the computations are not performed in real time, which would require unavoidable simplification to cope with computational performances. but the two domains are time-decoupled and communicate through a series of corrective iterations that, step by step, bring to a reliable time history representative of the reciprocal influence between the physical pantograph and virtual catenary.

Let us see in detail which are the instruments available nowadays for the analysis of pantograph-catenary interaction and are related with the proposed methodology.

Numerical simulations

Numerical simulations can reproduce the interaction between pantograph and catenary with a good level of approximation, thanks to the precise models developed both for the catenary and for the pantograph [6]. Since the simulations are performed offline, there are no limitations in the virtual domain dimension which, on the catenary side, is mainly discretized and modelled by means of finite elements. In this framework the pantograph model can be considered, in the simplest configurations, as a constant moving load underneath the contact wire. In more detailed representations the pantograph can be schematized with a lumped mass model, obtained by model identification procedures, in case considering possible non-linearities as additional forces acting on the masses of the model [7].

Increasing the computational burden, the numerical simulation can also represent the pantograph with a proper multibody model. In this case, the pantograph model communicates with the finite element catenary through the application of protocols that allows co-simulation of different software that communicate and interact at discrete time steps [8–10].

Hybrid Testing

In recent years a trending approach in the study of pantograph-catenary interaction involved the hybrid testing concept. Hybrid testing, more in specific Hardware-in-the-loop simulations, consists in the decoupling of the system in a virtual domain and a physical domain. This kind of approach allows to avoid the development of a numerical model for at least a part of the system, thus reducing the uncertainties in the overall simulation. In the pantograph-catenary application the virtual domain is represented by a model of the catenary and the physical one is represented by the real pantograph. The interaction perturbations are actively reproduced by actuators acting on the physical pantograph and the pantograph response is measured and sent back as input to the virtual domain.

One possibility is that the two domains, virtual and physical, are put in communications by means of real time computations, this is called real-time (or “closed-loop”) Hardware-in-the-loop. Real-time Hardware-in-the-loop guarantees the effective interaction between the two domains but, on the other side, can usually consider only simplified virtual domain. This because real time computations are limited by real time calculators’ performances. Thus, the main efforts in real time applications are devoted to the definition of a proper simplification of the catenary model [11].

An alternative to real-time Hardware-in-the-loop is the offline (or open-loop) Hardware-in-the-loop. In this case, the output of an offline simulation is given as input to the actuators exciting the physical pantograph. The simulation can in principle include any kind of virtual scenario, due to the need for real time calculations, but, on the other side, the interaction between the two domains is absent.

Hybrid system response convergence (HSRC)

The methodology proposed in this paper aims at the definition of a hybrid simulation able to merge the advantages of offline and real-time Hardware-in-the-loop simulations, giving the possibility to model any kind of virtual domain, like offline Hardware-in-the-loop, and, at the same time, reproducing an effective interaction between the virtual and physical domain, like real time Hardware-in-the-loop.

This objective is achieved through the application of the Hybrid system response convergence (HRSC) concept, developed by Fricke et al. and applied in the development of an innovative full-vehicle hybrid simulation in the automotive field [1,12]. The main idea is to have a recursive offline Hardware-in-the-loop able to establish an interaction between the hardware and the virtual domain through a series of corrective iterations.

2 Numerical application and analysis of the methodology

In this paragraph a first numerical assessment of the methodology is presented. This approach, even if fully numerical, is built up of the same elements that will take part in the application of the method. The only difference is that, in this phase, the physical pantograph is substituted by a numerical representation of the pantograph response. The numerical procedure is useful to address more in detail how the iterative loop is structured and how the different domains interact.

As already stated before, HSRC is an iterative hybrid testing procedure. The iterations are based on a recursive loop as shown in Figure 1, where on the left and right side are respectively placed the two main actors, the finite element simulation of the pantograph catenary interaction and the numerical model of the pantograph response, substituting in this phase the physical pantograph.

The procedure starts with the first iteration, in which, a finite element simulation of a pantograph passing underneath a finite element model of the catenary is performed. At the end of this simulation, the time history of the contact-point vertical-motion is set as input to the pantograph response model. The pantograph response is reproduced by a lumped mass model with 3 degrees of freedom (cf Figure 2).

The contact point vertical motion is applied as an imposed displacement to the degree of freedom corresponding to the pantograph collector. The resulting force of this imposed motion is evaluated and compared to the force obtained by the finite element simulation. The difference between the force obtained by the finite element simulation and the force obtained forcing the pantograph response model represent the first corrective input given to the finite element simulation. This load difference time history, in fact, is applied to the collector of the pantograph inside the finite element simulations. In this way the first iteration is concluded. The second iteration starts with the “corrected” finite element simulation. It is important to highlight that the load difference updating the FEM simulation at each iteration is not just the difference of the two forces, as obtained in the first iteration, but is a cumulative force difference containing all the corrective loads resulted from the previous iterations.

This is a loop general overview, now is time to define and analyse the various elements just cited in the loop description.

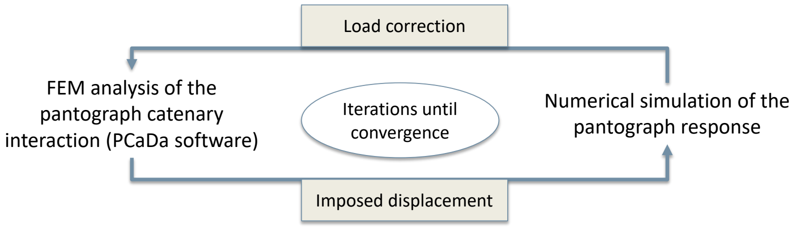


Figure 1: Iterative loop of the numerical validation of the methodology

Numerical simulation

The numerical simulation is performed considering a lumped mass model of a physical pantograph, passing underneath the finite element model of a catenary. The numerical simulations presented in this work have all been performed by means of the PCaDa software already described in previous works [7].

In the software the droppers are modelled as nonlinear elements, inducing a force displacement relationship extracted from laboratory tests.

The equation of motion of the overhead equipment in matrix form is:

$$\mathbf{M}_{\mathrm{c}}\underline{\ddot{\mathbf{x}}}_{\mathrm{c}} + \mathbf{C}_{\mathrm{c}}\underline{\dot{\mathbf{x}}}_{\mathrm{c}} + \mathbf{K}_{\mathrm{c}}\underline{\mathbf{x}}_{\mathrm{c}} = \underline{\mathbf{F}}_{\mathrm{cc}}\left(\underline{\mathbf{x}}_{\mathrm{c}}\right) + \underline{\mathbf{F}}_{\mathrm{cp}}\left(\underline{\mathbf{x}}_{\mathrm{c}}, \underline{\dot{\mathbf{x}}}_{\mathrm{c}}, \underline{\mathbf{x}}_{\mathrm{p}}, \underline{\dot{\mathbf{x}}}_{\mathrm{p}}, t\right) \tag{1}$$

where $\mathbf{M}_c$, $\mathbf{C}_c$ and $\mathbf{K}_c$ are the mass, damping and stiffness matrices representing the contribution of the messenger, contact wires and the registrations arms. $\underline{\mathbf{x}}_c$ contains the nodal coordinates of the catenary and $\underline{\mathbf{x}}_p$ contains the pantograph nodal coordinates. $\underline{\mathbf{F}}_{cc}$ and $\underline{\mathbf{F}}_{cp}$ are, respectively, the generalized non-linear forces due to the droppers and the generalized term due to the contact force exchanged with the pantograph.

On the other side, the equation of motion of the pantograph can be written as:

$$\mathbf{M}_{\mathrm{p}}\underline{\ddot{\mathbf{x}}}_{\mathrm{p}} + \mathbf{C}_{\mathrm{p}}\underline{\dot{\mathbf{x}}}_{\mathrm{p}} + \mathbf{K}_{\mathrm{p}}\underline{\mathbf{x}}_{\mathrm{p}} = \underline{\mathbf{F}}_{\mathrm{pp}}\left(\underline{\mathbf{x}}_{\mathrm{p}}\right) + \underline{\mathbf{F}}_{\mathrm{pc}}\left(\underline{\mathbf{x}}_{\mathrm{c}}, \underline{\dot{\mathbf{x}}}_{\mathrm{c}}, \underline{\mathbf{x}}_{\mathrm{p}}, \underline{\dot{\mathbf{x}}}_{\mathrm{p}}, t\right) \tag{2}$$

Again $\mathbf{M}_p$, $\mathbf{C}_p$ and $\mathbf{K}_p$ are the mass damping and stiffness matrices of the pantograph lumped mass model, those matrices depend on the physical pantograph behaviour the simulation wants to reproduce. $\underline{\mathbf{F}}_{pp}$ represents the non-linear terms associated to the non-linear pantograph components and $\underline{\mathbf{F}}_{pc}$ is the vector containing the generalized forces coming from the interaction with the catenary. The corresponding contact forces between the pantograph and the catenary are evaluated using a penalty method approach considering a contact element between the pantograph collector and the contact wire. The equations of motion of the two sub-systems are integrated at each time step.

Numerical pantograph response model

The numerical pantograph response model is the one representing, in this numerical phase, a surrogate of the physical pantograph in the hybrid procedure. Just as in the finite element simulation the pantograph is modelled as a multi-degrees of freedom system, depending on the pantograph typology. In Figure 2, two examples of pantograph models are shown.

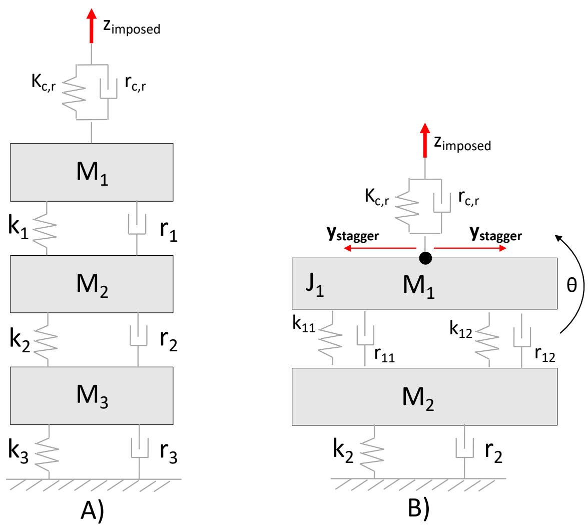


Figure 2: Example of different pantograph response models. Model A) is a 3 DoF lumped mass model considering a single collector. Model B) is a 3 DoF model considering a single collector with roll motion.

As already mentioned in the loop description, the model undergoes an imposed motion applied to the degree of freedom corresponding to the pantograph collector. The corresponding generated force at the collector degree of freedom is taken as output of this passage.

The difference between this force and the contact force estimated by the FEM simulation, at the same iteration, represents the coupling correction that is going to be part of the FEM simulation at the following iteration. It will be now explained the role and content of this corrective load difference.

Corrective load

The load correction in Figure 1 is the update passage that allows the finite element simulation and the pantograph response model to perform an effective communication. In fact, thanks to this passage, the finite element simulation is modified to better fit the contact force value obtained by the pantograph response. This modification consists in the application to the pantograph model, the one inside the finite element simulation, of an additional vertical force time history coming from the previous iteration, that modifies the overall simulation outcome. The value of the applied corrective vertical force is the result of the contact force difference coming from the two systems, the pantograph response model, and the finite element simulation.

The load correction in the FEM simulation is performed applying to the mass representing the collector the corrective force time history throughout the simulation. A schematization of this concept is shown in Figure 3.

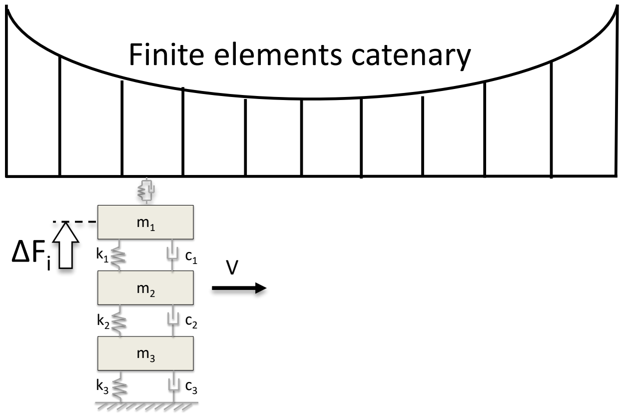


Figure 3: Simplified scheme of the application of the corrective load to the collector of the pantograph inside the finite element simulation

The expression of the ΔF_i at the end of the first iteration (i = 1) is computed as follows:

$$\Delta F_1(t) = F_{\mathrm{num},1}(t) - F_{\mathrm{pant},1}(t) \tag{3}$$

where $F_{\mathrm{num},1}$ and $F_{\mathrm{pant},1}$ are respectively the contact force coming from the finite element simulation and the contact force estimated from the imposed motion applied to the pantograph response model.

For the subsequent iterations, the delta force is considered as cumulative. It contains the corrective update of the previous iteration summed with the additional variation coming from the considered iteration.

For the i^{th} iteration:

$$\Delta F_{\mathrm{i}}(t) = \Delta F_{\mathrm{i}-1}(t) + F_{\mathrm{num,i}}(t) - F_{\mathrm{pant,i}}(t) \tag{4}$$

In this way the correction retains, as the iterations' number increases, all the force differences applied until that moment. The force correction is adjusted iteration by iteration and its value is increasingly able to tame the finite element simulation. After a certain number of iterations, the results coming from the pantograph response model and the finite element simulation can be considered converging to an acceptable result. Meaning that the two models have been merged and the results coming from the last iteration can be considered as representative of a communicating dynamica interaction. In a certain way the pantograph response model can then be considered as participating to the finite element simulation.

The quality of the result is assessed looking at the standard deviation of the difference between the forces coming from the finite element simulations and the pantograph response model. This quantity is also fundamental in the convergence evaluation of the iterative process.

Convergence in presence of pantograph parameter variations
The main functioning of the loop should now be clear, the missing information is how to understand when to stop the iterative cycle. In other words, the objective is to define whether proceeding with new iterations worth the additional computations or not. The solution was found analysing the trend of the standard deviation of the difference between the contact force obtained with the FEM simulation and the one obtained from the pantograph response model. Figure 4 gives an idea of the convergent trend for different simulations. Different cases are considered, i.e. "Nominal" where the pantograph response model corresponds to the model adopted for the FEM simulation, M+/-, K+/- and C+/- where increased/decreased values of the collector mass and of the stiffness and damping of the collector suspension is considered.

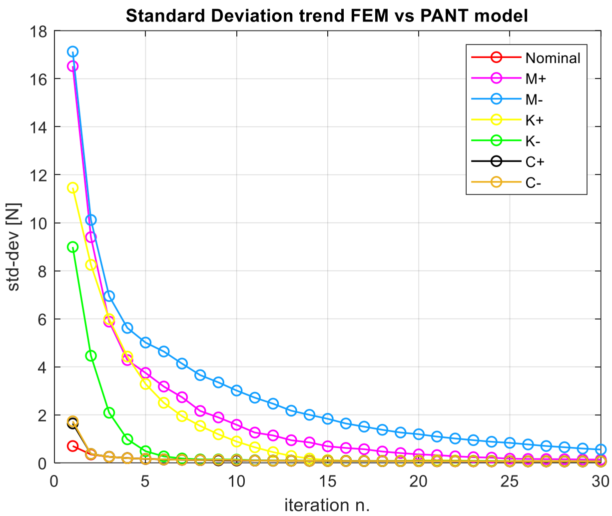


Figure 4 : Standard deviation trend of the difference between the contact force evaluated by the finite elements simulation and the one extracted from the pantograph model for each iteration

It is apparent that as soon as the iterations number increases the standard deviation value tends to converge to a minimum. Taking into consideration one of the curve of Figure 4, namely the one corresponding to the decreased collector mass M-, in Figure 5 the plots of the contact force comparison between the FEM and the pantograph numerical model are associated to the correspondent iteration in the standard deviation plot. Going from the first to the last iteration, it is apparent how the procedure brings to the effective coupling of the FEM simulation and the pantograph response model.

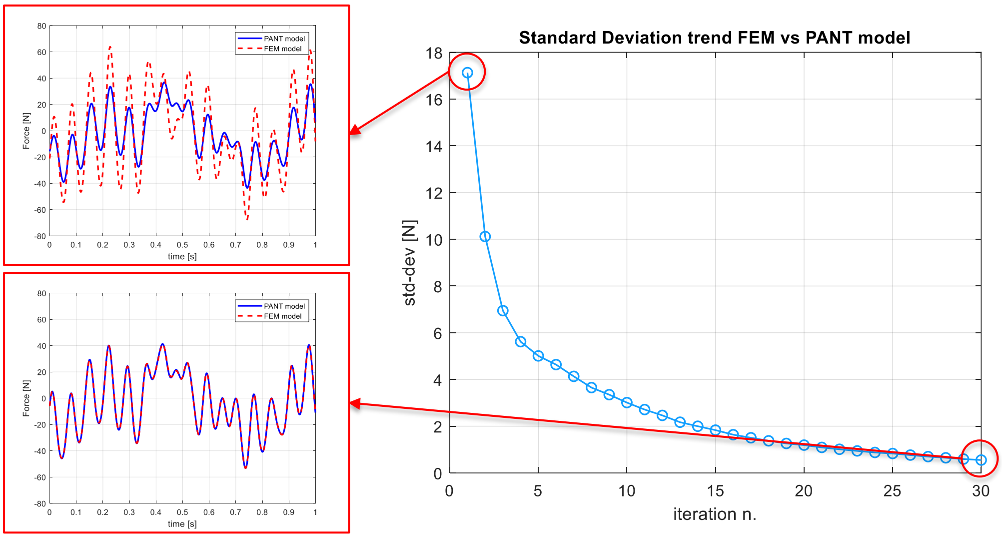
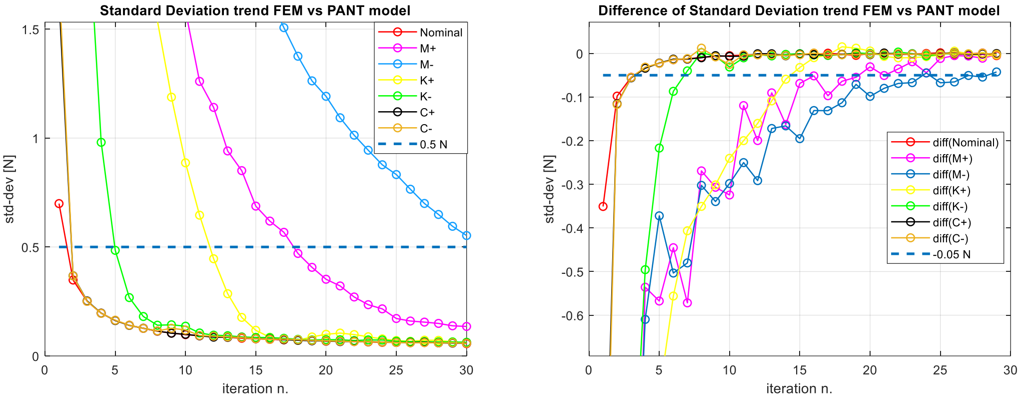


Figure 5 : Details of the 1st and 30th iterations for the M- variation. On the left the comparison between the contact force time histories of the FEM (red dashed line) and of the pantograph model (blue continuous line). On the right the standard deviation trend for the M- case of Figure 4

To understand the effect of the inclusion in the system of significative differences between the pantograph in the FEM simulation and the one modelling the pantograph response, many tests were performed considering the variations of the response model parameters. These tests allowed to understand the flexibility of the approach. This aspect is important since, passing to the experimental application, the physical pantograph and the one in the FEM simulation may present significant differences on account of pantograph non-modelled characteristics and non-linearities.

Figure 4 shows the convergence trend for the nominal simulation, where the pantographs of the FEM and of the numerical model are equal, and the cases performed varying the collector's degree of freedom parameters of ±50%. Investigating variations of the model parameters one at a time, it emerged that these are the only parameters whose variations are able to significantly alter the convergence trends. The plot highlights the fact that as soon as the numerical model differs from the pantograph model of the FEM the convergence behaviour change significantly. The analysis of the convergence can be deepened with the help of additional indicators.

To define and compare the different variation scenarios, three different indexes have been selected: the first one is the value assumed by the standard deviation trend, the one of Figure 4, at the 20th iteration; the second one is the number of iterations needed to the same trend to reach a value lower than 0.5 N; the last one is the number of iterations needed to the curve of the difference at consecutive iterations of the standard deviation to reach a value lower than 0.05 N. These threshold values can be individuated in the two plots of Figure 6 where the standard deviation trend and the difference of the standard deviation between consecutive iterations are shown.



a) b)

Figure 6 : a) Standard deviation trend and b) difference of the standard deviation trend for the parameters variations, the blue dashed lines are the selected comparison thresholds

The values coming from the analysis are reported in Table 1. As predictable, the nominal case converges to a very low value of standard deviation in the lowest number of iterations. On the other hand, the mass variations are responsible for the highest worsening of the procedure both in terms of residual value and convergence speed.

The analysis performed here allowed to have an idea of the possible difficulties the experimental application of the methodology could have encountered. The difference between the physical pantograph and the pantograph model in the FEM simulation is a source of convergence slowdown. The fact that, from the numerical analysis of the convergence, emerged that the collector degree of freedom has the highest correlation with the convergence worsening was a good omen in the study advance. In fact, dealing with the extrapolation of a pantograph model parameters, the estimation of the mass and stiffness of the collector's degree of freedom is usually affected by a low degree of uncertainty.

	Standard deviation at the 20th iteration [N]	N. of iterations to reach a standard deviation < 0.5 N	N. of iterations to reach a derivative of the std curve < |0.05|N
Nominal	0.068	2	4
m_1 + 50%	0.352	18	20
m_1 – 50%	1.192	31	24
k_1 + 50%	0.099	12	15
k_1 - 50%	0.075	5	7
c_1 + 50%	0.069	2	4
c_1 - 50%	0.070	2	4

Table 1: Convergence classification for the parameters' variations applied to the collector degree of freedom of the pantograph model

Convergence in presence of pantograph additional DoFs

An additional numerical case considered a low-speed applications pantograph, whose collector's constraints allows its roll motion (model B) in Figure 2). In the real application, this rolling motion is excited by the stagger imposed to the contact wire.

This stagger is reproduced and given as input to the pantograph model, which, this time is forced by an imposed horizontally moving displacement.

Since there is no possibility to include in the lumped mass model the moving displacement of the contact point, a solution is found in the addition of a spring-damper element on the top of mass m_1. The new model can be modified moving horizontally the spring-damper element following stagger signal coming from the finite element simulation. At the same time, the element can be forced on its top by the FEM catenary contact point motion.

The value of the damping and the stiffness $k_{c,r}$ and $r_{c,r}$ were chosen to reproduce a nearly rigid element, as an application of a penalty method, rather than to modify the imposed displacement effect on the model dynamics. The overall result is just as if the moving displacement would be imposed mass M_1.

The application of the methodology brings to the convergent trend of Figure 7, comparable with what was obtained in previous tests. This demonstrates that the methodology is in principle able to cope even with completely non-modelled dynamics on the pantograph side.

The numerical procedure was applied to a number of additional test cases, not addressed here for the sake of brevity, aimed at verifying the methodology performances when dealing with multiple collection, low speed applications and different pantograph models.

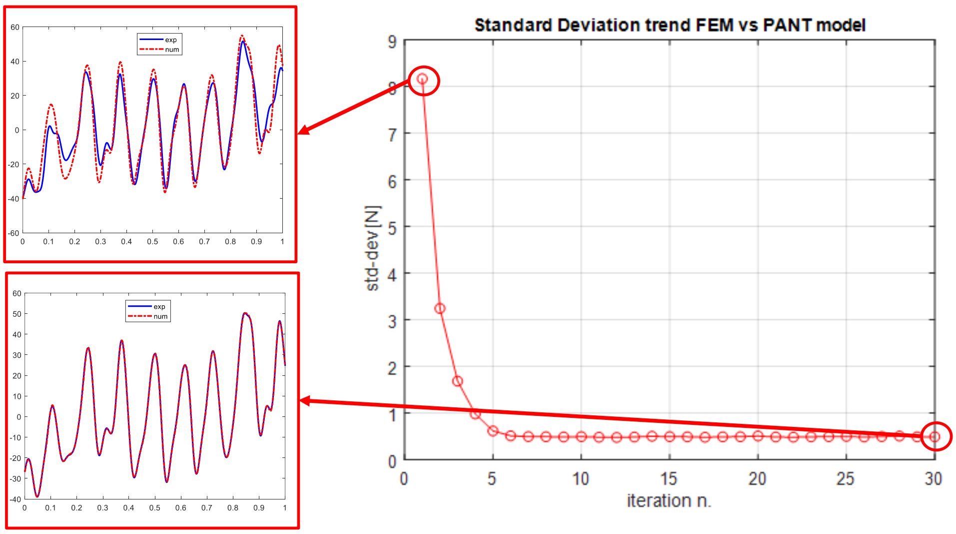


Figure 7 : Standard deviation of the difference between the contact force coming from the finite element simulation and the one extracted from the pantograph response model

3 Experimental application of the methodology

When applying the methodology to the experimental bench, the main difference with the previous numerical validating approach is the introduction in the iterative loop of the physical pantograph, excited by the bench actuators, in place of the numerical model of the pantograph response, as schematically shown in Figure 8. Obviously, this implies a series of complications: signal treatments are necessary to guarantee the absence of high frequencies disturbances and the effective synchronisation of the signals inside the loop.

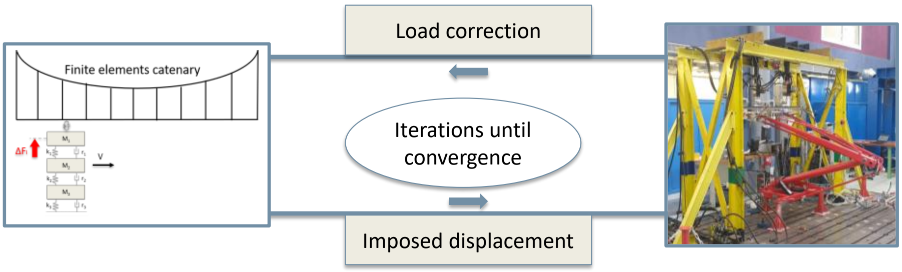


Figure 8: Scheme of the experimental application loop of the methodology

The experimental bench used to impose the displacement coming from the FEM simulation to the pantograph head is shown in Figure 8. It is made up of a main frame that supports the actuation systems. Two hydraulic actuators are responsible for the imposed vertical motion to the pantograph head. The two actuators are mounted on a slider that can be moved laterally, simulating the contact wire stagger, by means of a lateral actuation system supplied by an AC motor and a ball screw mechanism. The vertical actuators are equipped with load cells to measure the contact forces.

In the experimental application, the actual contact point motion is affected by the actuators' response that cannot be considered as ideal in the whole frequency range of interest, affecting the displacement both in terms of amplitude attenuation and time delays. To balance this detrimental effect, the reference imposed-motion is pre-filtered, considering an approximated inverse of the actuators' transfer function obtained through a series of monoharmonic tests in the 0-20 Hz frequency range.

This pre-filtering is possible on account of the open-loop nature of the methodology.

Experimental convergence

For a first analysis of the experimental application of the proposed methodology, reference is made to the case of an ATR95 25kV pantograph passing underneath the C270 high-speed catenary at 300 km/h [13]. The convergence is still the rule to establish the end of the iterations and the quality of the obtained results, which are very close to those obtained in the numerical validation of the procedure. Figure 9 shows the standard deviation trend resulting from 20 iterations. The final value reached by the curve can be considered sufficiently low. The trend does not present the regular slope of the curves obtained by the numerical applications of the methodology, as the one of Figure 4, but in any case the standard deviation value is subjected to a significative reduction going towards the 20th iteration and reaching the minimum value 0.57 N close to the values obtained in the numerical application of the procedure.

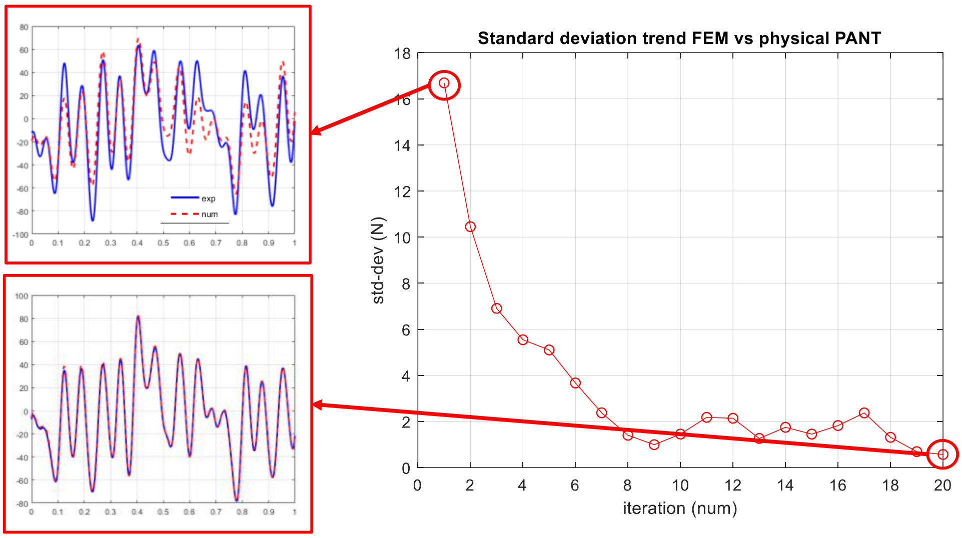


Figure 9: Standard deviation trend for the ATR95 25kV pantograph passing underneath the virtual model of the C270 high-speed catenary at 300km/h.

Looking at the contact force difference in time and frequency domain of the first and last iterations (Figure 10) it appears how the methodology moves from a significative force-difference signal to an almost zero constant value.

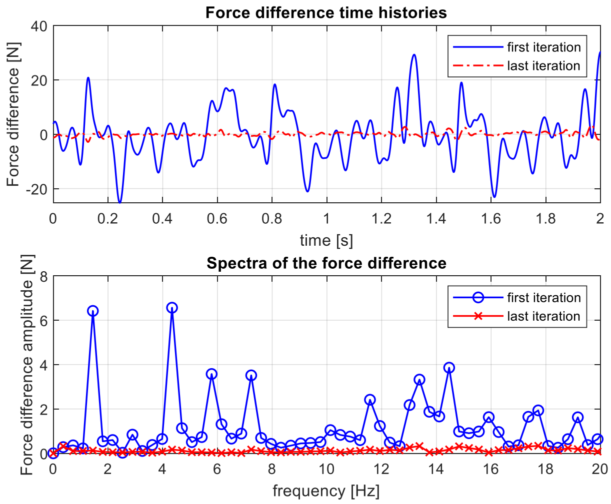


Figure 10: Force difference for the first iteration (blue continuous line) and the last iteration (red dashed line) of the experimental case.

The force-difference at the first iteration is the difference between a full numerical simulation, without any input modifications, and the force measured at the experimental bench resulting from the same contact point motion. From the bottom plot it can be noticed how the main source of difference between the virtual and the physical subsystems can be mainly attributed to the low frequencies. This quite is reasonable knowing that the non-linearities of the real pantograph are usually concentrated in this range of frequencies, below 8 Hz approximatively.

4 Conclusions

In this paper, a new methodology for the hybrid simulation of pantograph-catenary interaction based on the Hybrid System Response Convergence is proposed.

The methodology was preliminary verified with numerical experiments. The obtained results showed that the methodology is capable to reach an effective coupling between a virtual catenary and a pantograph response model through different corrective iterations. The convergence, and thus the coupling, is reached even with significant variation (±50 %) of the pantograph parameters associated to the collector degree of freedom. In this respect, the mass variations are responsible for the highest worsening of the procedure performances both in terms of residual value and convergence speed.

The convergence is even reached, at least in numerical experiments, when considering unmodelled dynamics of the pantograph, such as additional degrees of freedom.

A first example of laboratory application of the methodology was also presented, confirming the possibility to reach the desired convergence of the methodology and to get an effective coupling between a virtual catenary and a real pantograph.

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